Lecture - 23 + 24

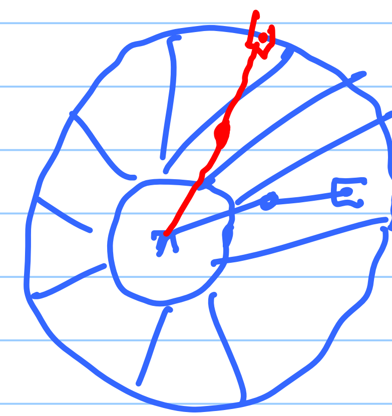
Note Title 10/25/2007

Types of singularities:

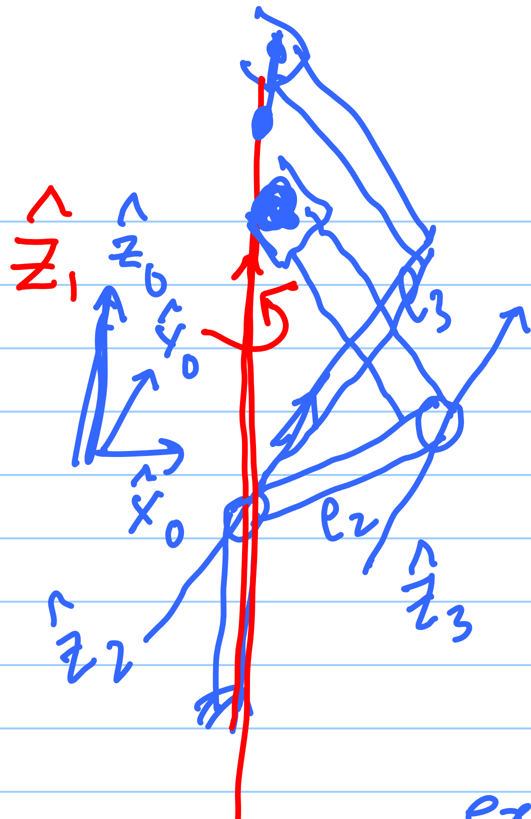
① Boundary Sing.:

easier to identify

$\theta_2 = 0, 180°$



② Interior : more diff. to char.



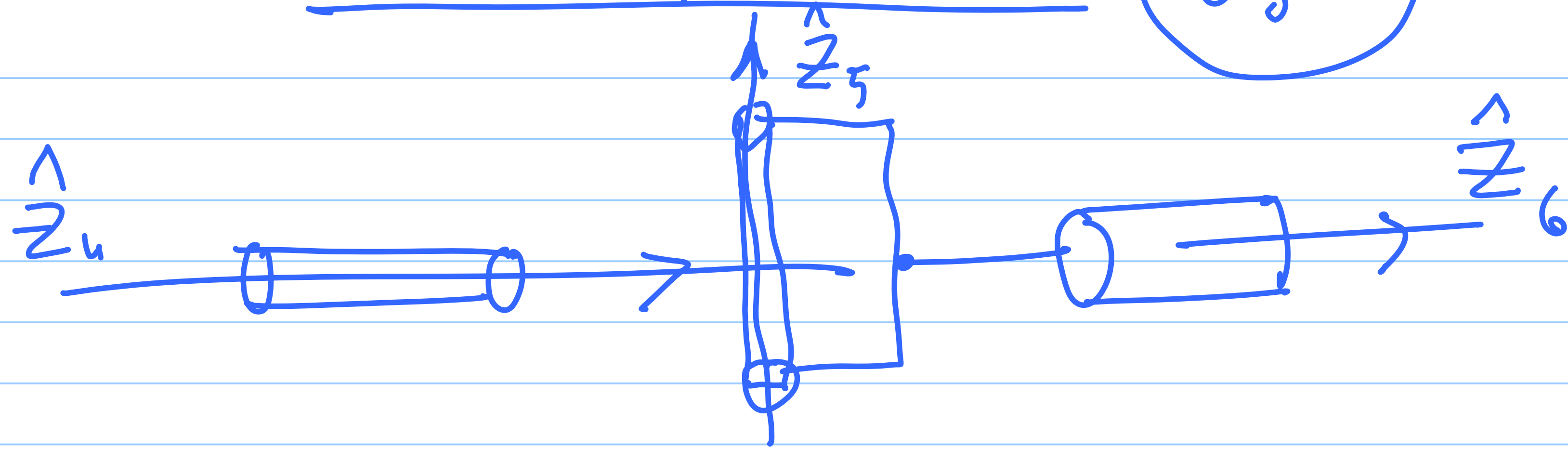
$$\underline{J(\theta_1, \theta_2, \theta_3)}$$

|| interior singularity

$$\begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} = J_1 \dot{\theta}_1 + J_2 \dot{\theta}_2 + J_3 \dot{\theta}_3$$

ex: derive J and compute exp. for sing. config.

Wrist of Puma: $\theta_5 = 0 \Rightarrow$ sing.



$\theta_3 = 0 \Longrightarrow$ Bound. sing.

We saw earlier: J for Puma has following structure

$$J = \left(\begin{array}{c|c} J_{11} & 0 \\ \hline J_{21} & J_{22} \end{array}\right)$$

$$\det(J) = \underbrace{\underline{\underline{\det(J_{11})}}}_{\underline{\text{first 3 joints}}} \times \underbrace{\underline{\det(J_{22})}}_{\underline{\underline{\text{last 3 joints}}}}$$

|| Sing. are therefore decoupled.

Following material is not in text.

Extension of "singularity" notion

What happens near singular configs.

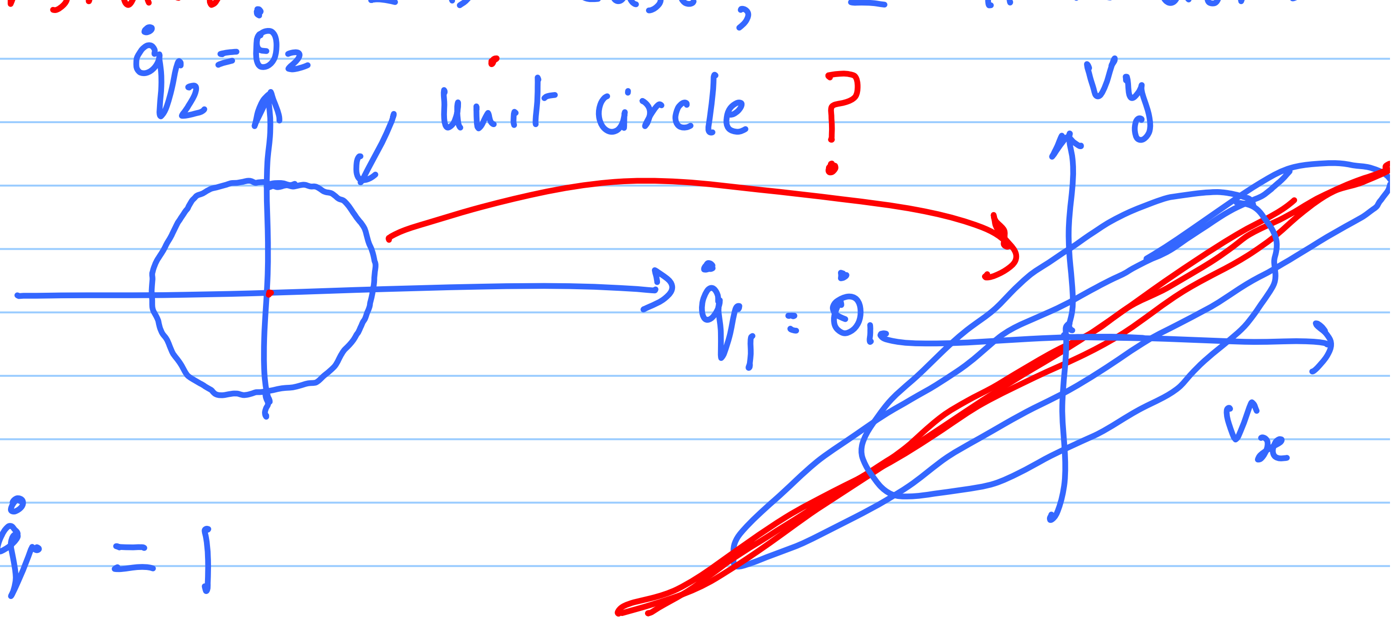
We need a "continuous" measure of how "good" a given config. is.

from the point of view of

achieving vel. at end-effector.

"manipulability":

Consider 2-D case, 2-link arm.



$|\dot{q}| = 1$

$\dot{q} = J^{-1} v$

$\dot{q}^T \dot{q} = 1$

$$\left(J^{-1}v\right)^T\left(J^{-1}v\right)=1$$

$$J=\begin{pmatrix} j_{11} & j_{12} \\ j_{21} & j_{22} \end{pmatrix}$$

$$\boxed{v^T\left(JJ^T\right)^{-1}v=1}$$

$$JJ^T=\begin{pmatrix} x & x \\ x & x \end{pmatrix}$$

$$\left(JJ^T\right)^{-1}=\begin{pmatrix} a & b \\ b & d \end{pmatrix}$$

$$\begin{pmatrix} v_x & v_y \end{pmatrix}\begin{pmatrix} a & b \\ b & d \end{pmatrix}\begin{pmatrix} v_x \\ v_y \end{pmatrix}=1$$

$$\boxed{a v_x^2 + 2b v_x v_y + d v_y^2 = 1}$$

manipulability = vol. of "ellipsoid" in end-eff. vel. space

$$= \sqrt{\det(J J^T)}$$

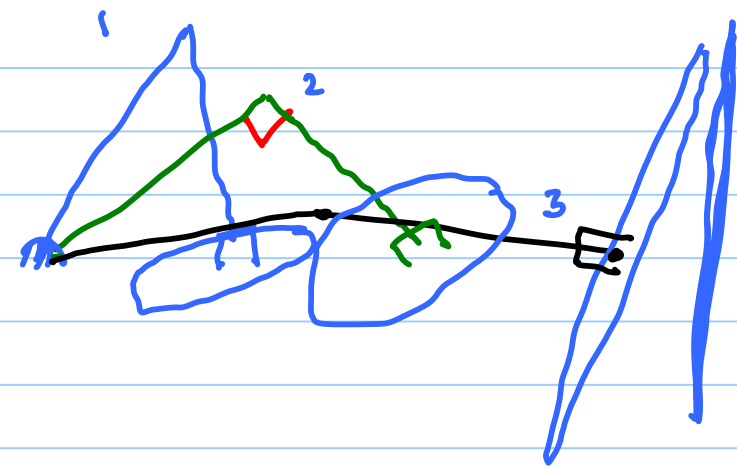
if J is square $= \det(J)^2$

at sing config. $\det(J) = 0$

example: 2-link arm

$$\boxed{\det(J) = \ell_1 \ell_2 s_2}$$

max at $\theta_2 = 90^\circ$



$$ {}^{0}v = {}^{0}J(\dot{q}) $$

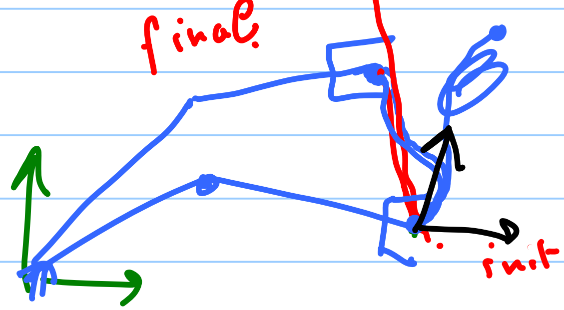
$$ {}^{N}v = {}^{N}J(\dot{q}) $$

J dep. on the frame w.r.t. which it is computed.

sing. are ind. of frame " " J is compute

|| Skip this year || Jacobians are also related to static forces

Next few lectures: Trajectory generation



1) Cartesian space Traj planning

traj. is specified in Cartesian parameter

2) joint space

traj:

$$\begin{pmatrix} \theta_1(t) \\ \theta_2(t) \end{pmatrix}$$ specify + generate

→ no sing. +

→ traj. in Cart. space may not be pretty

$(x, y, \phi)(t)$

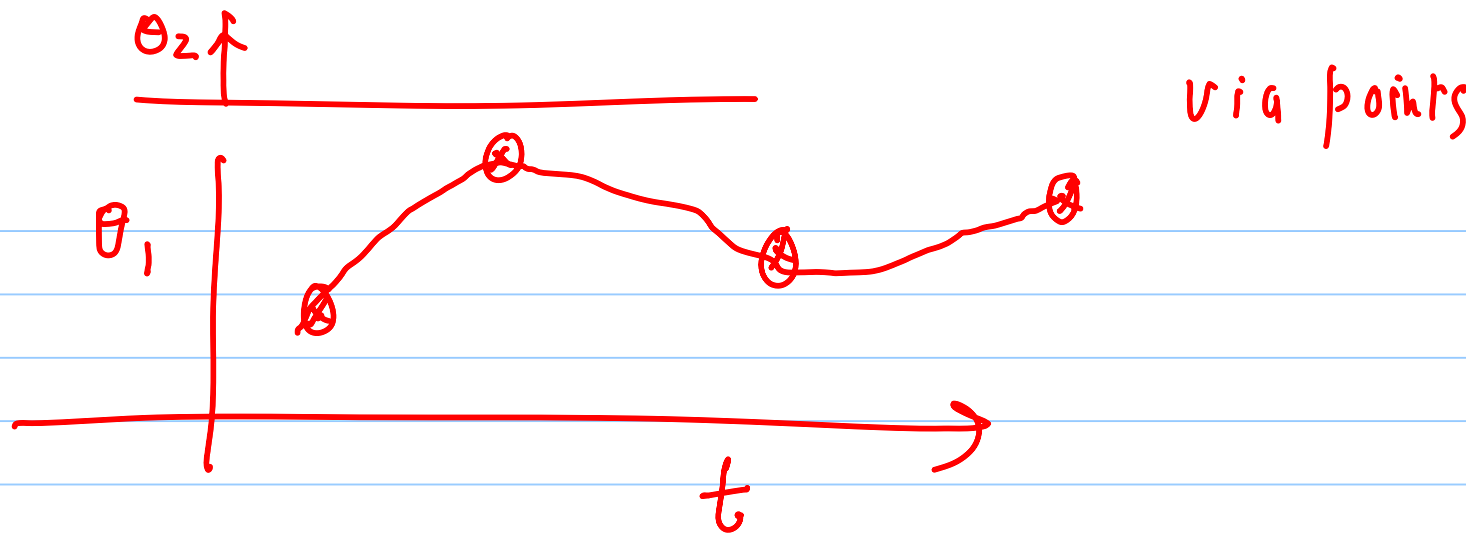
$$\begin{cases} x(t) \\ y(t) \\ \phi(t) \end{cases}$$ ← specify, generate

dis, adv

① intuitive, easier to visualize

② task demands if

③ singularities may occur. → will need to xferred to joint space


θ2
θ1
t
via points