Note Title 9/6/2007

Lecture 2+3

Applications of Robotics:

1) Manufacturing: soft automation
hard "

2) Personal Robotics:
   1) vaccum cleaning robot
   "Roomba"

3) space applications: "Canada Arm"
(Hazardous environments)

Mars Rover

4) Entertainment Robotics:

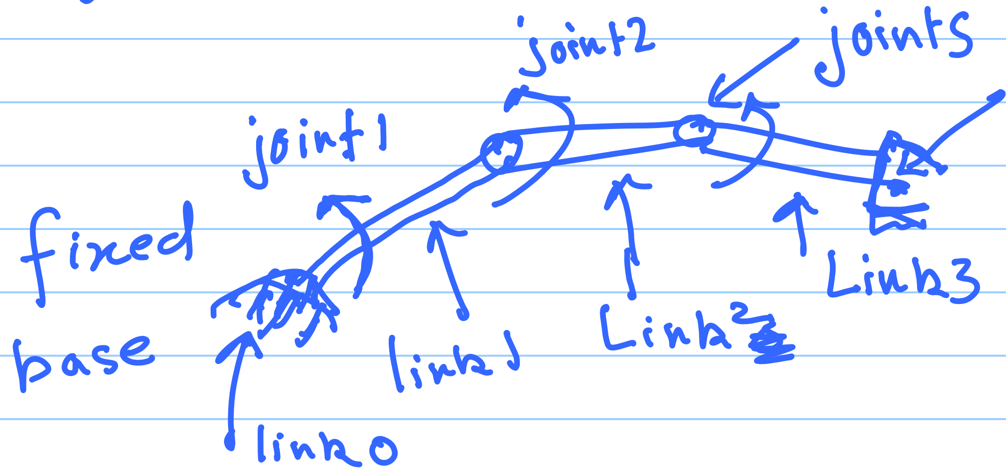
video games etc.

many motion generation / planning problems are similar in video games as in robotics.

A Brief preview of the course

i) mechanical manipulator arms

robot arm



"open kinematic chain"

ii) serial robot arm

one end of chain is free to move.

What do we do with it?

aim: Planning and control of the robot motion.

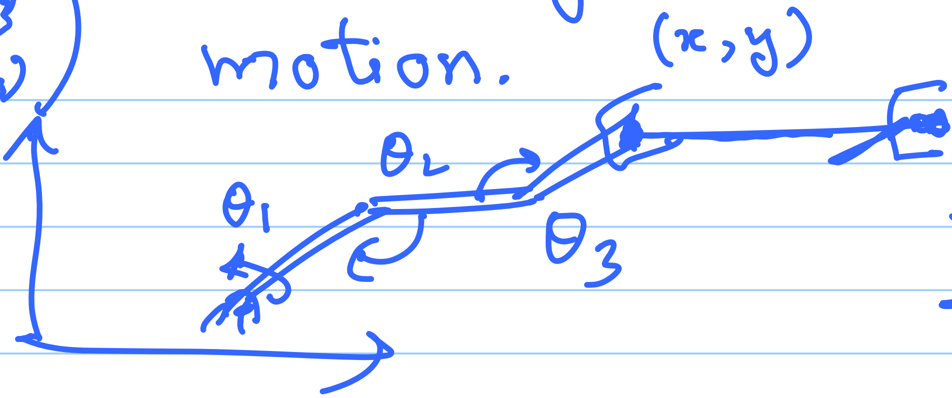
Sub-topics:

1. Description and representation of rigid body motions: how to specify motion / position / orientation of an object. co-ordinate frames translations, rotations

2. KINEMATICS: Relationship

$$\begin{pmatrix} f_1(\theta_1, \theta_2, \theta_3) \\ f_2(\theta_1, \theta_2, \theta_3) \\ f_- \end{pmatrix}$$

between joint motion & end-effector motion.



$$\underline{f}(\theta_1, \theta_2, \theta_3) \underset{\text{inverse}}{\overset{\text{forward}}{=}} (x, y)$$

Positional relationship

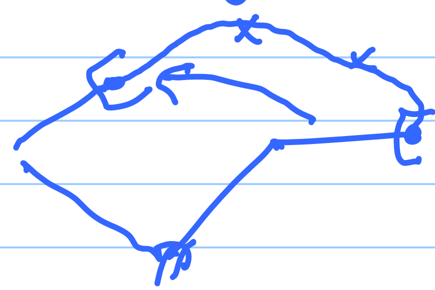
3) Dynamics : Relationship bet. joint motion & joint torques.

Differential equs.

$$\underline{f}(\theta_1, \theta_2, \theta_3, \dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_3, \ddot{\theta}_1, \ddot{\theta}_2, \ddot{\theta}_3) \underset{\text{forward}}{\overset{\text{inv.}}{=}} (\tau_1, \tau_2, \tau_3)$$

4) Control : track a given trajectory for the robot.

5) trajectory generation / planning



"exercise in fitting splines"

6) Robot Programming / Planning

1) joint level commands

2) Cartisian level "

3) object level " (World modelling)

4) task level

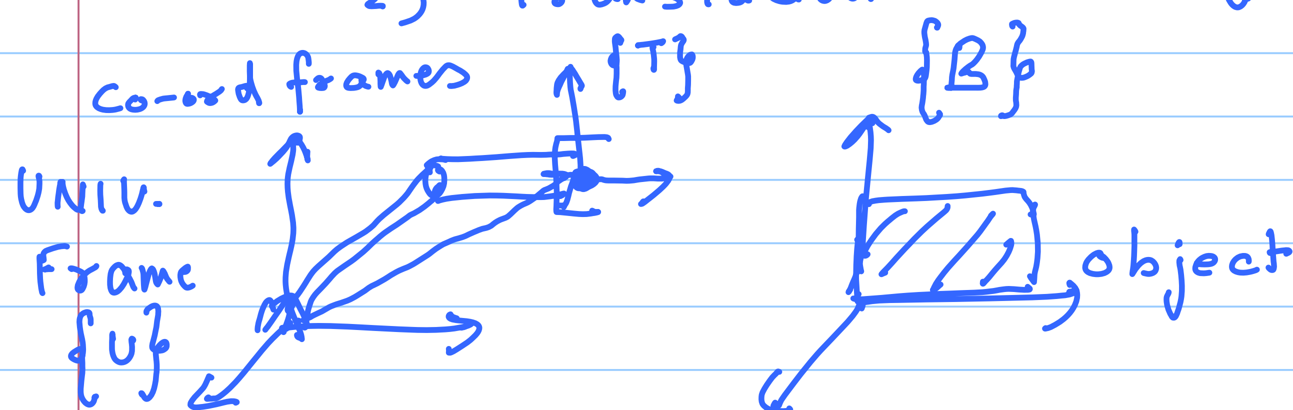
Representation of Rigid Body Motions (Euclidean Motion)

1) Rotation

2) Translation

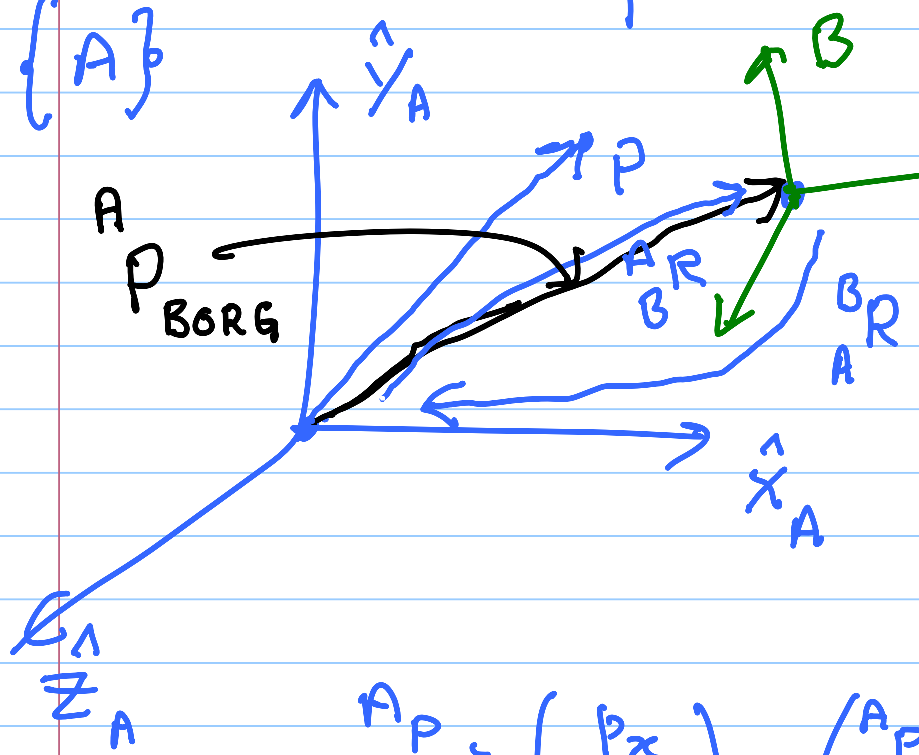
$$\{T\} = \left[{}^{U}P_{TORG},\ {}^{U}_{T}R\right]$$

Co-ord frames



Describing Position + orientation of one frame w.r.t. another.

(Right handed co-ord. systems)



$\{B\}$ w.r.t. $\{A\}$??

Points in space : P

$\Downarrow$

Vectors whose co-ord. are specified w.r.t a frame. ${}^{A}P = \begin{pmatrix} p_x \\ p_y \\ p_z \end{pmatrix}$

$${}^{A}P = \begin{pmatrix} p_x \\ p_y \\ p_z \end{pmatrix} = \begin{pmatrix} {}^{A}P \cdot \hat{X}_A \\ {}^{A}P \cdot \hat{Y}_A \\ {}^{A}P \cdot \hat{Z}_A \end{pmatrix}$$

${}^{A}P \cdot {}^{A}Q$

$$= ({}^{A}P)^{T}({}^{A}Q) \qquad P = p_x \hat{X}_A + p_y \hat{Y}_A + p_z \hat{Z}_A$$

$$= (p_x \; p_y \; p_z)\begin{pmatrix} q_x \\ q_y \\ q_z \end{pmatrix} = p_x q_x + p_y q_y + p_z q_z$$

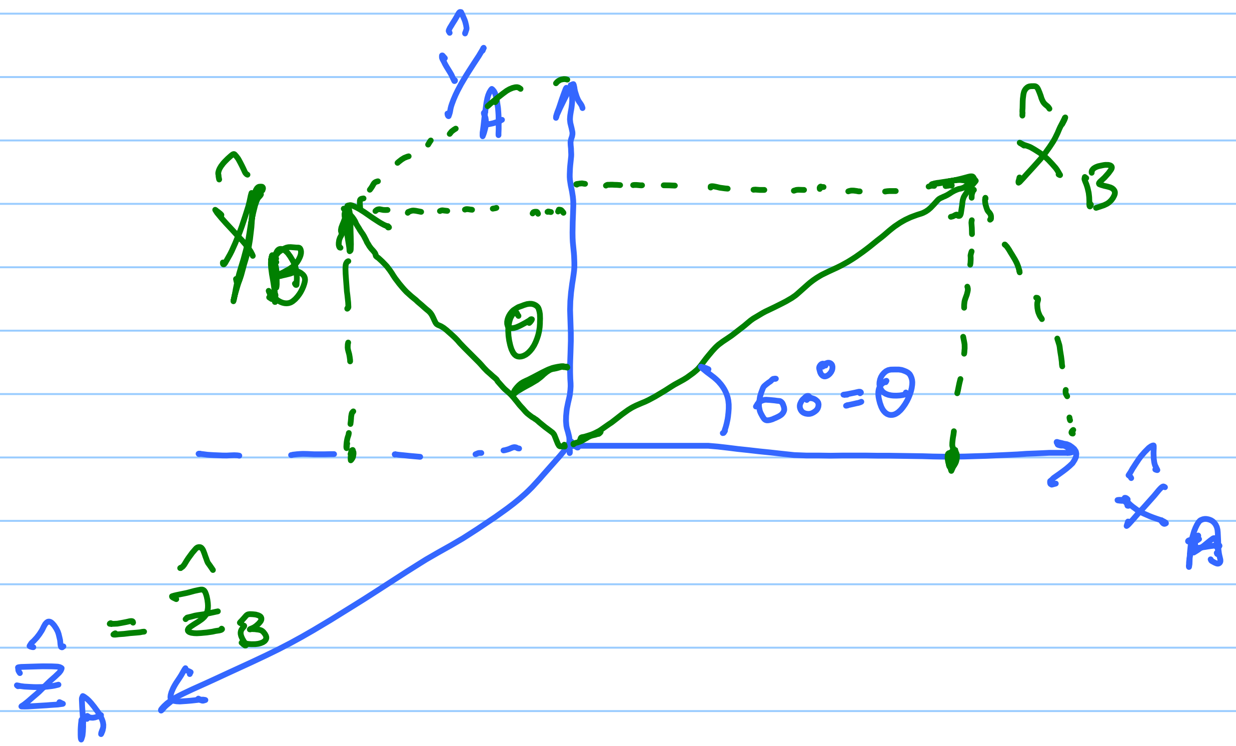
$\{B\}$ w.r.t. $\{A\}$:

1) Co-incident origins, differ in their orientation

$${}^{A}_{B}R = \left({}^{A}\hat{X}_B \;\middle|\; {}^{A}\hat{Y}_B \;\middle|\; {}^{A}\hat{Z}_B \right) = 3 \times 3 \text{ matrix}$$

Col. vector is co-ord. axis of B expressed w.r.t. A.

$$
{}^{A}_{B}R = \begin{pmatrix} \hat{X}_B \cdot \hat{X}_A & \hat{Y}_B \cdot \hat{X}_A & \hat{Z}_B \cdot \hat{X}_A \\ \hat{X}_B \cdot \hat{Y}_A & \hat{Y}_B \cdot \hat{Y}_A & \hat{Z}_B \cdot \hat{Y}_A \\ \hat{X}_B \cdot \hat{Z}_A & \hat{Y}_B \cdot \hat{Z}_A & \hat{Z}_B \cdot \hat{Z}_A \end{pmatrix}
$$



$c\theta$: $\cos\theta$

$s\theta$: $\sin\theta$

$$
{}^{A}_{B}R = \begin{array}{ccc} {}^{A}\hat{X}_B & {}^{A}\hat{Y}_B & {}^{A}\hat{Z}_B \end{array}
$$

$$
{}^{A}_{B}R = \begin{pmatrix} c\theta & -s\theta & 0 \\ s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}
$$

Properties of "Rotation" matrices ${}^{A}_{B}R$

1) col. vectors are unit vectors
2) " " " mutually orthogonal
3) $\det({}^{A}_{B}R) = +1$

$SO(3)$

<u>orthonormal matrices:</u> $SO(N)$

by inspection, rows of ${}^{A}_{B}R$ are unit vectors of $\{A\}$ w.r.t. $\{B\}$

$$ {}^{A}_{B}R = \begin{pmatrix} {}^{B}\hat{X}_A^T \\ \hline {}^{B}\hat{Y}_A^T \\ \hline {}^{B}\hat{Z}_A^T \end{pmatrix} = \left({}^{B}_{A}R\right)^T $$

This suggests that

$$ \left({}^{A}_{B}R\right)^{-1} = \left({}^{B}_{A}R\right) = \left({}^{B}_{A}R\right)^T \longleftarrow $$ Key property

You can verify that

$$\left({}^{A}_{B}R\right)\left({}^{A}_{B}R\right)^{T} = I_{3\times 3}$$

In general, for $\{A\}$, $\{B\}$

$$\{B\} = \left\{{}^{A}P_{BORG},\ {}^{A}_{B}R\right)$$

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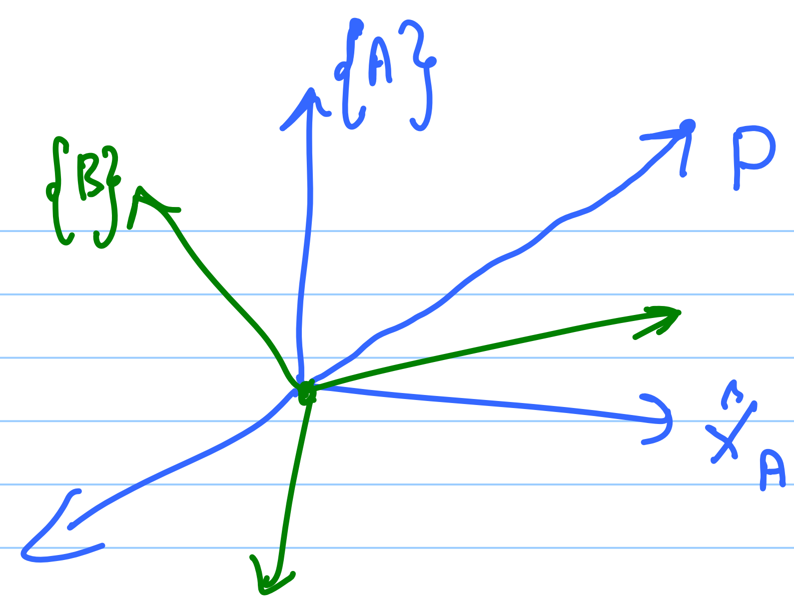
how to relate a point (vector) w.r.t. two diff. frames ?



$${}^{A}P \overset{?}{=} {}^{B}P$$

$${}^{A}P = \begin{pmatrix} {}^{?}P \cdot {}^{?}\hat{X}_A \\ {}^{?}P \cdot {}^{?}\hat{Y}_A \\ {}^{?}P \cdot {}^{?}\hat{Z}_A \end{pmatrix} = \begin{pmatrix} {}^{B}\hat{X}_A \cdot {}^{B}P \\ {}^{B}\hat{Y}_A \cdot {}^{B}P \\ {}^{B}\hat{Z}_A \cdot {}^{B}P \end{pmatrix}$$

$$= \begin{pmatrix} {}^{B}\hat{x}_A^{T} \; {}^{B}p \\ {}^{B}\hat{y}_A^{T} \; {}^{B}p \\ {}^{B}\hat{z}_A^{T} \; {}^{B}p \end{pmatrix}$$

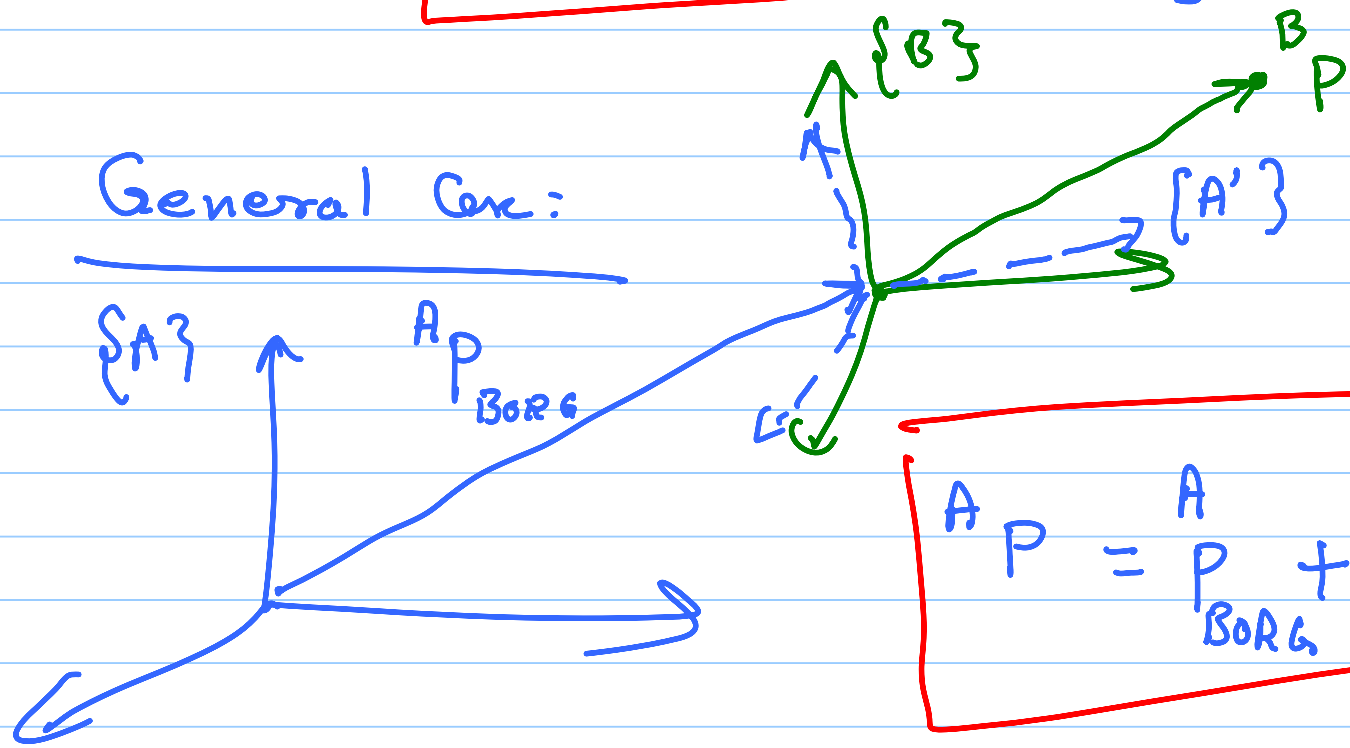
$$= \begin{pmatrix} {}^{B}\hat{x}_A^{T} \\ {}^{B}\hat{y}_A^{T} \\ {}^{B}\hat{z}_n^{T} \end{pmatrix} {}^{B}p$$

$$= \left({}^{B}_{A}R\right)^{T} \; {}^{B}p$$

$${}^{A}P = \boxed{{}^{A}_{B}R \; {}^{B}P}$$

pre-multiply with ${}^{A}_{B}R$

General Case:



$$\boxed{{}^{A}P = {}^{A}P_{BORG} + {}^{A}_{B}R \; {}^{B}P}$$

For Notational simplicity:

$$\begin{pmatrix} {}^{A}P \\ \hline 1 \end{pmatrix} = \left(\begin{array}{c|c} {}^{A}_{B}R & {}^{A}P_{BORG} \\ \hline \underline{0} & 1 \end{array}\right) \begin{pmatrix} {}^{B}P \\ \hline 1 \end{pmatrix}$$

$$\begin{pmatrix} {}^{A}P \\ \hline 1 \end{pmatrix} = {}^{A}_{B}T \begin{pmatrix} {}^{B}P \\ \hline 1 \end{pmatrix}$$

4x4 $\longrightarrow$ homogeneous transformation