Note Title — 10/18/2007

Lecture 20

LVE :

$$ {}^{A}V_{Q} = \overbrace{{}^{A}V_{BORG}}^{\text{Lin vel. of origin of \{B\} rel. to \{A\}}} + \overbrace{{}^{A}\Omega_{B} \times {}^{A}_{B}R\,{}^{B}Q}^{\text{Lin. vel. due to rotation of \{B\} rel. to \{A\}}} + {}^{A}_{B}R \underbrace{{}^{B}V_{Q}}_{\text{vel. of Q rel. to \{B\}}} $$

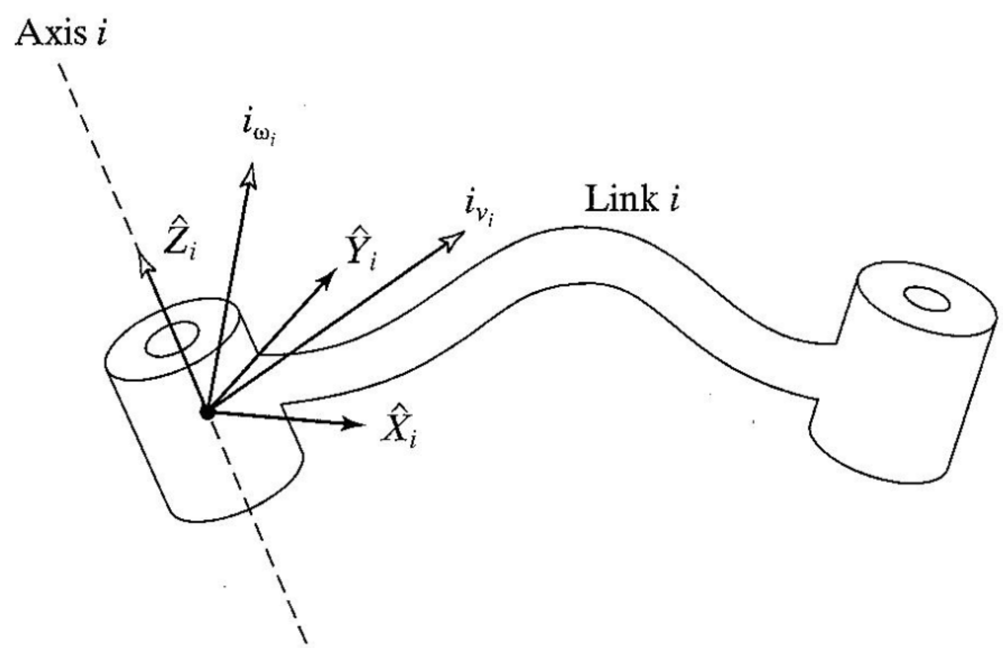
$$ {}^{A}\Omega_{C} = {}^{A}\Omega_{B} + \overbrace{\underbrace{{}^{A}_{B}R}_{\uparrow \text{ purely coord xform.}}\,{}^{B}\Omega_{C}}^{{}^{A}\left({}^{B}\Omega_{C}\right)} $$

lin vel. of orig of $\{i\}$: $v_i = {}^0({}^0V_i)$

$\omega_i = {}^0({}^0\Omega_i)$

${}^i v_i = {}^i_0R\, v_i$

${}^i\omega_i = {}^i_0R\, \omega_i$



Assume we have evaluated v_i, ω_i,

we will derive expr. for v_{i+1}, ω_{i+1}

Corr. to $L\Omega E$:

$\{A\} = \{0\}$

$\{B\} = \{i\}$

$\{C\} = \{i+1\}$

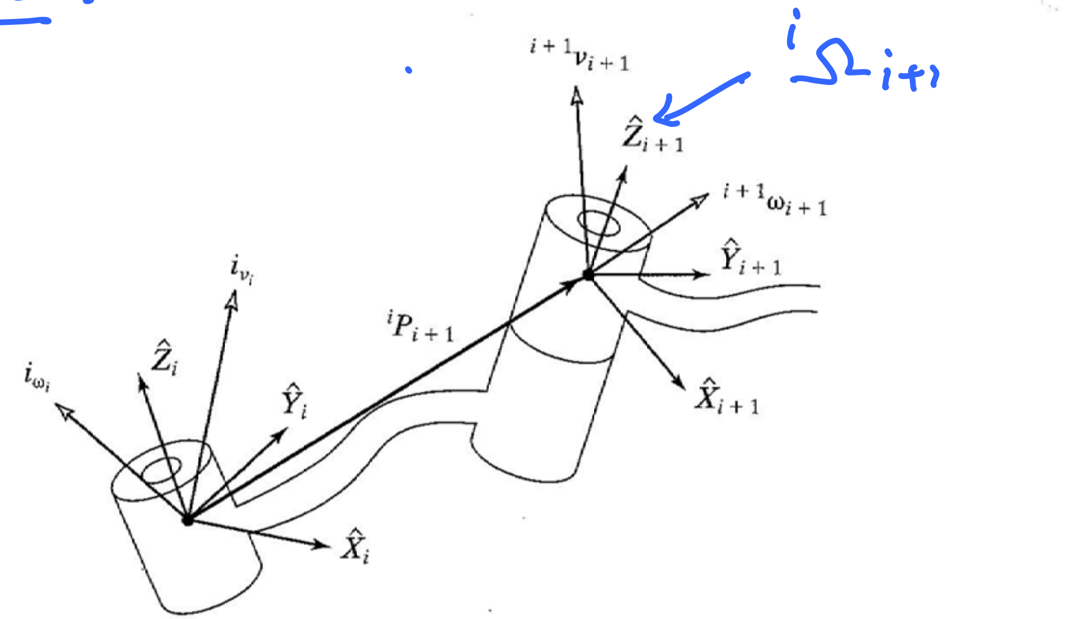


FIGURE 5.7: Velocity vectors of neighboring links.

[2]Remember that linear velocity is associated with a point, but angular velocity is associated with a

a) rev joint (i+1)

$$\omega_{i+1} = \omega_i + {}^{0}\left({}^{i}\Omega_{i+1}\right)$$

$$= \omega_i + {}^{0}_{i}R\left({}^{i}_{i+1}R\begin{pmatrix}0\\0\\\dot{\theta}_{i+1}\end{pmatrix}\right)$$

$$= \omega_i + {}^{0}_{i}R\,{}^{i}_{i+1}R\,\dot{\theta}_{i+1}\,{}^{i+1}\hat{Z}_{i+1}$$

$${}^{i+1}\omega_{i+1} = {}^{i+1}_{0}R\,\omega_{i+1} = \boxed{{}^{i+1}_{i}R\,{}^{i}\omega_i + \dot{\theta}_{i+1}\,{}^{i+1}\hat{Z}_{i+1}}$$

b) prism. joint (i+1) : $${}^{i+1}\omega_{i+1} = \boxed{{}^{i+1}_{i}R\,{}^{i}\omega_i}$$

LINEAR VEL:

$$ {}^{A}V_{Q} = {}^{A}V_{BORG} + {}^{A}_{B}R\,{}^{B}V_{Q} + {}^{A}\Omega_{B} \times {}^{A}_{B}R\,{}^{B}Q $$

$v_{i+1} = ?$

$\{A\} = \{0\}$

$\{B\} = \{i\}$

$Q = P_{i+1}$

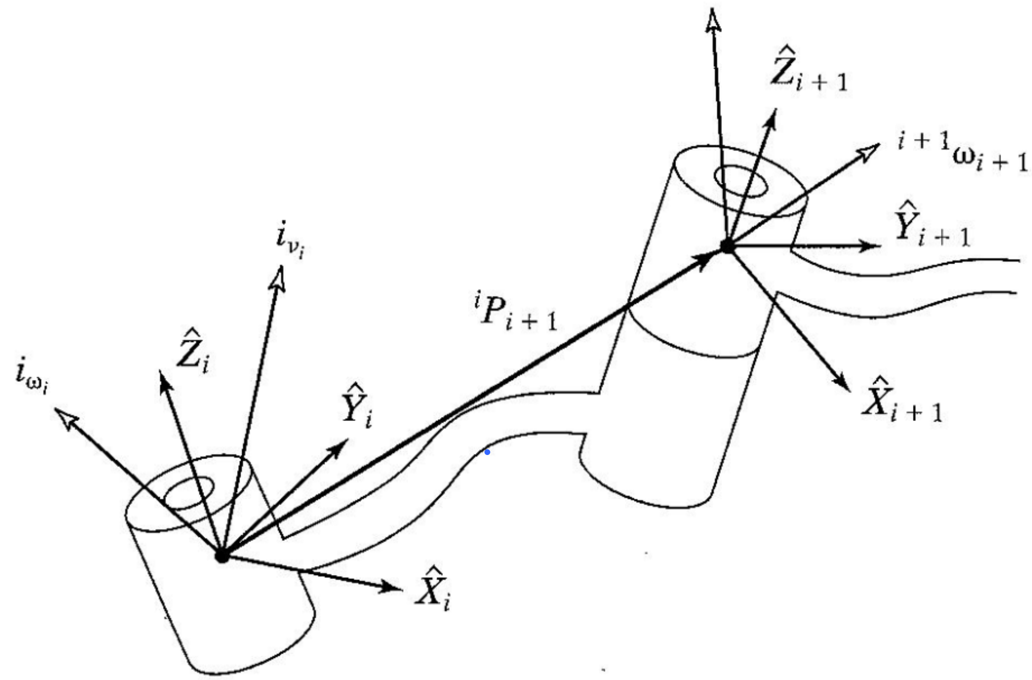


FIGURE 5.7: Velocity vectors of neighboring links.

[2] Remember that linear velocity is associated with a point, but angular velocity is associated with a

1) <u>Rev. joint</u> : $v_{i+1} = v_i + \underline{0} + \omega_i \times {}^{0}_{i}R\, {}^{i}P_{i+1}$

$R[\vec{a} \times \vec{b}] = R\vec{a} \times R\vec{b}$

$${}^{i+1}v_{i+1} = {}^{i+1}_{0}R\, v_{i+1} = \boxed{{}^{i+1}_{i}R \left[{}^{i}v_i + {}^{i}\omega_i \times {}^{i}P_{i+1} \right]}$$

2) <u>Prism. joint</u> : $${}^{i+1}v_{i+1} = \boxed{I + \dot{d}_{i+1}\, {}^{i+1}\hat{z}_{i+1}}$$

We will apply the above eqns to a planar 2-link arm:

ill make use of the link transformations, we compute them:

$$ {}^0_1T = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad {}^0_1R $$

$$ {}^1_2T = \begin{bmatrix} c_2 & -s_2 & 0 & l_1 \\ s_2 & c_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad {}^1_2R \tag{5.49} $$

$$ {}^2_3T = \begin{bmatrix} 1 & 0 & 0 & l_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad {}^2_3R $$

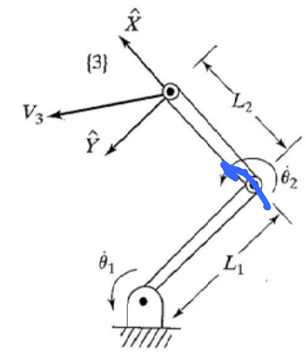


FIGURE 5.8: A two-link manipulator.

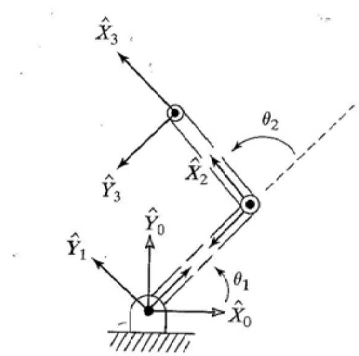


FIGURE 5.9: Frame assignments for the two-link manipulator.

Applying above eqns:

$${}^{0}v_0 = \underline{0} \qquad {}^{0}\omega_0 = \underline{0}$$

$${}^{1}\omega_1 = {}^{1}_{0}R\,{}^{0}\omega_0 + \dot{\theta}_1 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{pmatrix}$$

$${}^{1}v_1 = \underline{0}$$

$${}^{2}\omega_2 = {}^{2}_{1}R\,{}^{1}\omega_1 + \dot{\theta}_2 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 0 \\ \dot{\theta}_1 + \dot{\theta}_2 \end{pmatrix}$$

similarly continuing

$$ {}^2v_2 = {}^2_1R \left(\cancelto{0}{{}^1v_1} + {}^1\omega_1 \times {}^1P_2 \right)$$

$$= {}^2_1R \begin{pmatrix} 0 \\ \ell_1 \dot{\theta}_1 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} \ell_1 s_2 \dot{\theta}_1 \\ \ell_1 c_2 \dot{\theta}_1 \\ 0 \end{pmatrix}$$

skipping a few steps $\;{}^{3}V_3 \ldots$

${}^{3}\omega_3 \ldots$

$$ {}^{3}v_3 = \begin{pmatrix} l_1 s_2 \dot{\theta}_1 \\ (l_1 c_2 + l_3)\dot{\theta}_1 + l_3 \dot{\theta}_2 \\ 0 \end{pmatrix} $$

$$ {}^{3}\omega_3 = \begin{pmatrix} 0 \\ 0 \\ \dot{\theta}_1 + \dot{\theta}_2 \end{pmatrix} $$

sometimes: $v_3 = {}^0_3R\,{}^3v_3$ $\qquad {}^0_3R = {}^0_1R\,{}^1_2R\,{}^2_3R$

$$\omega_3 = {}^0_3R\,{}^3\omega_3$$

$$\begin{pmatrix} v_3 \\ \omega_3 \end{pmatrix} = \begin{pmatrix} v_{3x} \\ v_{3y} \\ \hline \omega_{3z} \end{pmatrix} = \begin{bmatrix} -l_1 s_1 \dot\theta_1 - l_2 s_{12}(\dot\theta_1+\dot\theta_2) \\ l_1 c_1 \dot\theta_1 + l_2 c_{12}(\dot\theta_1+\dot\theta_2) \\ \dot\theta_1+\dot\theta_2 \end{bmatrix}$$

$\rightarrow$ rel. is linear in $\underline{\underline{\dot\theta_1, \dot\theta_2}}$

$$\begin{pmatrix} v_{3x} \\ v_{3y} \\ \omega_{3z} \end{pmatrix} = \underbrace{\begin{pmatrix} -l_1 s_1 - l_2 s_{12} & -l_2 s_{12} \\ l_1 c_1 + l_2 c_{12} & l_2 c_{12} \\ 1 & 1 \end{pmatrix}}_{3 \times 2} \begin{pmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{pmatrix}$$

3×1 $\qquad$ 3×2 $\qquad$ 2×1

General Case

6×1 $\qquad$ $6 \times N$ $\qquad$ $\underbrace{}_{N \times 1}$

${}^{0}J$ Jacobian Matrix

1) Linear rel $\Leftrightarrow$ Jacobian Matrix

2) J is time-varying / configuration dep

3) J dep. on the frame w.r.t. which

end-eff. vel. are being expressed

Jacobians: (Math)

$$\underline{Y} = F(\underline{X}) \qquad \underline{X} = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$$

$$\begin{pmatrix} y_1 \\ \vdots \\ y_m \end{pmatrix} = \begin{pmatrix} f_1(x_1, x_2 \cdots x_n) \\ f_2(x_1, x_2 \cdots x_n) \\ \vdots \\ f_m(x_1, \cdots x_n) \end{pmatrix}$$

$$\frac{d}{dt}\underline{Y} = \dot{\underline{Y}} = \begin{pmatrix} \dot{y}_1 \\ \vdots \\ \dot{y}_m \end{pmatrix} = \begin{matrix} \frac{\partial f_1}{\partial x_1}\dot{x}_1 + \frac{\partial f_1}{\partial x_2}\dot{x}_2 \cdots + \frac{\partial f_1}{\partial x_n}\dot{x}_n \\ \frac{\partial f_m}{\partial x_1}\dot{x}_1 + \frac{\partial f_m}{\partial x_2}\dot{x}_2 \cdots + \frac{\partial f_m}{\partial x_n}\dot{x}_n \end{matrix}$$

$$= \underbrace{\begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \vdots & & \vdots \\ \frac{\partial f_m}{\partial x_1} & \cdots & \frac{\partial f_m}{\partial x_n} \end{pmatrix}}_{m \times n} \begin{pmatrix} \dot{x}_1 \\ \vdots \\ \dot{x}_n \end{pmatrix}$$

$$S(\Omega) = \underline{\underline{\dot{R} R^T}} \qquad \underline{\dot{y}} = J_{m \times n} \, \underline{\dot{X}}$$

$$\begin{pmatrix} {}^0 v_N \\ \hline {}^0 \omega_N \end{pmatrix} \qquad {}^0 v_N = \frac{d}{dt} \, {}^0 p_N \Rightarrow$$

lin. vel. is easily derived via this

for ang. vel. → direct. calc. via symb. route. manip. is cumbersome/prob.

Geometric || DIRECT COMPUTATION FOR Robot arms

$$\begin{pmatrix} {}^{0}v_N \\ \hline {}^{0}\omega_N \end{pmatrix} = \underline{J} \begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \vdots \\ \dot{q}_N \end{pmatrix} = \begin{pmatrix} J_v \\ \hline J_\omega \end{pmatrix} \underline{\dot{q}}$$

$$= \dot{q}_1 J_1 + \dot{q}_2 J_2 \cdots \dot{q}_N J_N$$

$$\begin{pmatrix} \Delta P_{Nx} \\ \Delta P_{Ny} \\ \Delta P_{Nz} \\ \hline k_x \Delta\theta \\ k_y \Delta\theta \\ k_z \Delta\theta \end{pmatrix} = \Delta q_1 J_1 + \Delta q_2 J_2 + \cdots + \Delta q_N J_N$$

$$J_i = \begin{pmatrix} J_{iv} \\ \hline J_{i\omega} \end{pmatrix}$$

1) ANGULAR VEL:

$$^{0}\omega_N = {}^{0}\left({}^{0}\Omega_N \right)$$

$$= {}^0\Omega_1 + {}^0_1R\left({}^1\Omega_2\right) + \cdots + {}^0_iR\left({}^i\Omega_{i+1}\right)$$
$$+ \cdots {}^0_{N-1}R\left({}^{N-1}\Omega_N\right)$$

$$J = \left(\frac{J_v}{J_\omega}\right)$$

$${}^i\Omega_{i+1} = \begin{cases} 0 & \text{prism} \\ \dot{\theta}_{i+1} \; {}^{i+1}\hat{z}_{i+1} & \end{cases}$$

$$= \rho \, \dot{q}_{i+1} \; {}^{i+1}\hat{z}_{i+1}$$

$\rho = 0$ prism

$\cdot = 1$ rev

$${}^0\omega_N = \sum_{i=1}^{N} \rho \, \dot{q}_i \; {}^0_iR \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$= \sum_{i=1}^{N} \underbrace{\dot{q}_i}\; \underbrace{\rho \; {}^0_iR \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}}$$

$$\begin{pmatrix} k_x \, \Delta\theta / \Delta t \\ k_y \, \Delta\theta / \Delta t \\ k_z \, \Delta\theta / \Delta t \end{pmatrix} = \sum_{i=1}^{N} \underbrace{\frac{\Delta q_i}{\Delta t}} \; \underbrace{\rho \; {}^0_iR \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}}_{J_{\omega_i}}$$

3rd way of Calc. $\underline{\underline{J}}$ $\longrightarrow$

$$J_{i\omega} = \left(\rho \times \text{3rd col. of } {}^0_iR \right)$$

$$= \text{3rd col. of } {}^0_iR \quad \text{(Rev. Joint)}$$

$$= \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad \text{(prismatic joint)}$$