Note Title 11/11/2007

Lecture - 17 + 18

$q_i = \theta_i$ rev

d_i prism

"Differential Kinematics"

$${}^{0}_{N}T = \left[\begin{array}{c|c} {}^{0}_{N}R & {}^{0}P_{NORG} \\ \hline 0 & 1 \end{array}\right],$$

Velocity of joints $\rightleftarrows$ Vel. of End-Eff / Tool

$$\begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \vdots \\ \dot{q}_n \end{pmatrix} \qquad \Big(\underbrace{\dot{p}_x, \dot{p}_y, \dot{p}_z}_{{}^{0}\dot{P}_{NORG}}, \text{ang velocity} \Big)$$

1. how to represent velocity of a rigid body (frame)
2. rel. bet. joint vel & end-eff. velocity
3. apply it to a robot

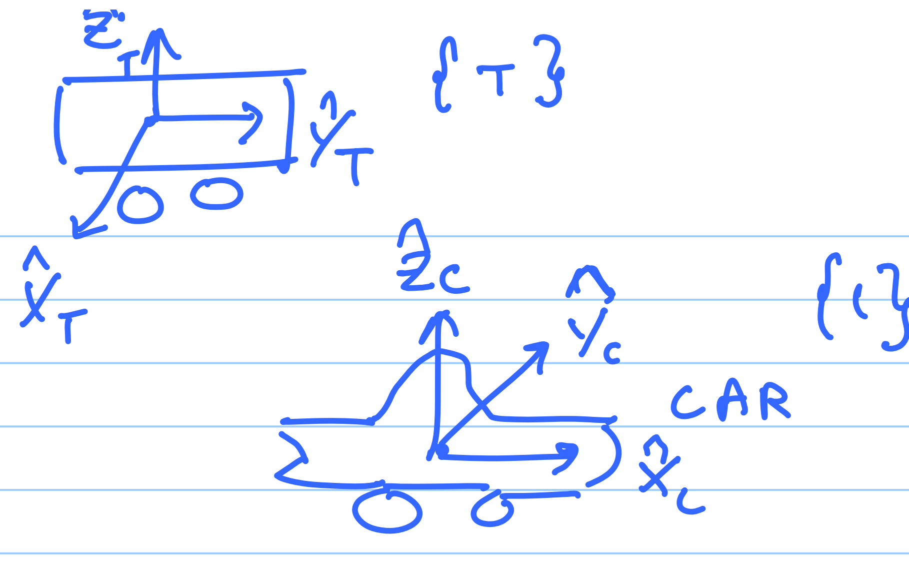
<u>Notation follows text:</u> $^{A}Q = {}^{A}_{B}T \, {}^{B}Q$

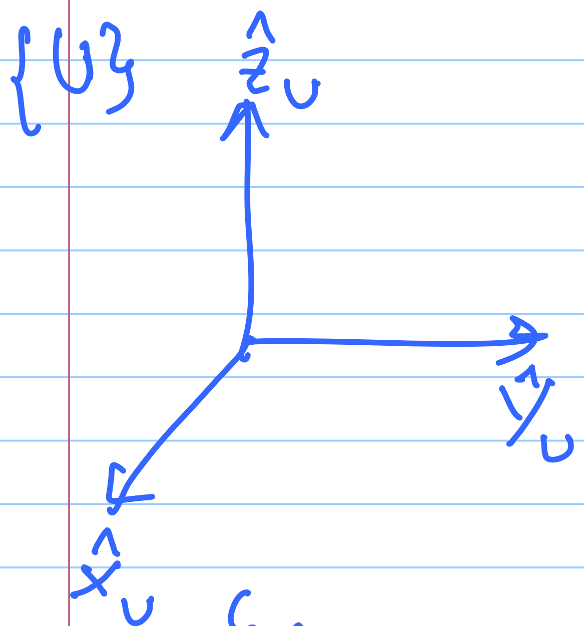
<u>Linear Velocity:</u> 1) associated with a point in space. Say it is Q

$$^{A}\left({}^{B}V_{Q}\right) = \underbrace{{}^{A}_{B}R}_{\text{pure co-ord trans.}} \left(\underbrace{\mathop{\mathrm{Lt.}}_{\Delta t\to 0} \frac{{}^{B}Q_{t+\Delta t} - {}^{B}Q_{t}}{\Delta t}}_{\text{derivative}}\right)$$

(vel. vectors are free vector)

$$^{U}\left({}^{U}V_{CORG}\right) = v_{C} \qquad \{U\},\ \{C\}$$





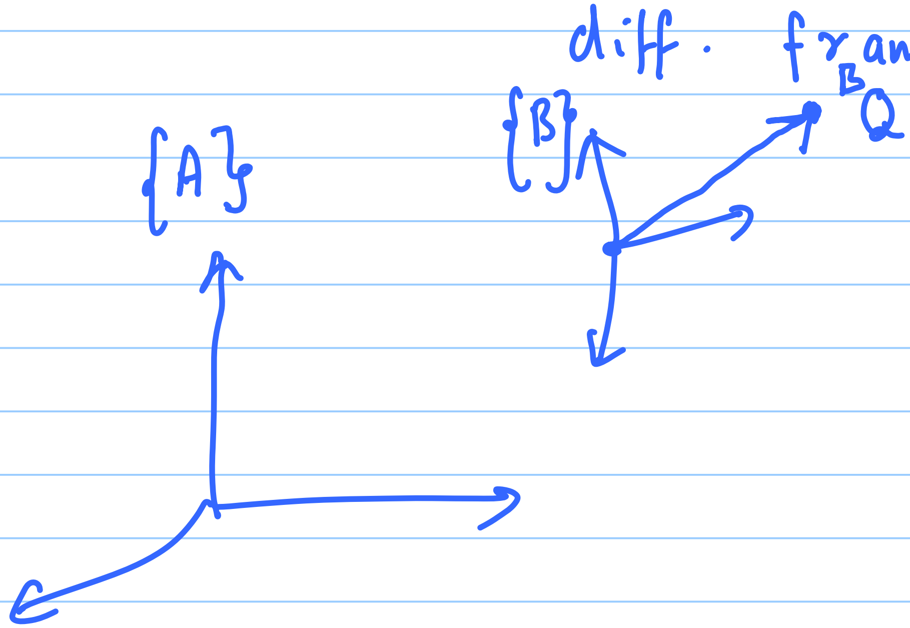
$${}^{U}V_{TORG} = \begin{pmatrix} 0 \\ 200 \\ 0 \end{pmatrix}$$

$$- \, {}^{U}V_{CORG} = \begin{pmatrix} 0 \\ 150 \\ 0 \end{pmatrix}$$

$${}^{C}\left({}^{C}V_{TORG}\right) = \begin{pmatrix} 50 \\ 0 \\ 0 \end{pmatrix}$$

Relationship between velocities across diff. frame

$\{A\}, \{B\}, {}^{B}Q$



$${}^{A}V_{Q} \stackrel{?}{=} {}^{B}V_{Q}$$

1) Assume ${}^{A}_{B}R$ is fixed.

$${}^{A}Q = {}^{A}P_{BORG} + {}^{A}_{B}R\,{}^{B}Q$$

taking $\frac{d}{dt}$:

$${}^{A}V_{Q} = {}^{A}\dot{Q} = {}^{A}\dot{P}_{BORG} + {}^{A}_{B}R\,{}^{B}\dot{Q}$$

$$= {}^{A}V_{BORG} + {}^{A}_{B}R\,{}^{B}V_{Q}$$

2) $\{A\}, \{B\}$ have same origin.

$${}^{A}Q = {}^{A}_{B}R\,{}^{B}Q \qquad {}^{A}_{B}R(t)$$

$$ {}^{A}V_{Q} = {}^{A}\dot{Q} = {}^{A}_{B}\dot{R}\,{}^{B}Q + {}^{A}_{B}R\,{}^{B}\dot{Q} $$

$$ = {}^{A}_{B}\dot{R}\,\underbrace{{}^{B}_{A}R\,{}^{A}Q} + \cdots $$

$$ = \underbrace{{}^{A}_{B}\dot{R}\left({}^{A}_{B}R\right)^{T}}\,{}^{A}Q $$

DIGRESSION

$$ = S\left({}^{A}\Omega_{B}\right)\cdot{}^{A}Q $$

$$ = {}^{A}\Omega_{B}\times{}^{A}Q + {}^{A}_{B}R\,{}^{B}\dot{Q} $$

|| ang. vel. vector

$$\dot{R} R^T = ?$$

$$R R^T = I$$

$$\dot{R} R^T + R \dot{R}^T = 0 \qquad (AB)^T = B^T A^T$$

$$\underbrace{\dot{R} R^T}_{S} = - R \dot{R}^T \qquad (A^T)^T = A$$

$$= - (\underbrace{\dot{R} R^T}_{S})^T$$

$$S + S^T = 0$$

S is skew-symm.

$$S = \begin{pmatrix} 0 & -\Omega_z & \Omega_y \\ \Omega_z & 0 & -\Omega_x \\ -\Omega_y & \Omega_x & 0 \end{pmatrix} \qquad S(\Omega)$$

$$\underline{\underline{S}}\,\vec{P} = \begin{pmatrix} \Omega_x \\ \Omega_y \\ \Omega_z \end{pmatrix} \times \vec{P}$$

Ang. vel Ω: Phys. Interpretation

$\dot{R} R^T$

Angle-axis rep

$R_k(\theta) \quad \hat{k}, \theta$

$$\dot{R} = \lim_{\Delta t \to 0} \frac{R_{t+\Delta t} - R_t}{\Delta t} \qquad \hat{K},\ \Delta\theta$$

$$R_{t+\Delta t} = R_K(\Delta\theta) R_t$$

$$\dot{R} = \lim_{\Delta t \to 0} \frac{\left[R_K(\Delta\theta) - I \right]}{\Delta t} R_t$$

that the equivalent rotation matrix is

$$R_K(\theta) = \begin{bmatrix} k_x k_x v\theta + c\theta & k_x k_y v\theta - k_z s\theta & k_x k_z v\theta + k_y s\theta \\ k_x k_y v\theta + k_z s\theta & k_y k_y v\theta + c\theta & k_y k_z v\theta - k_x s\theta \\ k_x k_z v\theta - k_y s\theta & k_y k_z v\theta + k_x s\theta & k_z k_z v\theta + c\theta \end{bmatrix}.$$

Where $c\theta = \cos\theta$, $s\theta = \sin\theta$, ...

$$\frac{R_k(\Delta\theta)}{\Delta t} = \begin{bmatrix} 0 & -k_z \Delta\theta/\Delta t & k_y \Delta\theta/\Delta t \\ k_z \Delta\theta/\Delta t & 0 & -k_x \Delta\theta/\Delta t \\ -k_y \Delta\theta/\Delta t & k_x \Delta\theta/\Delta t & 0 \end{bmatrix}$$

$\lim_{\theta \to 0} \sin\theta = \theta$

$\lim_{\theta \to 0} c\theta \to 1$

$\lim_{\theta \to 0} v\theta = 1 - \cos\theta \to 0$

$$= \begin{bmatrix} 0 & -k_z \dot{\theta} & k_y \dot{\theta} \\ k_z \dot{\theta} & 0 & -k_x \dot{\theta} \\ -k_y \dot{\theta} & k_x \dot{\theta} & 0 \end{bmatrix}$$

$S(\Omega) \quad \dot{R} R^T =$

$$\begin{pmatrix} \Omega_x \\ \Omega_y \\ \Omega_z \end{pmatrix} = \begin{pmatrix} k_x \dot{\theta} \\ k_y \dot{\theta} \\ k_z \dot{\theta} \end{pmatrix}$$

rot. of $k_x \dot{\theta}$ and $\hat{x}$

for infinitesimal rotations, order does not matter

exercise $\rightarrow$ $$R_{k_1}(\Delta\theta_1)\, R_{k_2}(\Delta\theta_2) = R_{k_2}(\Delta\theta_2)\, R_{k_1}(\Delta\theta_1)$$