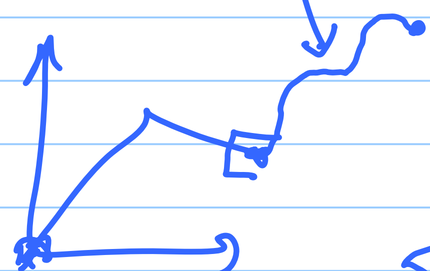
Lecture 24

|| We will come back to "dynamics" (ch. 6) after traj. planning (ch. 7)

Trajectory Planning:

motion or trajectory



How to compute the desired "trajectory" along which the robot moves?

1) specify (user interface): How will an operator/user specify the desired motion?

2) internal representation of the entire motion // ← functions
(in the computer)

3) "generate the actual trajectory"
"Sampling" the internal rep. at appropriate sampling rates.
20-50 Hz rates

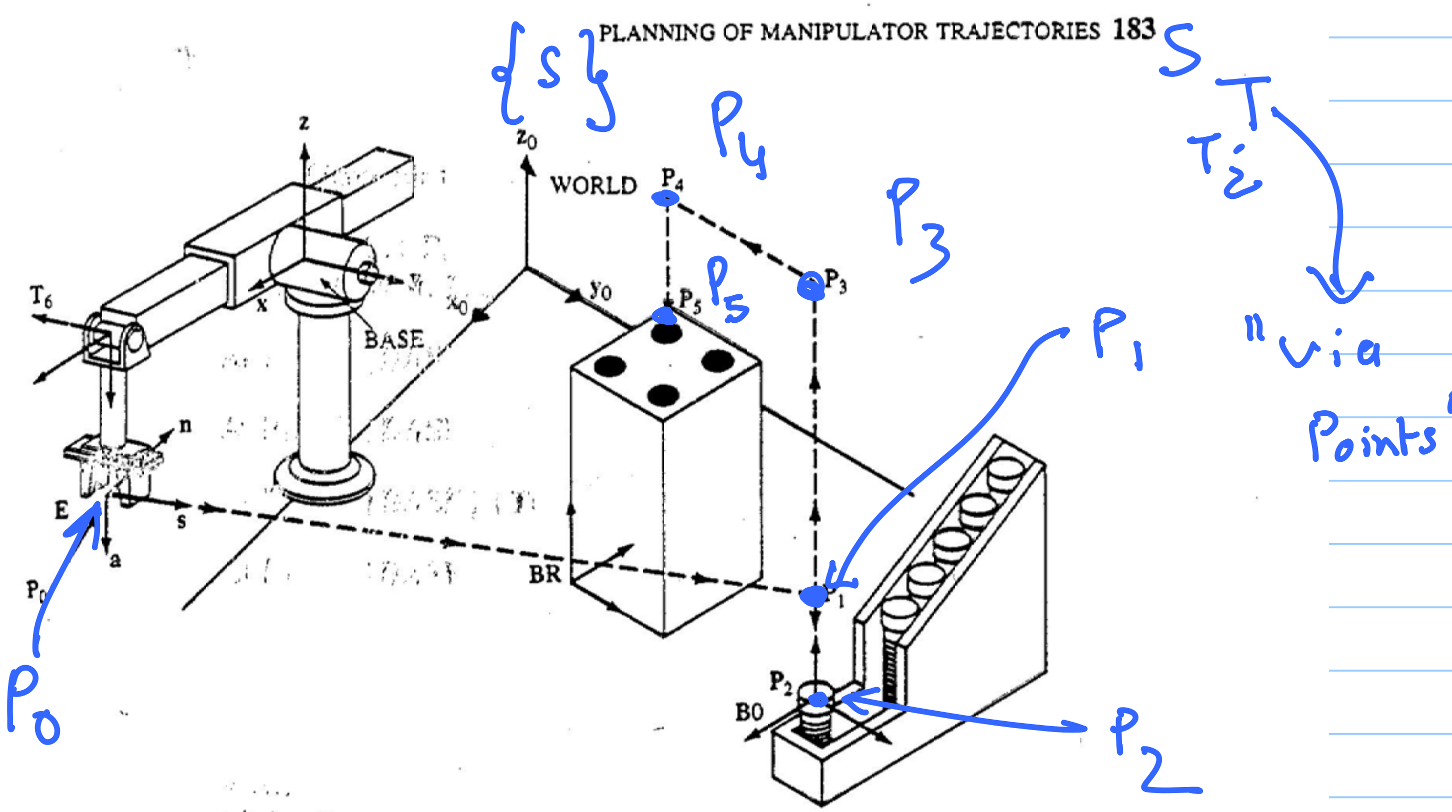


Figure 4.6 Figure for the example.

Traj. specification: in terms of intermediate frames ${}^{S}_{T_i}T$, called via points. start point, goal point

problematic
↓ Cartesian space $\longrightarrow$ trajectory plann.

1. motion in between via points is a basic primitive: such as a straight line

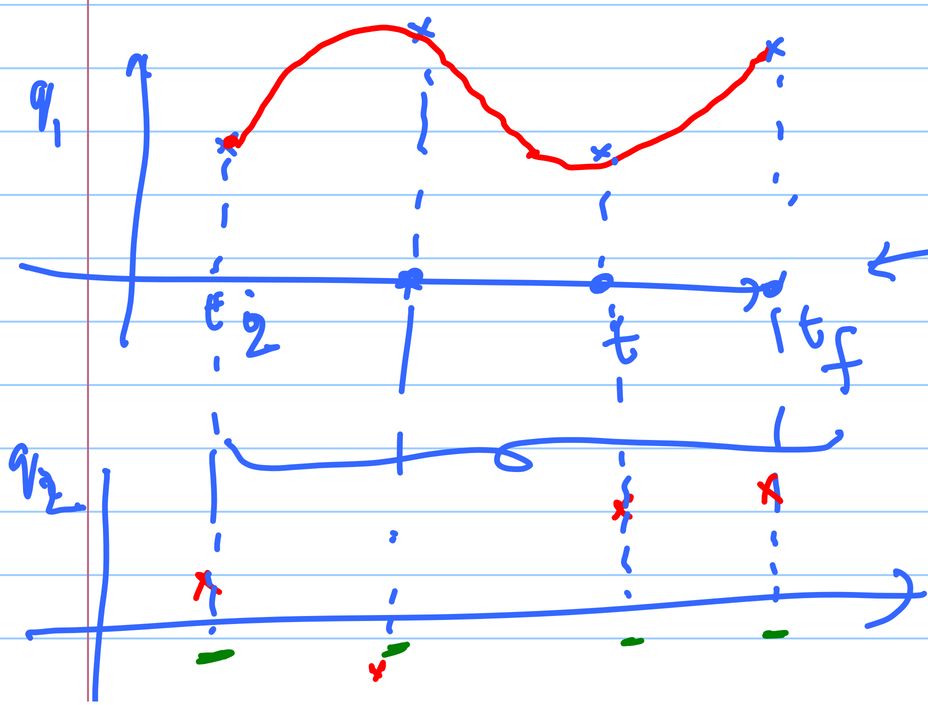
↓ easier
joint space $\longrightarrow$ trajectory planning.

2. no constraints on motion in between.

$${}^{S}T_{T_2} \xrightarrow[\text{or "SOLVE"}]{\text{Inv. KIN}} \begin{pmatrix} q_{i1} \\ q_{i2} \\ \vdots \\ q_{in} \end{pmatrix} = \underline{q_2}$$

trajectory: → a time function of joint variable

$\underline{q}(t) : t = (t_i, t_f)$



each segment is assigned a time interval.

∘ smooth function to interpolate between these points"

Smoothness Constraints: Continuous velocity

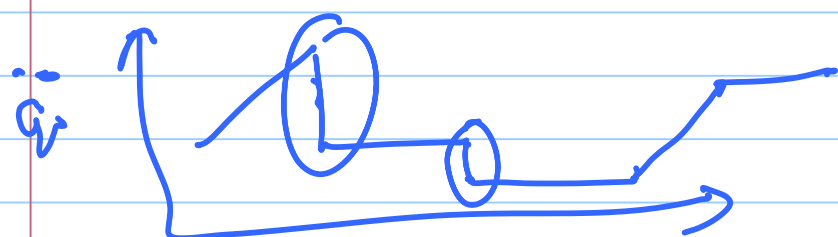
C^1 ✓

A function in C^k : k^{th} derivative is continuous

Continuous acceleration

✓ C^2

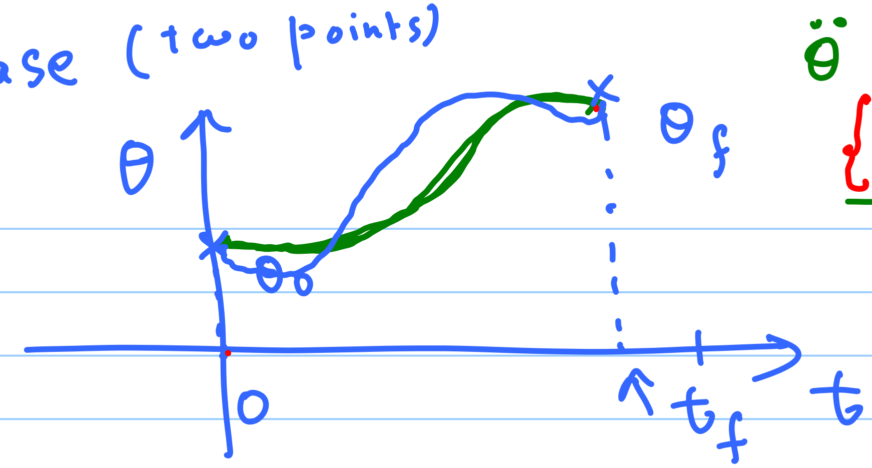
C^0, C^1, C^2 . . . - - $\boxed{C^\infty}$

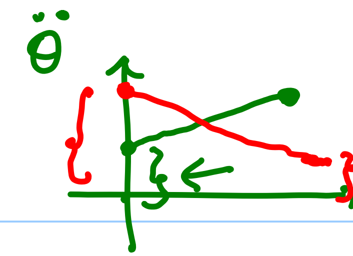


C^1 or C^2 ← more desirable

Traj planning: C^2 or (C^1) function interpolating via points.

1. Simple Case (two points)





Constraints :

pos. — $\theta(t)\big|_{t=0} = \theta_0$

$\theta(t)\big|_{t=t_f} = \theta_f$

choose a "class" of functions to "interpolate"

↳ [polynomials of degree "n"]

vel. $\dot{\theta}(t)\big|_{t=0} = 0$

$\dot{\theta}(t)\big|_{t=t_f} = 0$

$$\theta(t) = \sum_{i=0}^{n} a_i\, t^i = a_0 + a_1 t + a_2 t^2 + \dots$$

⇓

4 Constraint

||

constraint : smoothness, interpolation

$\Downarrow$

fit a cubic

\# of constraints = # of co-effs to be determined.

= n+1 for an n-deg polynomial

$$\theta(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

$$\begin{cases} \theta(t)\big|_{t=0} = \theta_0 = a_0 \\ \dot{\theta}(t)\big|_{t=0} = 0 = a_1 \\ \theta(t)\big|_{t=t_f} = \theta_f = a_0 + a_1 t_f + a_2 t_f^2 + a_3 t_f^3 \\ \dot{\theta}(t)\big|_{t=t_f} = 0 = a_1 + 2a_2 t_f + 3a_3 t_f^2 \end{cases}$$

4 eqns. in 4 unknowns (co-effs):
(linear in co-effs a_i) Solve, and we get

$$\left\{\begin{aligned} a_0 &= \theta_0 \\ a_1 &= 0 \\ a_2 &= \frac{3}{t_f^2}(\theta_f - \theta_0) \\ a_3 &= -\frac{2}{t_f^3}(\theta_f - \theta_0) \end{aligned}\right.$$

if we reqd: cont. of acc. (C^2) :

$$\Longrightarrow \begin{cases} \ddot{\theta}\big|_{t=0} = 0 \\ \ddot{\theta}\big|_{t=t_f} = 0 \end{cases}$$

2 more constraints

total # of const. = 6 $\Longrightarrow$ <u>fifth deg. poly</u>

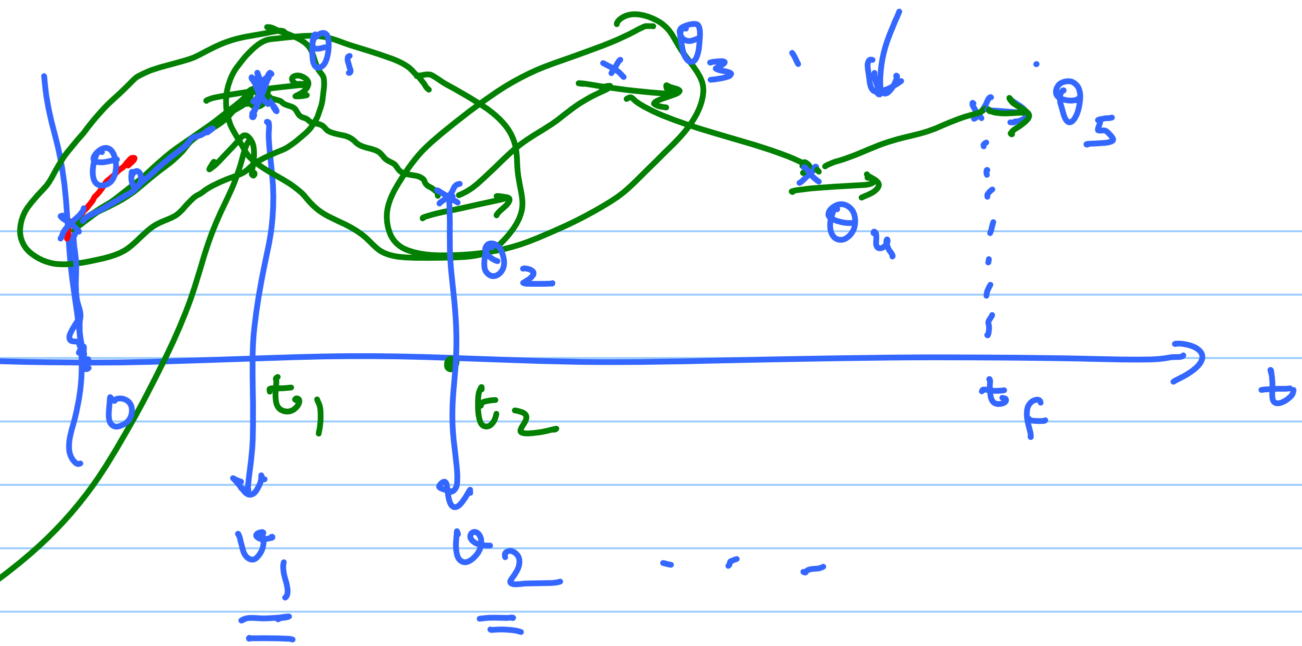
high deg. polys $\Longrightarrow$ more smoothness
but more
"extraneous"
motion of the
joint.

<u>general case :</u>

$\backslash\backslash$ 7^{th} order poly $\Longleftarrow$ 8 const. $\cancel{\Longrightarrow}$ $\left\{ \begin{array}{l} \text{pos. consts.} : 6 \\ \text{start vel.} = 0 : 1 \\ \text{final vel} = 0 : 1 \end{array} \right.$ / <u>six points:</u> fit one single poly

$i \rightarrow$ ith via point

be careful of ambiguity with joint number



how to specify v_i's :

(1) user specifies

(2) "automatic way of choosing it"

v_i = average of the two slopes at the via point i

$$\frac{\dfrac{\theta_1-\theta_0}{t_1} + \dfrac{\theta_2-\theta_1}{t_2-t_1}}{2}$$

fit pairwise cubic

3 $\begin{cases} \text{vel. is continuous} \leftarrow \text{"}n-1\text{" cons} \\ \text{acc. is continuous} \leftarrow \text{"}n-1\text{" const.} \end{cases}$

"piecewise C^2 functions"

"splines" or spline functions