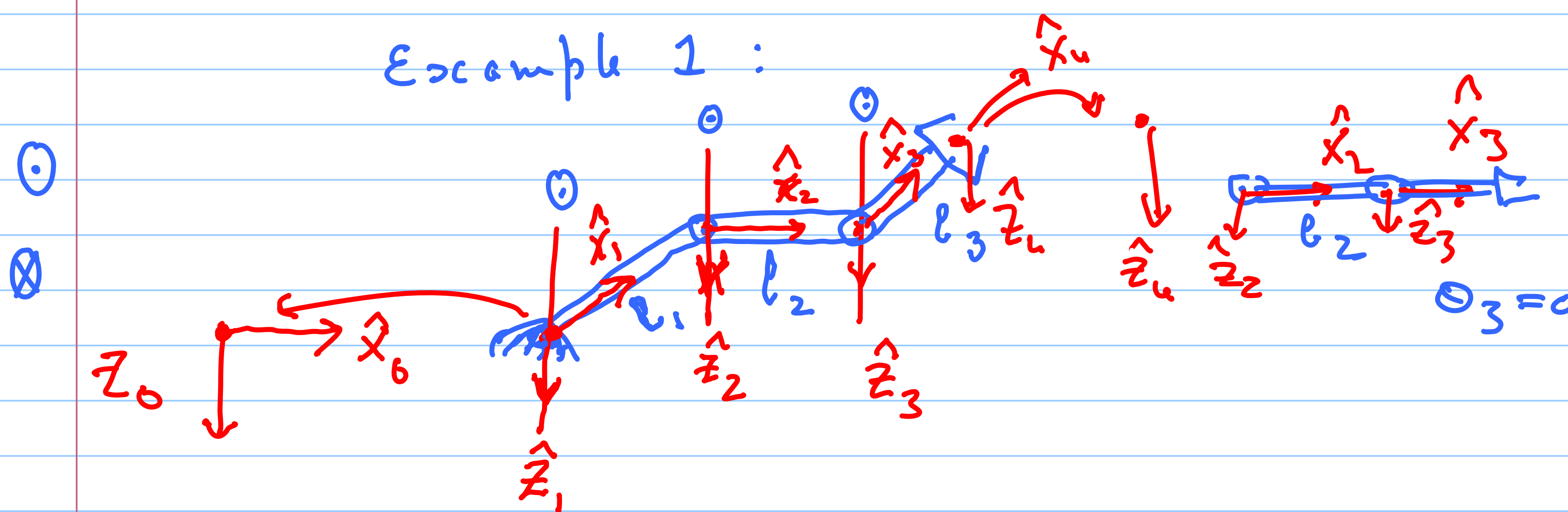
Note Title 9/25/2007

Lecture - 10

1) D-H frame assignment

Example 1 :



1) assign $\hat{z}_i$ to joint-axis i.

2) assign $\hat{x}_1$ to common $\perp$ bet. $\hat{z}_1$ + $\hat{z}_2$.

$\Rightarrow a_1 = l_1, \ \alpha_1 = 0$ (since $\hat{z}_1 \parallel \hat{z}_2$)

3) origin of $\{1\}$: since $\hat{z}_1 \parallel \hat{z}_2 \Rightarrow$ common $\perp$ is not unique. choice in orig. of $\{1\}$.

We choose origin of $\{1\}$ to be in the plane of the arm.

4) assign $\hat{x}_2$ to common $\perp$ bet $\hat{z}_2$ & $\hat{z}_3$.

$\underbrace{\hat{z}_2 \parallel \hat{z}_3}_{} \Rightarrow \alpha_2 = 0, \ a_2 = l_2$

$\Downarrow$ common $\perp$ not unique.

choose origin of $\{2\}$ to make $d_2 = 0$

$\Rightarrow$ origin of $\{2\}$ also lies in the plane of the arm.

5) $\{3\}$: last frame. No next $\hat{z}_i$ to det. common $\perp$.

assign $\hat{x}_3 \parallel \hat{x}_2$ when $\theta_3 = 0$.

assign origin of $\{3\}$ to make $d_3 = 0$

$\Rightarrow$ it lies in the plane of the arm.

6) $\{0\}$: choose $\hat{z}_0 \parallel \hat{z}_1 \Rightarrow \alpha_0 = 0$ (co-incident)

$\hat{x}_0$: $\perp$ bet. $\hat{z}_0$ + $\hat{z}_1$

$\hat{x}_0$ aligns with $\hat{x}_1$ when $\theta_1 = 0$

origin: choose $d_1 = 0$

$\Rightarrow$ in the same plane as the arm.

7) if we were to assign a tool frame (end-effector) at the end-effector ?

Choose it to maximize the # of D-H para with zero value.

parameters: a_{i-1}, α_{i-1}, θ_i, d_i

In this case $\hat{z}_4 \parallel \hat{z}_3$, $\hat{x}_4 \parallel \hat{x}_3 \Rightarrow \theta_4 = 0$, in the plane of the arm. $\Downarrow$ $d_4 = 0$

$\alpha_3 = 0$

D-H para. table for planar 3-joint arm

$^{i-1}_{i}T = [\alpha_{i-1}, a_{i-1}, \theta_i, d_i]$ fun. of

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	0	ℓ_1	0	θ_2
3	0	ℓ_2	0	θ_3
4	0	ℓ_3	0	0

$$^{0}_{3}T = {}^{0}_{1}T \; {}^{1}_{2}T \; {}^{2}_{3}T = \left(\begin{array}{c|c} ^{0}_{3}R & ^{0}P_{3ORG} \\ \hline \underline{0} & 1 \end{array}\right)$$

$${}^{0}_{3}R = \begin{pmatrix} c_{123} & -s_{123} & 0 \\ s_{123} & c_{123} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$c_{ijk} = \cos(\theta_i + \theta_j + \theta_k)$

← forw. kin for planar arm.

$${}^{0}P_{3ORG} = \begin{pmatrix} l_1 c_1 + l_2 c_{12} \\ l_1 s_1 + l_2 s_{12} \\ 0 \end{pmatrix}$$

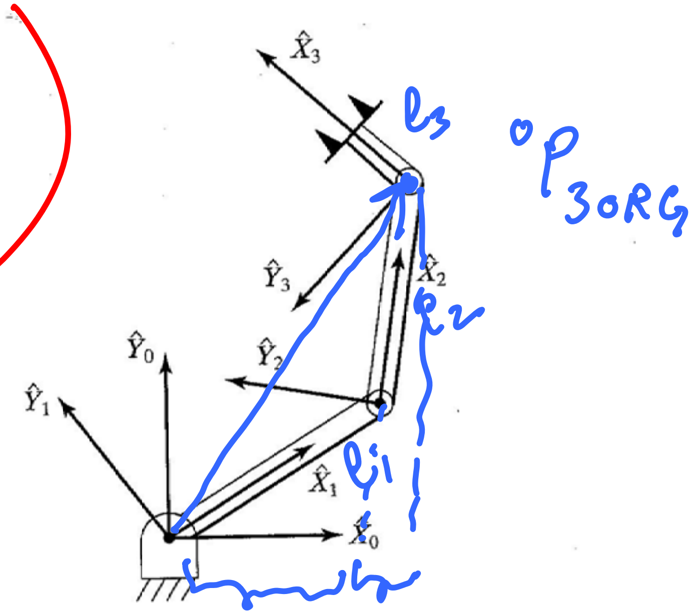


FIGURE 3.7: Link-frame assignments.

<u>2nd Example</u> :

. <u>PUMA</u> 560 arm (6-dof)
or 6R

① $\hat{z}_1$ & $\hat{z}_2$ intersect.

$\Rightarrow \boxed{a_1 = 0}$

choice of dir for $\hat{x}_1$.

we chose $\hat{x}_1 = -(\hat{z}_1 \times \hat{z}_2)$

$\therefore \boxed{\alpha_1 = -90^0}$

origin of {1} : at int. of $\hat{z}_1$ & $\hat{z}_2$

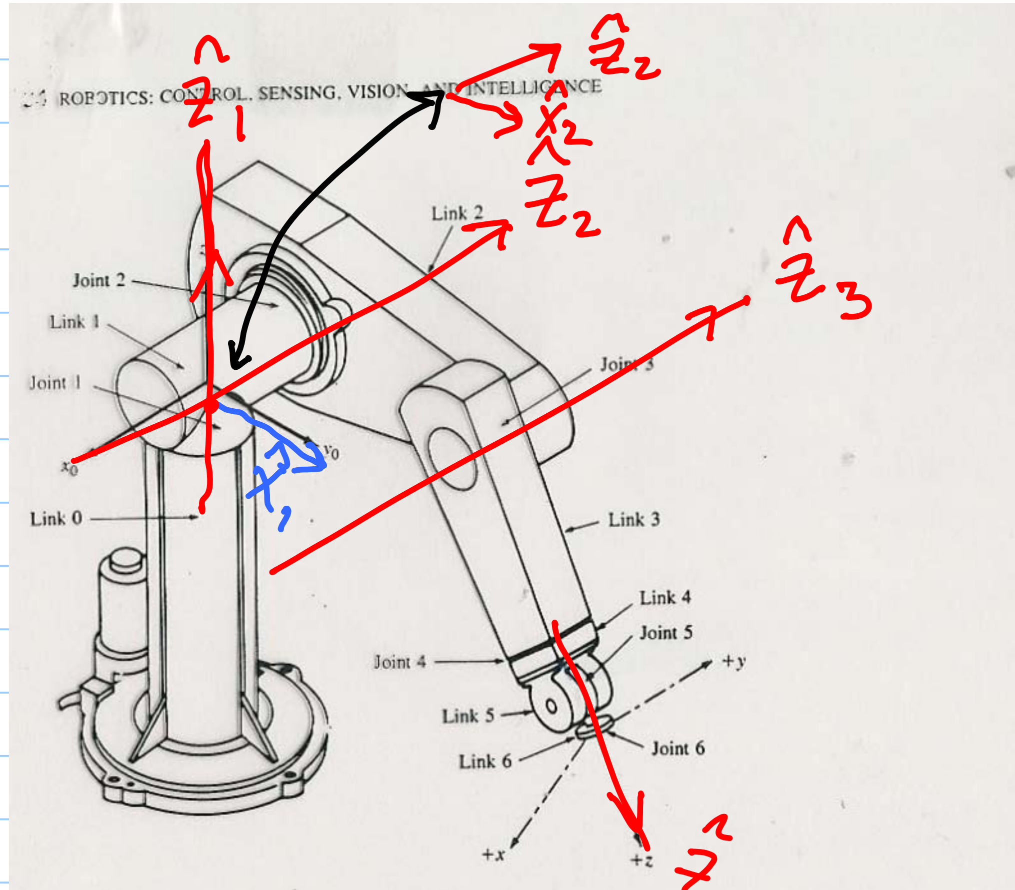


Figure 2.8 A PUMA robot arm illustrating joints and links.

② $\hat{z}_2$ & $\hat{z}_3$ are ∥.

$\alpha_2 = 0$

$\hat{x}_2$: common ⊥ bet $\hat{z}_2$ & $\hat{z}_3$

origin of {2} is coinc. with that of {1} to make $d_2 = 0$

a_2 = nonzero

3) origin of $\{3\}$: Common $\perp$ bet. $\hat{z}_3$ + $\hat{z}_4$ intersects $\hat{z}_3$.

$a_3 \neq 0$.

$d_3 \neq 0$.

$\alpha_3 = -90^\circ$

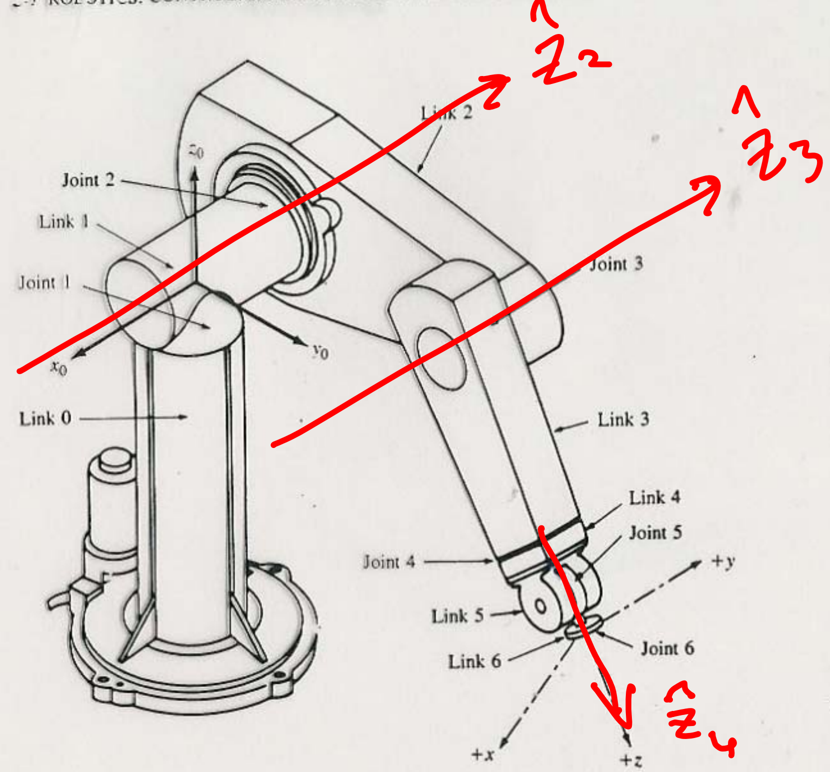


Figure 2.8 A PUMA robot arm illustrating joints and links.

4) $\hat{z}_4 \times \hat{z}_5 = x_4$, $\alpha_4 = 90^\circ$, $a_4 = 0$, $d_4 \neq 0$ joint axis 4 + 5 intersect

5) joint axes 5, 6 intersect. $a_5 = 0$, $d_5 = 0$

$\hat{x}_5 = -(\hat{z}_5 \times \hat{z}_6)$ $\alpha_5 = -90^\circ$

$z_4 = z_4$

$Y_4 = -z_5$

$z_4 = z_6$

6) $d_6 = 0$, ~~a_6~~

$\hat{x}_6 = \hat{x}_5$ when $\theta_6 = 0$

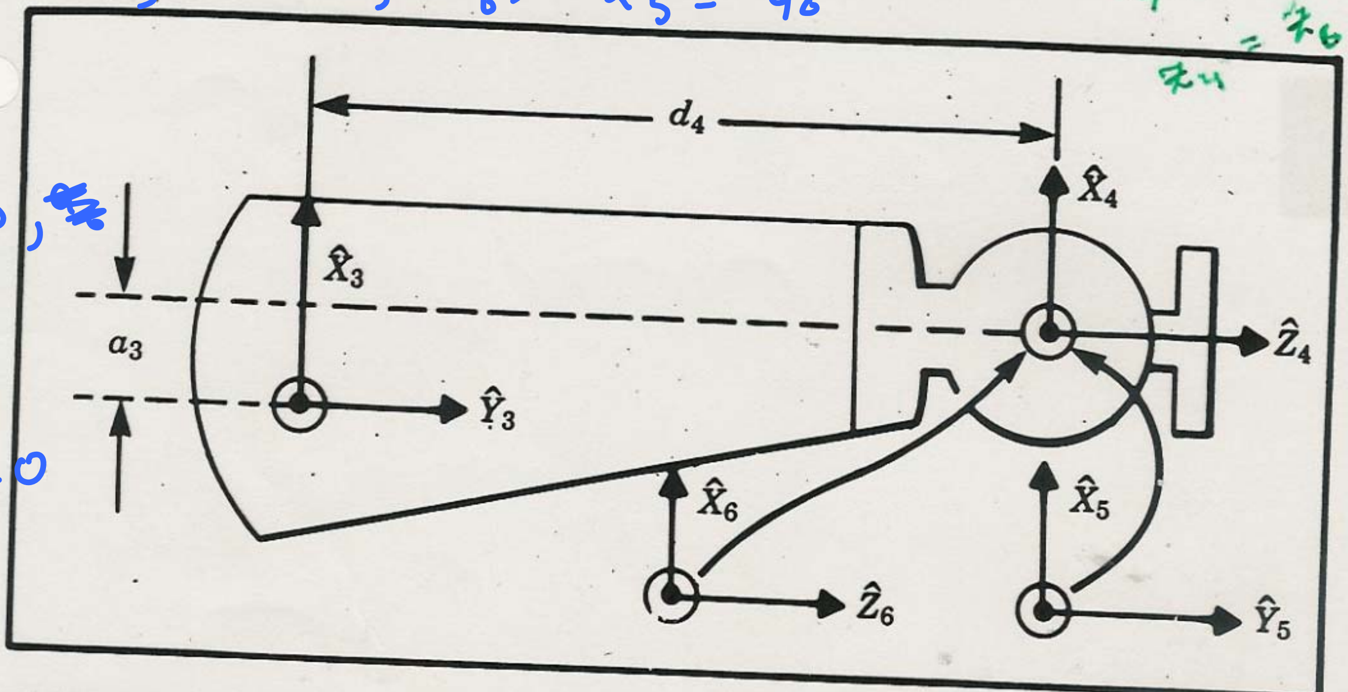


FIGURE 3.19 Kinematic parameters and frame assignments for the forearm of the PUMA 560 manipulator.

$\{0\}$ coincides with $\{1\}$ when $\theta_1 = 0$

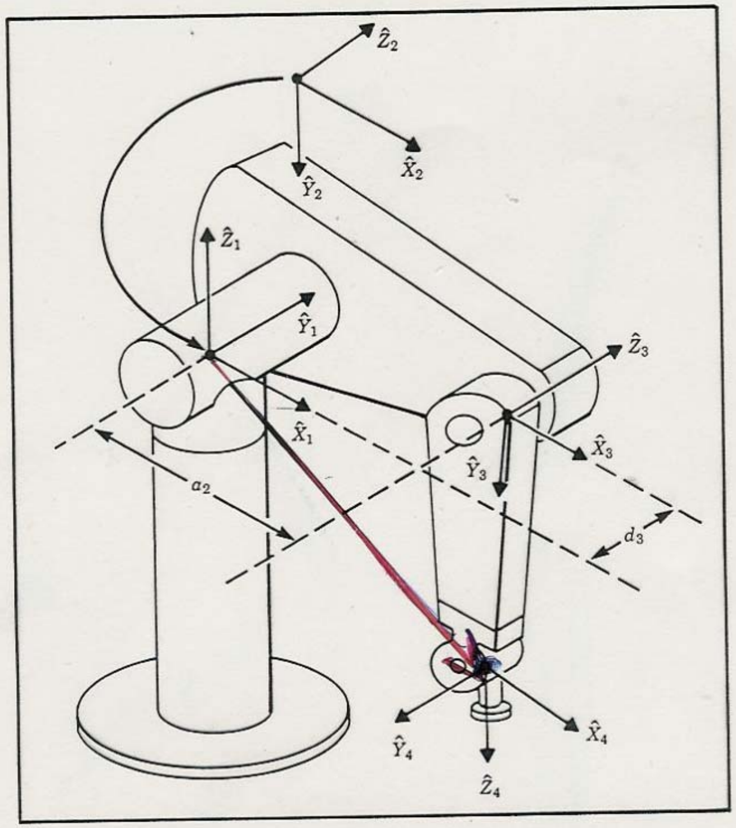


FIGURE 3.18 Some kinematic parameters and frame assignments for the PUMA 560 manipulator.

$z_4 = z_4$

$Y_4 = -z_5$

$z_4 = z_6$

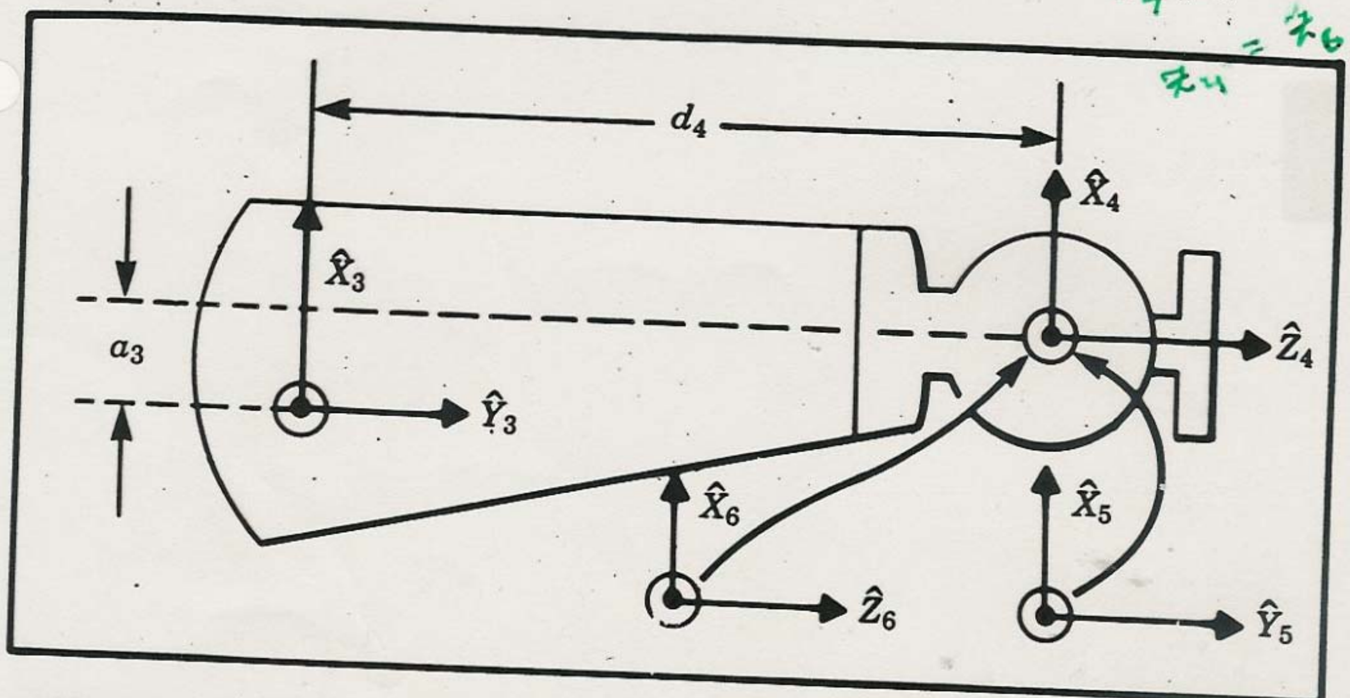


FIGURE 3.19 Kinematic parameters and frame assignments for the forearm of the PUMA 560 manipulator.

$${}^{0}_{6}T = {}^{0}_{1}T \, {}^{1}_{2}T \cdots {}^{5}_{6}T$$

This matrix can be easily computed in closed form.

see below for ${}^{i-1}_{i}T$ matrices

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	$-90°$	0	0	θ_2
3	0	a_2	d_3	θ_3
4	$-90°$	a_3	d_4	θ_4
5	$90°$	0	0	θ_5
6	$-90°$	0	0	θ_6

FIGURE 3.21 Link parameters of the PUMA 560.

variable parameter

$${}^{0}_{6}T = \left(\begin{array}{c|c} {}^{0}_{6}R & {}^{0}P_{6ORG} \\ \hline \underline{0} & 1 \end{array}\right)$$

(r_{ij})

$\begin{pmatrix} p_x \\ p_y \\ p_z \end{pmatrix}$

see Eq. 3.14 in text for ${}^{0}_{6}T$ matrix

Two salient pts: re ${}^{0}_{6}T$ matrix

① p_x, p_y, p_z dep. only on $\theta_1, \theta_2, \theta_3$

↳ because last 3 joints intersect (wrist structure) — first 3 joints

② orient. matrix: only c_{23} & s_{23} entries

since joint 2 & 3 are parallel.

Using (3.6), we compute each of the link transformations:

$$
{}^0_1T = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},
$$

$$
{}^1_2T = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s\theta_2 & -c\theta_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},
$$

$$
{}^2_3T = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & a_2 \\ s\theta_3 & c\theta_3 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix},
$$

$$
{}^3_4T = \begin{bmatrix} c\theta_4 & -s\theta_4 & 0 & a_3 \\ 0 & 0 & 1 & d_4 \\ -s\theta_4 & -c\theta_4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},
$$

$$
{}^4_5T = \begin{bmatrix} c\theta_5 & -s\theta_5 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s\theta_5 & c\theta_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},
$$

$$
{}^5_6T = \begin{bmatrix} c\theta_6 & -s\theta_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s\theta_6 & -c\theta_6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{3.9}
$$

We now form ${}^0_6T$ by matrix multiplication of the individual link matrices. While forming this product, we will derive some subresults that will be useful when solving the inverse kinematic problem in Chapter 4. We start by multiplying ${}^4_5T$ and ${}^5_6T$; that is,

$$
{}^4_6T = {}^4_5T\,{}^5_6T = \begin{bmatrix} c_5c_6 & -c_5s_6 & -s_5 & 0 \\ s_6 & c_6 & 0 & 0 \\ s_5c_6 & -s_5s_6 & c_5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{3.10}
$$

where c_5 is shorthand for $\cos\theta_5$, s_5 for $\sin\theta_5$, and so on.[6] Then we have

$$
{}^3_6T = {}^3_4T\,{}^4_6T = \begin{bmatrix} c_4c_5c_6 - s_4s_6 & -c_4c_5s_6 - s_4c_6 & -c_4s_5 & a_3 \\ s_5c_6 & -s_5s_6 & c_5 & d_4 \\ -s_4c_5c_6 - c_4s_6 & s_4c_5s_6 - c_4c_6 & s_4s_5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{3.11}
$$

[6] Depending on the amount of space available to show expressions, we use any of the following three forms: $\cos\theta_5$, $c\theta_5$, or c_5.

In General: given a robot with N joint (N-dof)

we now know how to assign $\{i\}$ at each joint, and hence compute

$$ {}^{0}_{N}T = \left(\begin{array}{c|c} {}^{0}_{N}R & {}^{0}P_{N\,ORG} \\ \hline \underline{0} & 1 \end{array}\right) $$

joint - parameters: θ_i - revolute

d_i - prismatic

q_i = joint-var.

$$\underline{q} = \begin{pmatrix} q_1 \\ \vdots \\ q_N \end{pmatrix} \underset{\text{INV KIN}}{\overset{\text{forw. KIN}}{\rightleftarrows}} {}^{0}_{N}T \text{ matrix}$$

joint-space

joint-vector

$$\left(\begin{array}{c|c} {}^{0}_{N}R & {}^{0}P_{NORG} \\ \hline \underline{0} & 1 \end{array} \right)$$

3 ind. parameters

6 ind parameters

$\alpha, \beta, \gamma \rightarrow$ Euler angle

$p_x, p_y, p_z \rightarrow$ Cart.

Cartesian space

Note: D-H Notation : variations exist

$\hat{z}_i \longleftrightarrow i^{th}$ joint axis

$\hat{z}_{i-1} \longleftrightarrow i^{th}$ " "

Generalization of forw. kin. :

determine "where" is the Tool w. r. t. Station frame ?

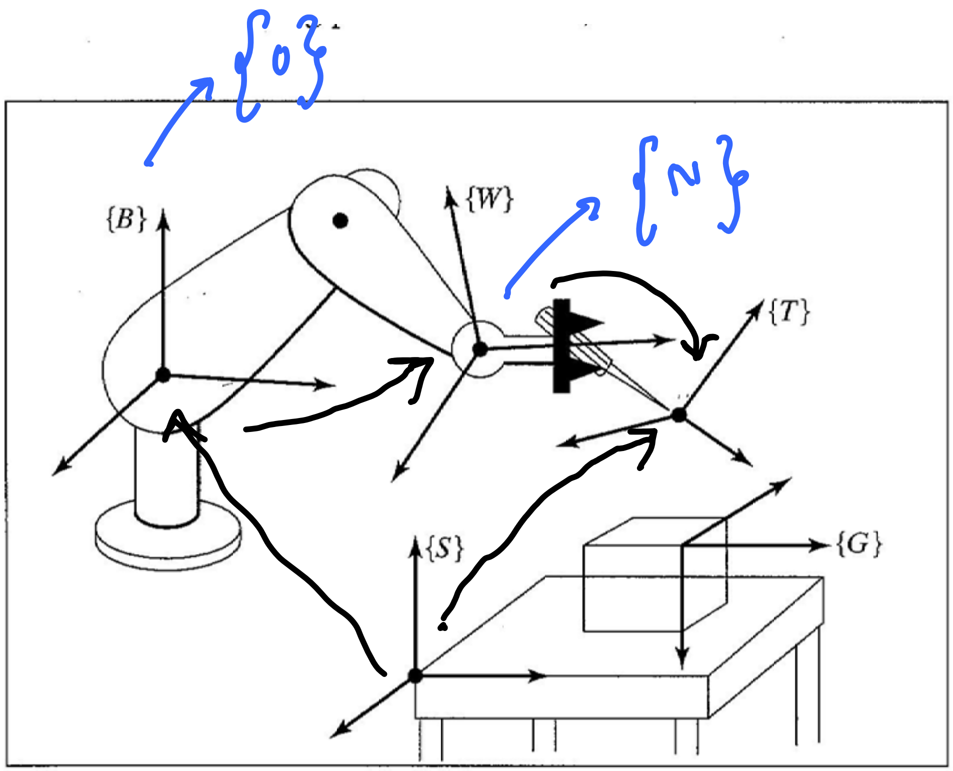


FIGURE 3.27: The standard frames.

given $\begin{pmatrix} q_1 \\ \vdots \\ q_N \end{pmatrix}$,

and,

Assume ${}^{B}_{S}T$, ${}^{W}_{T}T$ is known

Determine ${}^{S}_{T}T$

WHERE $\left(\underline{q},\ {}^{B}_{S}T,\ {}^{W}_{T}T\right)$: returns ${}^{S}_{T}T$

$$\underline{q} = \begin{pmatrix} q_1 \\ \vdots \\ q_N \end{pmatrix}$$

Using Xform arithmetic:

follow "upper" set of arrows.

$$\boxed{{}^{S}_{T}T = \left[{}^{B}_{S}T\right]^{-1}\ {}^{B}_{W}T\ {}^{W}_{T}T}$$

for. kin.