Note Title 10/4/2007

Lecture 14+15

Given ${}^{0}_{3}T$, compute $\theta_1, \theta_2, \theta_3$. 3 link planar arm

$\downarrow$ (x, y, ϕ) are known.

$$\begin{pmatrix} c\phi & -s\phi & 0 & x \\ s\phi & c\phi & 0 & y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

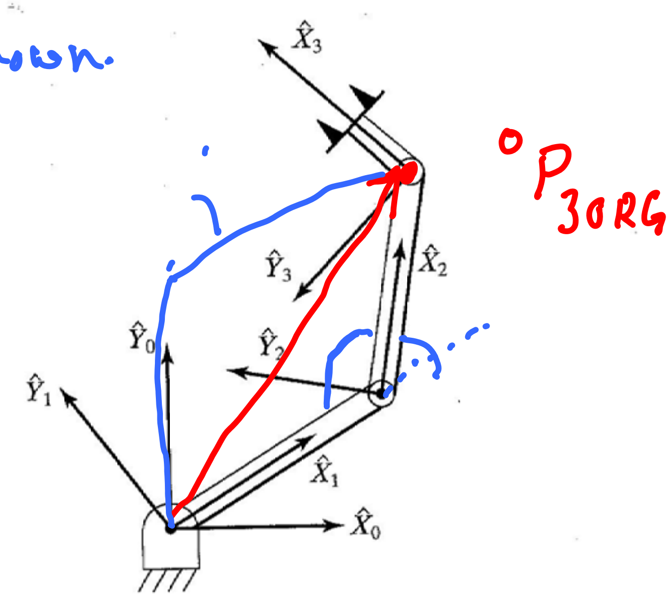


FIGURE 3.7: Link-frame assignments.

we had derived ${}^{0}_{3}T$

$$= \left[\begin{array}{ccc|c} c_{123} & -s_{123} & 0 & l_1 c_1 + l_2 c_{12} \\ s_{123} & c_{123} & 0 & l_1 s_1 + l_2 s_{12} \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

Comparing elem. by elem. :

$$l_1 c_1 + l_2 c_{12} = x \quad (1)$$

$$l_1 s_1 + l_2 s_{12} = y \quad (2)$$

$$\theta_1 + \theta_2 + \theta_3 = \phi \quad (3)$$

sq. + add 1 + 2 :

$$\ell_1^2 + \ell_2^2 + 2\ell_1\ell_2 c_1 c_{12} + 2\ell_1\ell_2 s_1 s_{12} = x^2 + y^2$$

$$\Leftrightarrow \ell_1^2 + \ell_2^2 + 2\ell_1\ell_2 (c_1 c_{12} + s_1 s_{12}) = x^2 + y^2$$

$$\Leftrightarrow \ell_1^2 + \ell_2^2 + 2\ell_1\ell_2 (\cos(\cancel{\theta_1} + \theta_2 - \cancel{\theta_1})) = x^2 + y^2$$

$$\Leftrightarrow \cos\theta_2 = \frac{x^2 + y^2 - (\ell_1^2 + \ell_2^2)}{2\ell_1\ell_2} \quad \# 2$$

two solns. θ_2^*, $\theta_2^{**} = -\theta_2^*$

To solve for θ_1: sub. θ_2^*, θ_2^{**} resp.

in (1) + (2) :

$$(l_1 + l_2 c_2) c_1 - (l_2 s_2) s_1 = x$$

$$(l_1 + l_2 c_2) s_1 + (l_2 s_2) c_1 = y$$

use # 6 to solve for θ_1. one soln. for θ_1 for each θ_2.

$$\theta_3 = \phi - (\theta_1 + \theta_2)$$

|| Overall, two solns for inv. kin.

Special case: what if ~~x, y~~ $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

Only way this would be satisfied (see sp. case for eqn #6)

if $l_1 + l_2 c_2 = 0$ $\qquad$ $l_2 s_2 = 0$

$\Downarrow$ $\qquad\qquad\qquad$ $\Downarrow$

$c_2 = -\frac{l_1}{l_2}$ $\qquad\qquad$ $\theta_2 = 0, \pi \ldots$

$\Downarrow$

$\theta_2 = 0$ Not possible

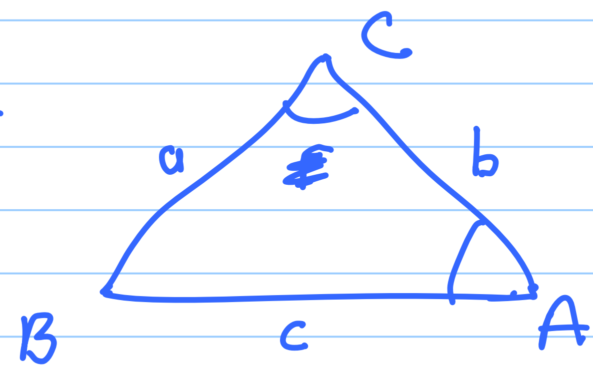
$\theta_2 = \pi$ yes, but only if $l_1 = l_2$

$$\boxed{\ell_1 = \ell_2, \quad \theta_2 = \pi, \quad \theta_1 = \text{arbitrary}}$$

<u>Method II :</u> Geometric approach

(application of Cosine law)

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$



easy to follow.

<u>see text.</u>

|| Inv. Kin for 6-dof robots

<u>closed form soln:</u> "SOLVABLE"

1995 "ALL six-dof robots with prism/revolute joints are solvable." "does use numerical technique"

<u>Earlier:</u> 1970's Pieper's PhD thesis

ALL rev. joint robots with <u>three</u> <u>consecutive axes intersecting</u> are solvable.

# of solns: table 4.5

~~dep.~~ varies as a func. of a_i's

$a_1 = a_3 = a_5 = 0 \quad \leq 4$ solns

|| no a_i ~~is~~ zero. 16 solns

PUMA: 8 possible solns. see text

fig for 4 (position only) solns.

(Fig 4.4, page 105)

Use Pieper's method to carry out inv. kin for a robot with last three joint axes intersecting.

Looking at the structure of (PUMA): (Fig 3.18 in text)

1. length of ${}^{0}P_{4ORG}$ is independent of θ_1
2. z co-ord of ${}^{0}P_{4ORG}$ " " " θ_1
3. ${}^{0}P_{4ORG}$ dep. on $\theta_1, \theta_2, \theta_3$

$\Rightarrow$ ① & ② will give two eqns in two unknowns, θ_2 & θ_3

hopefully we will be able to solve them.

KNOWN

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = {}^{0}P_{4ORG} = {}^{0}_{1}T\,{}^{1}_{2}T\,\underbrace{{}^{2}_{3}T\,{}^{3}P_{4ORG}}_{= \begin{pmatrix} f_1(\theta_3) \\ f_2(\theta_3) \\ f_3(\theta_3) \end{pmatrix}}$$

from ${}^{i-1}_{i}T$ matrix, we obtain

$$^{3}P_{4ORG} = \begin{pmatrix} a_3 \\ -s\alpha_3 d_4 \\ c\alpha_3 d_4 \end{pmatrix}$$

where

... (3.6) for ${}^3P_4$ in (4.44) yields the following expressions fo

$$f_1 = a_3 c_3 + d_4 s\alpha_3 s_3 + a_2,$$

$$f_2 = a_3 c\alpha_2 s_3 - d_4 s\alpha_3 c\alpha_2 c_3 - d_4 s\alpha_2 c\alpha_3 - d_3 s\alpha_2,$$

$$f_3 = a_3 s\alpha_2 s_3 - d_4 s\alpha_3 s\alpha_2 c_3 + d_4 c\alpha_2 c\alpha_3 + d_3 c\alpha_2.$$

$$\underbrace{{}^0P_{4ORG} = {}^0_1T\,{}^1_2T \begin{pmatrix} f_1(\theta_3) \\ f_2(\theta_3) \\ f_3(\theta_3) \end{pmatrix}} = \begin{pmatrix} g_1(\theta_2,\theta_3) \\ g_2(\theta_2,\theta_3) \\ g_3(\theta_2,\theta_3) \end{pmatrix}$$

Using (3.6) for ${}^0_1T$ and ${}^1_2T$ in (4.43) we obtain

$$ {}^0P_{4ORG} = \begin{bmatrix} c_1 g_1 - s_1 g_2 \\ s_1 g_1 + c_1 g_2 \\ g_3 \\ 1 \end{bmatrix} \tag{4.46} $$

where

$$ \begin{aligned} g_1 &= c_2 f_1 - s_2 f_2 + a_1, \\ g_2 &= s_2 c\alpha_1 f_1 + c_2 c\alpha_1 f_2 - s\alpha_1 f_3 - d_2 s\alpha_1, \\ g_3 &= s_2 s\alpha_1 f_1 + c_2 s\alpha_1 f_2 + c\alpha_1 f_3 + d_2 c\alpha_1. \end{aligned} \tag{4.47} $$

1

We now write an expression for the magnitude squared of ${}^0P_{4ORG}$, which is seen from (4.46) to be

$$ r = g_1^2 + g_2^2 + g_3^2 \tag{4.48} $$

or, using (4.47) for the g_i, we have

$$ r = f_1^2 + f_2^2 + f_3^2 + a_1^2 + d_2^2 + 2d_2 f_3 + 2a_1(c_2 f_1 - s_2 f_2). \tag{4.49} $$

We now write this equation, along with the Z component equation from (4.46), as a system of two equations in the form

$$ \begin{aligned} r &= (k_1 c_2 + k_2 s_2) 2a_1 + k_3, \\ z &= (k_1 s_2 - k_2 c_2) s\alpha_1 + k_4, \end{aligned} \tag{4.50} $$

where

$$
{}^{0}P_{4ORG} = {}^{0}_{1}T \underbrace{\begin{pmatrix} g_1(\theta_2,\theta_3) \\ g_2(\theta_2,\theta_3) \\ g_3(\theta_2,\theta_3) \\ 1 \end{pmatrix}}
$$

$$
= \begin{pmatrix} c_1 g_1 - s_1 g_2 \\ s_1 g_1 + c_1 g_2 \\ g_3 \\ 1 \end{pmatrix}
$$

$\sqrt{x^2+y^2+z^2}$ $=$ $r = g_1^2 + g_2^2 + g_3^2$ $=$ see eqn. 4.9 above

$\|$

length of ${}^0p_{4ORG}$

$$\boxed{\begin{aligned} r &= g_1^2 + g_2^2 + g_3^2 \\ z &= g_3 \end{aligned}} \Longrightarrow$$

r, z both known

z-comp of ${}^0p_{4ORG}$

we get from 4.50 above $\longrightarrow$

$$\left(\frac{r - k_3}{2a_1}\right)^2 + \left(\frac{z - k_4}{s\alpha_1}\right)^2 = k_1^2 + k_2^2$$

$\underline{\underline{\text{in } \theta_3}}$ $\Longrightarrow$ use $u = \tan \theta_3/2$: 4^{th} order eqn in u

$\Rightarrow$ 4 possible solns. for $\underline{\underline{\theta_3}}$

for Last 3 joint-var: $\theta_4, \theta_5, \theta_6$

One can see that

Rot. around $\hat{Z}_4$, $\hat{Z}_5$ and $\hat{Z}_6$ are

the same as Z-Y-Z Euler rotation $R_{Z'Y'Z'}(\alpha, \beta, \gamma)$

applied to frame $\{4\}_{\theta_4=0}$.

with the caveat that

$$\boxed{\begin{aligned} \alpha &= \theta_4 \\ \beta &= -\theta_5 \\ \gamma &= \theta_6 \end{aligned}}$$

use Eqn. (2.74) in Ch 2 (Inv. of Euler Rotation) for ${}^4_6R$)

$\{4\}_{\theta_4} = 0$

$4' \parallel$

$${}^4_6R = \left({}^0_4R\right)^{-1}_{\theta_4=0} \left({}^0_6R\right)$$

KNOWN since $\theta_1, \theta_2, \theta_3$ are solved for.

known

$z_4 = z_4$

$Y_4 = -z_5 = z_6$

z_4

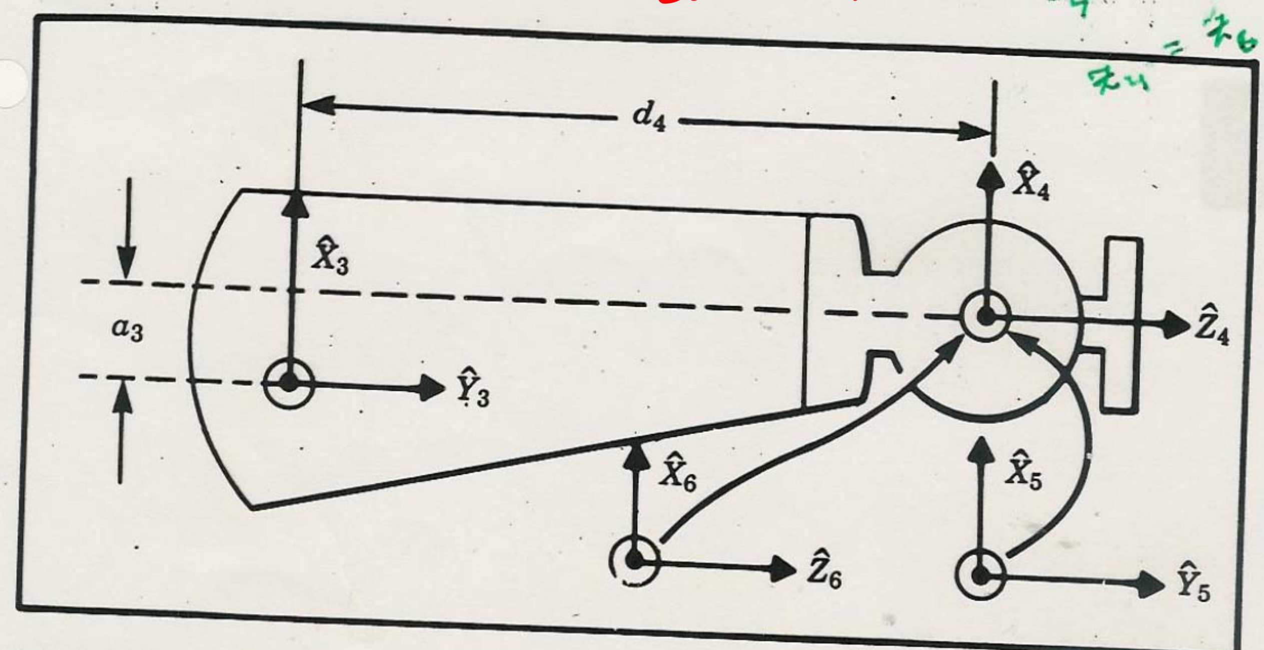


FIGURE 3.19 Kinematic parameters and frame assignments for the forearm of the PUMA 560 manipulator.

Two Soln. for inv. Euler comput.

$\alpha = \alpha^*, \quad \alpha^* + 180^\circ$ $\qquad \alpha = 0$

$\beta = \beta^*, \quad -\beta^*$ $\qquad \beta = 45^\circ$

$\gamma = \gamma^*, \quad \gamma^* + 180^\circ$ $\qquad \gamma = 90^\circ$

$\downarrow \qquad\qquad \downarrow$

Soln I $\qquad$ flipped soln. (soln II)

for these values illustrated with Tinker Toy in class

Hence, overall eight possible soln. for inv. KIN. (last three joint axes intersect)