Lecture 16

Note Title 10/10/2005

INVERSE KIN:

Given ${}^{0}_{N}T$, compute $\begin{pmatrix} \theta_1 \\ \vdots \\ \theta_N \end{pmatrix}$

or ${}^{B}_{W}T$

Last Lecture: Pieper's method

6-dof revolute joints

with 3 consecutive

intersecting axes.

<u>Inv Kin for PUMA arm:</u> i) apply Pieper's method

exercise

→ 2) alternative method

This lecture

'e wish to solve

$^{i-1}_{i}T$ ← form is known

$$ {}^0_6T = \begin{bmatrix} r_{11} & r_{12} & r_{13} & p_x \\ r_{21} & r_{22} & r_{23} & p_y \\ r_{31} & r_{32} & r_{33} & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

known

$$ = \boxed{{}^0_1T(\theta_1)}\,{}^1_2T(\theta_2)\,{}^2_3T(\theta_3)\,{}^3_4T(\theta_4)\,{}^4_5T(\theta_5)\,{}^5_6T(\theta_6) $$

general strategy: separate variables by "taking" each matrix one by one to left side

at least one

$$\underline{\underline{\left[{}^{0}_{1}T(\theta_1)\right]^{-1}}}\left[{}^{0}_{6}T\right] = {}^{1}_{2}T \cdots\cdots {}^{5}_{6}T$$

known

has a constant element

$$= {}^{1}_{6}T(\theta_2 \ldots \theta_6)$$

Look for a constant element in R.H.S. and compare it to the corresponding element in LHS (func. of θ_1)

We see that element $\underline{\underline{(2,4)}}$ on R.H.S., i.e., in ${}^{1}_{6}T$ is d_3 [see eqn. 3.13 on page 82 in text]

$$\begin{bmatrix} c_1 & s_1 & 0 & 0 \\ -s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & p_x \\ r_{21} & r_{22} & r_{23} & p_y \\ r_{31} & r_{32} & r_{33} & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = {}^1_6T, \tag{4.56}$$

(2.12) developed in Chapter 3. This simple

$[{}^0_1T]^{-1}$

$$\boxed{-s_1 p_x + c_1 p_y = d_3} \leftarrow 1$$

use Trig. eqn # 5 in our list to solve for θ_1. two solns for θ_1.

Now, Comparing elem. (1,4) & (3,4) on both sides : (L.H.S. is known since θ_1 is already solved for)

$$\begin{cases} c_1 p_x + s_1 p_y = a_2 c_2 + a_3 c_{23} - d_4 s_{23} & (2) \\ p_z = -(a_3 s_{23} + a_2 s_2 + d_4 c_{23}) & (3) \end{cases}$$

Square & add (1), (2), (3)

we get

4 $\rightarrow$ $$a_3 c_3 - d_4 s_3 = \frac{p_x^2 + p_y^2 + p_z^2 - a_2^2 - a_3^2 - d_3^2 - d_4^2}{2 a_2}$$

$$(=) \quad a_3 c_3 - d_4 s_3 = K$$

Trig eqn. # 5 in our list.

2 solns for θ_3,

Having solved for θ_3, you could now sub for θ_1, θ_3 in eqn. 2 + 3, and solve for θ_2. Alternatively, go to next step and look at:

$$\left[{}^0_3T\right]^{-1}\left[{}^0_6T\right] = {}^3_6T$$

elem $(1,4) = a_3$

elem $(2,4) = d_4$

(see eqn. 3.11 on page 81)

or

$$\begin{bmatrix} c_1c_{23} & s_1c_{23} & -s_{23} & -a_2c_3 \\ -c_1s_{23} & -s_1s_{23} & -c_{23} & a_2s_3 \\ -s_1 & c_1 & 0 & -d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & p_x \\ r_{21} & r_{22} & r_{23} & p_y \\ r_{31} & r_{32} & r_{33} & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = {}^3_6T, \qquad (4.70)$$

where ${}^3_6T$ is given by equation (3.11) developed in Chapter 3. Equating the $(1,4)$ elements from both sides of (4.70), as well as the $(2,4)$ elements, we get

(1,4) on LHS

(3,4) on LHS

$$\begin{aligned} c_1c_{23}p_x + s_1c_{23}p_y - s_{23}p_z - a_2c_3 &= a_3, \\ -c_1s_{23}p_x - s_1s_{23}p_y - c_{23}p_z + a_2s_3 &= d_4, \end{aligned} \qquad (4.71)$$

These equations may be solved simultaneously for s_{23} and c_{23}, resulting in

$\Longrightarrow$ eqn. # 3 in Trig. eqns

$$\begin{aligned} s_{23} &= \frac{(-a_3 - a_2c_3)p_z + (c_1p_x + s_1p_y)(a_2s_3 - d_4)}{p_z^2 + (c_1p_x + s_1p_y)^2}, \\ c_{23} &= \frac{(a_2s_3 - d_4)p_z - (-a_3 - a_2c_3)(c_1p_x + s_1p_y)}{p_z^2 + (c_1p_x + s_1p_y)^2}. \end{aligned} \qquad (4.72)$$

Since the denominators are equal and positive, we solve for the sum of θ_2 and θ_3 as

$$\theta_{23} = \text{Atan2}[(-a_3 - a_2c_3)p_z - (c_1p_x + s_1p_y)(d_4 - a_2s_3), \qquad (4.73)$$

$\hookrightarrow$ given one soln. for $\theta_2+\theta_3$, given (θ_1, θ_3) soln.

get $\theta_2 = (\theta_2+\theta_3) - (\theta_3)$

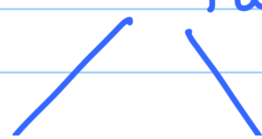
we will get four solns. $(\theta_1, \theta_2, \theta_3)$

proceed like this further as in text, and solve for $\theta_4, \theta_5, \theta_6$ also.

[NOT CARRIED OUT IN LECTURE
READ IT]

Note: flipped soln: for wrist, i.e $\theta_4, \theta_5, \theta_6$

two sets of soln. was illustrated with "Tinker Toy"



$$0 = \theta_4^* \qquad \theta_4^* + 180^\circ = 180^\circ$$
$$45^\circ = \theta_5^* \qquad -\theta_5^* = -45^\circ$$
$$90^\circ = \theta_6^* \qquad \theta_6^* + 180 = 270^\circ$$

Similar to forw. KIN, one would like to generalize the $\underline{\text{INV KIN}}$ to $\underline{\text{SOLVE}}$

↓ WRIST Frame — Given ${}^{B}_{W}T$ ↓ Tool Frame — Given ${}^{S}_{T}T$

$\underbrace{}_{\text{SOLVE}}$

INV KIN

$${}^{B}_{W}T \rightarrow \begin{pmatrix} \theta_1 \\ \vdots \\ \theta_N \end{pmatrix} \qquad {}^{S}_{T}T \rightarrow \begin{pmatrix} \theta_1 \\ \vdots \\ \theta_6 \end{pmatrix}$$

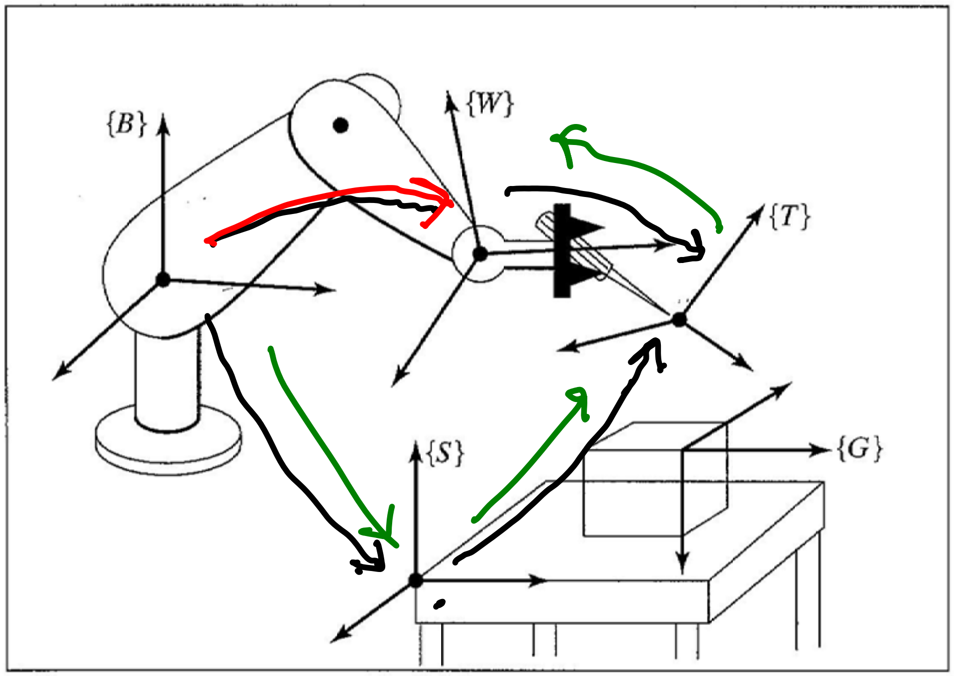


FIGURE 3.27: The standard frames.

$${}^{B}_{W}T \; {}^{W}_{T}T = {}^{B}_{S}T \; {}^{S}_{T}T$$

$${}^{B}_{W}T = {}^{B}_{S}T \; {}^{S}_{T}T \left[{}^{W}_{T}T \right]^{-1}$$

KNOWN

<u>SOLVE :</u> ① Compute ${}^{B}_{W}T$

Given ${}^{S}_{T}T$ ② Use INVKIN to get $\begin{pmatrix} \theta_1 \\ \vdots \\ \theta_N \end{pmatrix}$

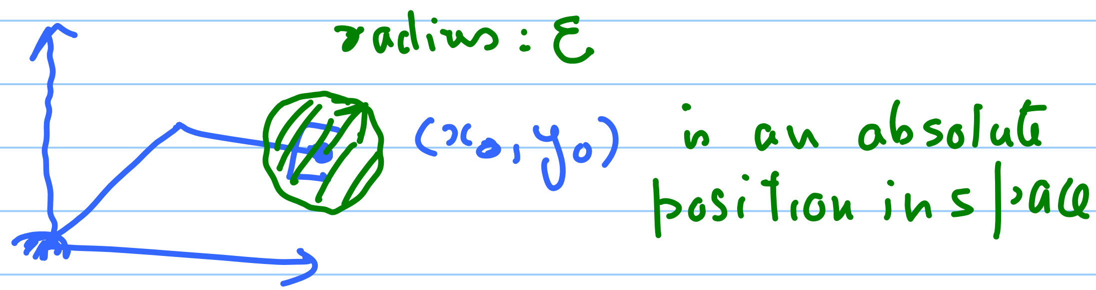
Repeatability and Accuracy:

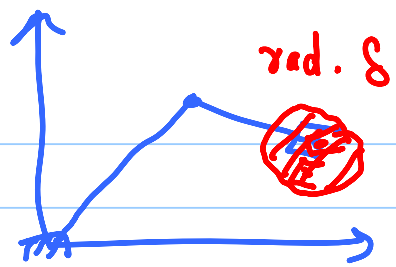
For any "positioning device":

ε : accuracy : w. r. t. an absolute measurement

δ : repeatability: w. r. t. one attempt to another

Normally $\delta \ll \varepsilon$





on repeated attempts,
how close are diff.
positions, within
a circle of rad. δ

"No absolute position"
only relative to other attempts.

In Robotics:

Repeatability

taught points: robot is physically (manually) moved to desired joint values,

in joint space $\begin{pmatrix}\theta_1\\ \theta_2\end{pmatrix}$ the joint values are remembered and then played again & again.

Accuracy

Computed points : in Cartesian space move to (x, y)

requires an inverse kinematics capability. robot controller has to comput $\begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix}$ corr. to (x, y). This will be affected by accuracy of knowledge of D-H param.