Note Title 11/6/2007

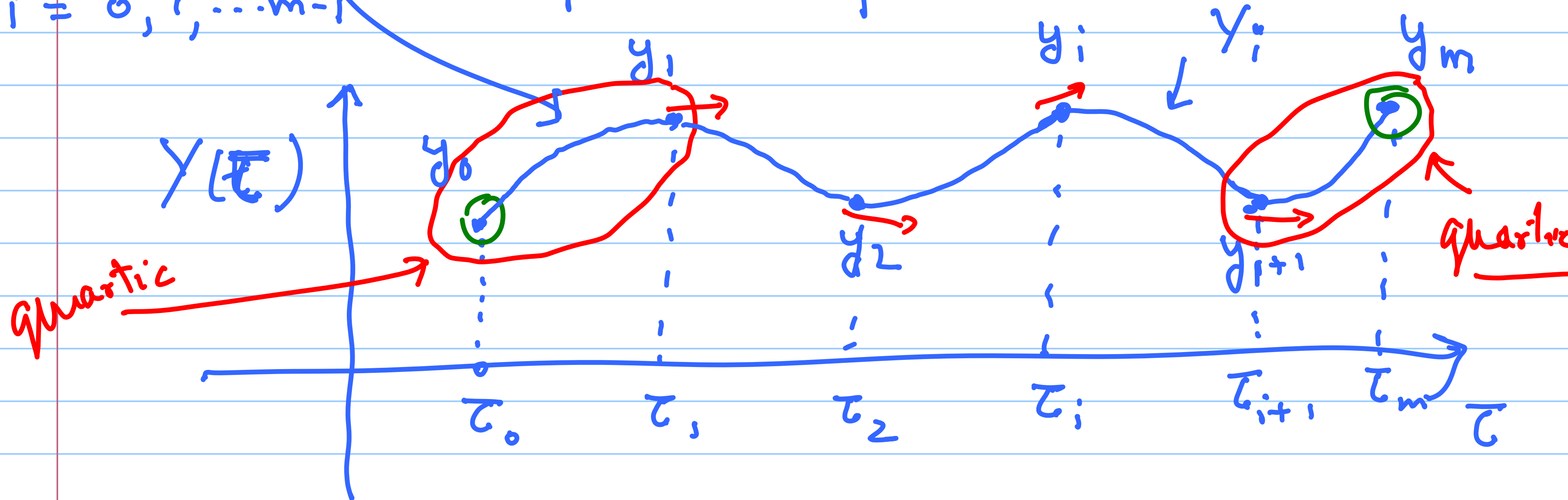
Lecture-28

Not in text

* "Continuity of vel. + acc. at interior points"

$Y_i(\tau)$ "spline formulation"

$i = 0, 1, \ldots m-1$



normalized time $\longrightarrow$ $$t = \frac{\tau - \tau_i}{\tau_{i+1} - \tau_i}$$

$i = 0, \cdots m-1$ $$Y_i = a_i + b_i t + c_i t^2 + d_i t^3$$

$$\frac{dY_i}{dt} = Y_i' \qquad \frac{dY_i}{d\tau} = \dot{Y}_i$$ $\leftarrow$ vel., acc.

$$\dot{Y}_i = \frac{Y_i'}{\tau_{i+1} - \tau_i} = \frac{Y_i'}{h_i} \qquad h_i = \tau_{i+1} - \tau_i$$

$$\ddot{Y}_i = \frac{Y_i''}{h_i^2}$$

Constraints:

1) Position: $Y_i(0) = \boxed{y_i = a_i}$ 1

$Y_i(1) = \boxed{y_{i+1} = a_i + b_i + c_i + d_i}$ 2

2) velocity: $\dot{Y}_i(0) = v_i = \frac{b_i}{h_i}$

$\dot{Y}_i(1) = v_{i+1} = \frac{b_i + 2c_i + 3d_i}{h_i}$

$$\Longleftrightarrow \quad v_i h_i = b_i \qquad (3)$$

$$v_{i+1} h_i = b_i + 2c_i + 3d_i \qquad (4)$$

solve for a_i, b_i, c_i, d_i from (1) − (4):

$y_{i+1} - y_i = \Delta_i$,

we get:

$$(5) \quad \begin{cases} a_i = y_i \\ b_i = v_i h_i \\ c_i = 3\Delta_i - 2v_i h_i - v_{i+1} h_i \end{cases}$$

$$\Big\lfloor\; d_i = -2\Delta_i + v_i h_i + v_{i+1} h_i$$

we will now add "acceleration continuity" constraints

$$\Longrightarrow \quad \ddot{y}_i(1) = \ddot{y}_{i+1}(0)$$

$i = 0, \ldots, m-2$

$$\frac{2c_i + 6d_i}{h_i^2} = \frac{2c_{i+1}}{h_{i+1}^2}$$

sub. from 5 into above eqn.,

we get :

$i = 0, \ldots m-2$

$$\frac{2}{h_i} v_i + \left(\frac{4}{h_i} + \frac{4}{h_{i+1}} \right) v_{i+1} + \frac{2}{h_{i+1}} v_{i+2} = 6 \left[\frac{\Delta_i}{h_i^2} + \frac{\Delta_{i+1}}{h_{i+1}^2} \right]$$

<u>end conditions:</u> start, and final vel $= 0$

$$\left. \begin{aligned} v_0 &= 0 \\ v_m &= 0 \end{aligned} \right\}$$

$\Rightarrow$ we can solve for v_i's : $i = 0, \ldots m$

having solved for v_i, use (5)

to determine a_i, b_i, c_i, d_i

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this formulation (cubic segments) will result in non-zero acc. ~~at~~ start, and end points.

for zero start/end acc., use quartics as the first/last seg. || Exercise to carry this out

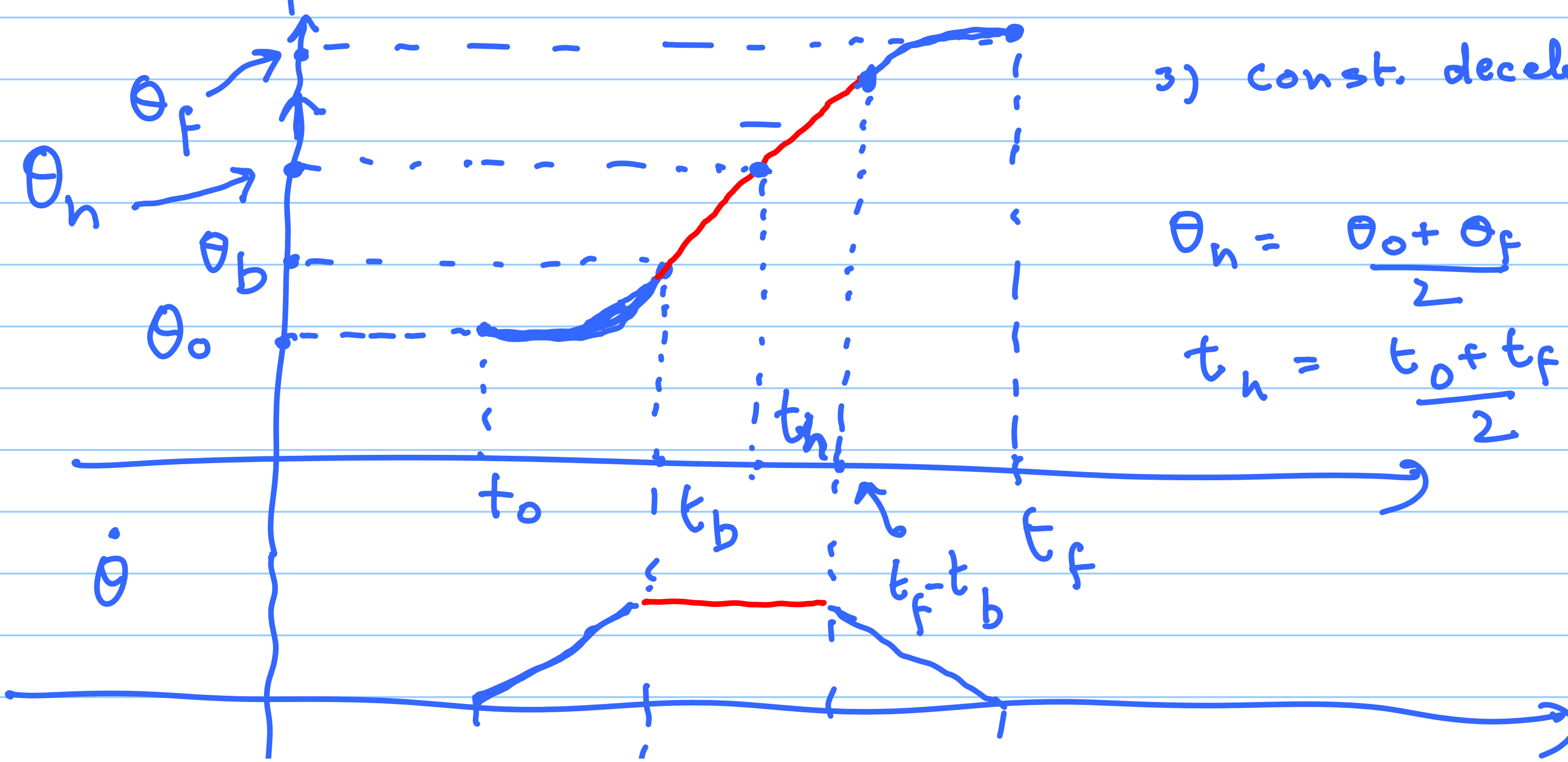
"Simpler way" → but "discontinuous acceleration!"

"parabolic blends":

1) const. acc.

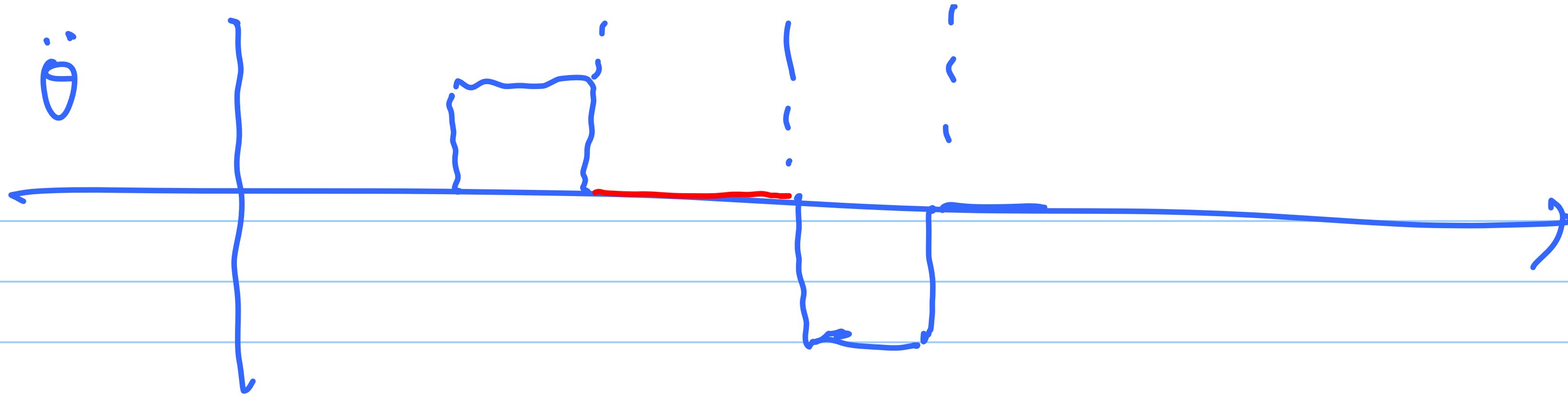
2) const. vel. (zero acc)

3) const. deceleration



$$\theta_h = \frac{\theta_0 + \theta_f}{2}$$

$$t_h = \frac{t_0 + t_f}{2}$$



Eqns for parabolic blending:

2 point Case:

Two choices:

① specify t_b

or

② acc. $\ddot{\theta}$ during blending

$\ddot{\theta}$ is specified: (known)

assume $t_0 = 0$

1 $$\ddot{\theta}\, t_b = \frac{\theta_h - \theta_b}{t_h - t_b}$$

2 $$\theta_b = \theta_0 + \frac{1}{2} \ddot{\theta}\, t_b^2$$

Comb. 1 + 2

$$\ddot{\theta} t_b^2 - 2\ddot{\theta} t_h t_b + (\theta_f - \theta_0) = 0$$

since $t_0 = 0$, $t_h = \frac{t_f}{2}$

mistake in lecture.

$$\Leftrightarrow \ddot{\theta} t_b^2 - \cancel{2}\,\ddot{\theta} t_f \cdot t_b + (\theta_f - \theta_0) = 0$$

$$t_b = \frac{t_f}{2} - \frac{\sqrt{\ddot{\theta}^2 t_f^2 - 4\ddot{\theta}(\theta_f - \theta_0)}}{2\ddot{\theta}}$$

Note: for soln. to exist,

$$\ddot{\theta} \geqslant \frac{4(\theta_f - \theta_0)}{t_f^2}$$

Note: This assumes $\ddot{\theta}$ is +ve

$(\theta_f > \theta_0)$

similar situation if $\ddot{\theta}$ is -ve

$(\theta_f < \theta_0)$.

For multiple via points,
essentially repeat the above

procedure! see page 212–216.

I expect you to cover it yourself.