Note Title 11/8/2007

# Lecture - 29+30

We know how to interpolate a set of via points via smooth cubic/parabolic functions.

Traj. planning :

(1) Joint - space

(2) Cartesian space

JOINT SPACE TRAJ PLANNER

set of via points $^{0}_{N}T_{i}$ → Inv. KIN → set of via points $\underline{q}_i = \begin{pmatrix} q_{1i} \\ \vdots \\ q_{Ni} \end{pmatrix}$ → Spline interpolation → Sample

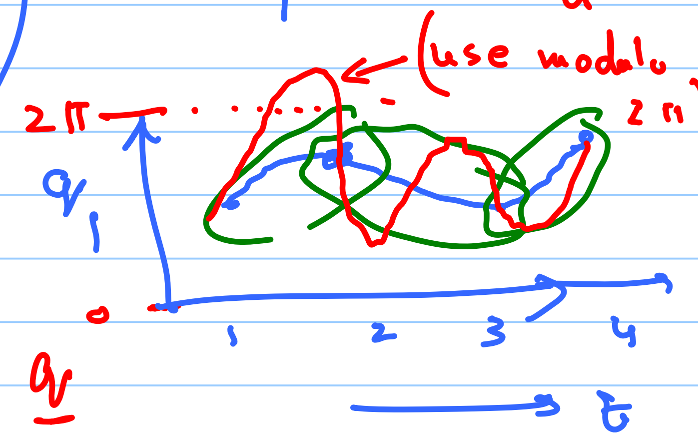
↑ internal rep.

10–30 msec ↓ $\underline{\dot{q}}_d$

① Simple to implement

② no idea of how the end-eff. would move

$\underline{q}_d$, $\underline{\dot{q}}_d$ → robot + controller → $\underline{q}$



## Cartesian space Traj planning:

$\longrightarrow$ interpolation in Cartesian space $\longrightarrow$ sample $\xrightarrow{X_d}$ INV. KIN $\longrightarrow \underline{q}_d$

splines, in Cartesian parameters

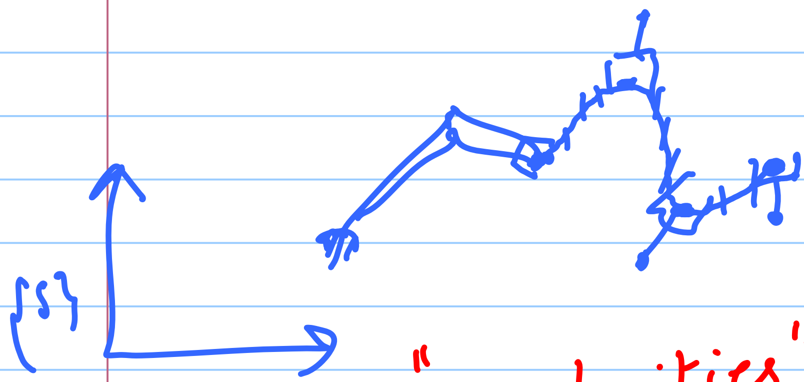
$\dot{\underline{q}}_d$, $\underline{q}_d$

robot control

${}^0_N T_i \rightarrow {}^0P_{N_i}$, ${}^0_N R_i \xrightarrow{\text{Eq. angle-axis}} (\hat{K}_i\ \theta_i)$

${}^0P_{N_i}$: 3 para. $\quad (\hat{K}_i\ \theta_i)$: 3 para.

$X = \begin{pmatrix} p_x \\ p_y \\ p_z \\ k_x\theta \\ k_y\theta \\ k_z\theta \end{pmatrix}$




$\begin{pmatrix} x \\ y \\ \phi \end{pmatrix}$

$\dot{q} = J^{-1}\dot{x}$

$\ddot{q} = \dot{J}^{-1}\dot{x} + J^{-1}\ddot{x}$

"singularities" → problem in Cartesian sp. Traj planners

→ "trap" singularity situations
"detect and stop"

Problems:

① type I: path may go through unreachable workspace.

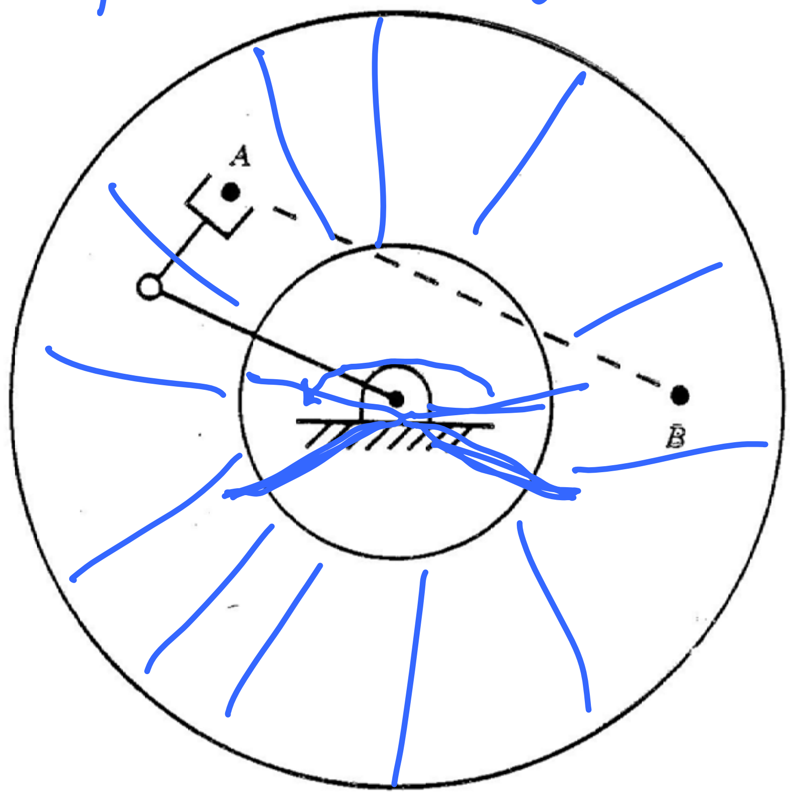



FIGURE 7.13 Cartesian path problem type 1

Problems of type 1: intermediate points unreachable

Can be detected in advance, provided you can charac. the robot workspace.

example 5.5, page 153

$$\dot{\theta}_1 = \frac{c_{12}}{l_1 s_2} \leftarrow \uparrow \infty \text{ as } \theta_2 \to \pi$$

$$\dot{\theta}_2 = \frac{-c_1}{l_2 s_2} = \frac{c_{12}}{l_1 s_2}$$

$l_1 \neq l_2$

type 2

singularities

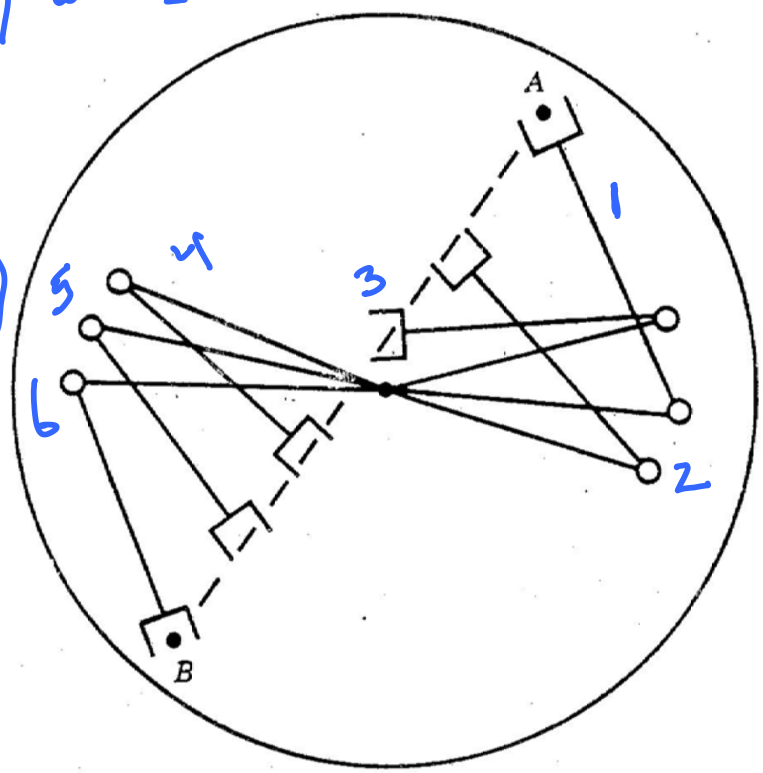



FIGURE 7.14 Cartesian path problem type 2

**Type 3:** "joint limits" : may divide your robot workspace into disconnected regions.



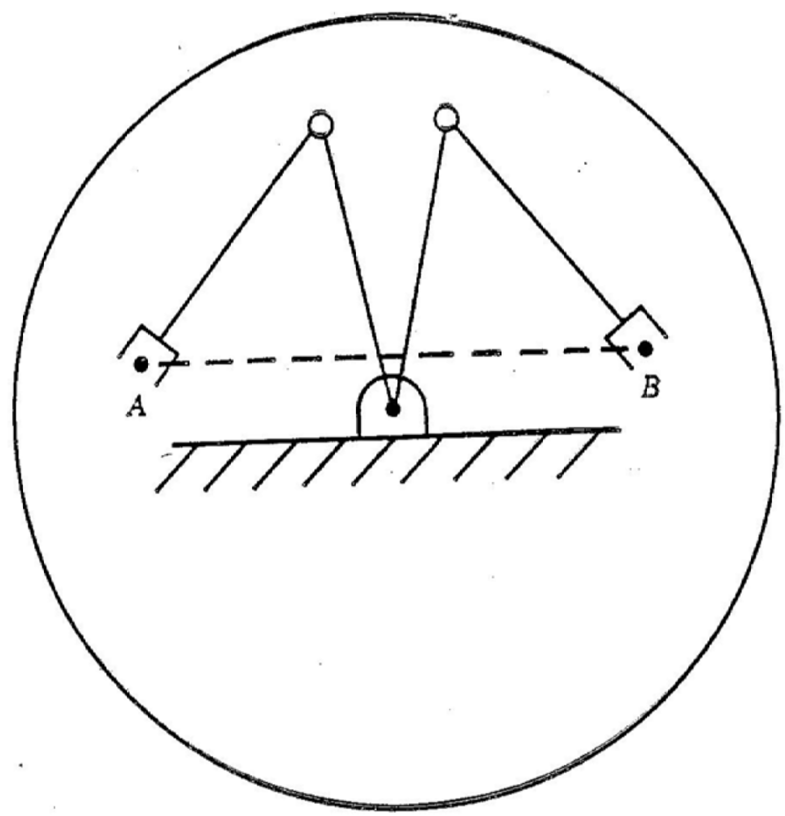



FIGURE 7.15 Cartesian path problem type 3

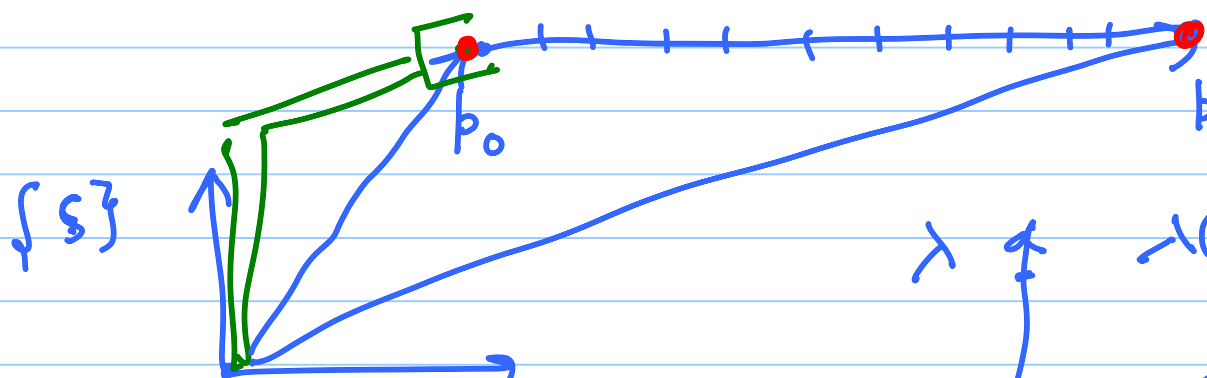
NOT IN TEXT

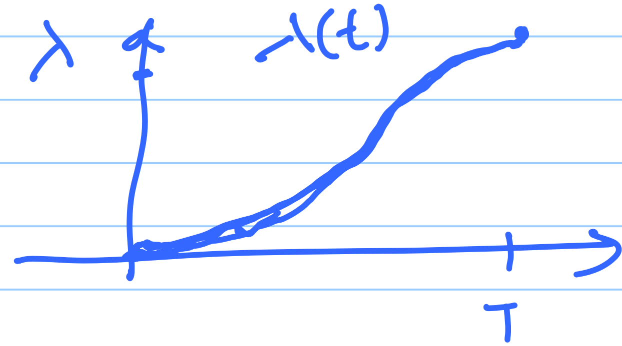
## Cartesian space traj planning :

follow a prescribed path in Cartesian space.

$\lambda \in [0,1]$

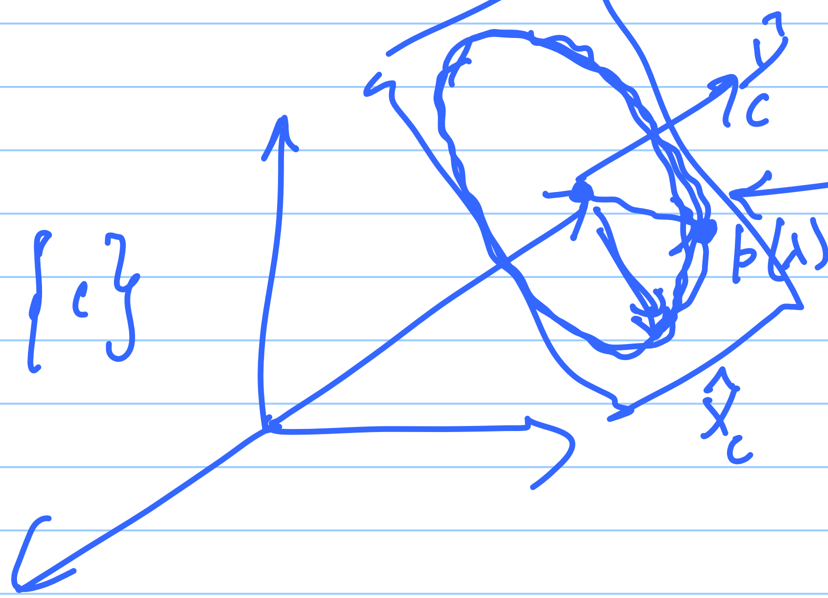
$$p(\lambda) = p_0 + \lambda (p_1 - p_0)$$




spline

$p(\lambda(t))$ : $p(t)$




paramet. circle w.r.t. $\lambda$

$$ {}^{c}p(\lambda) = \begin{bmatrix} r\cos(\lambda/r) \\ r\sin(\lambda/r) \\ 0 \end{bmatrix} $$

Rotations: Smooth rotation around a "fixed" axis.

$R_0 \; (t=0)$ $\quad$ $\underline{\underline{R_1}} \; (t=t_f)$

$$\underline{\underline{R(t)}} = R_0 \cdot \underline{\underline{R_D(t)}}$$

Drive Rotation

$$R(t)\Big|_{t=0} = R_0$$

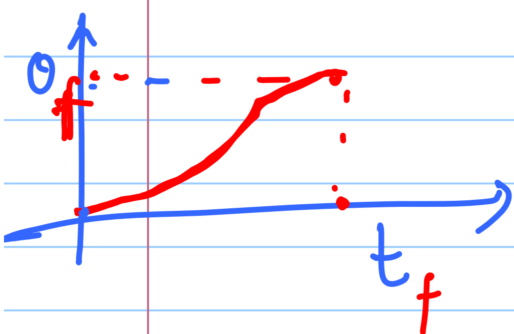
$$R(t)\Big|_{t=t_f} = R_1$$

$$R_1 = R_0 \, R_D(t_f)$$

$$\Rightarrow \quad R_D(t_f) = \boxed{R_0^T R_1}$$

$$\downarrow$$

$$R_{\hat{k}}(\theta) \longrightarrow \text{gives me } \hat{k}, \; \theta_f$$



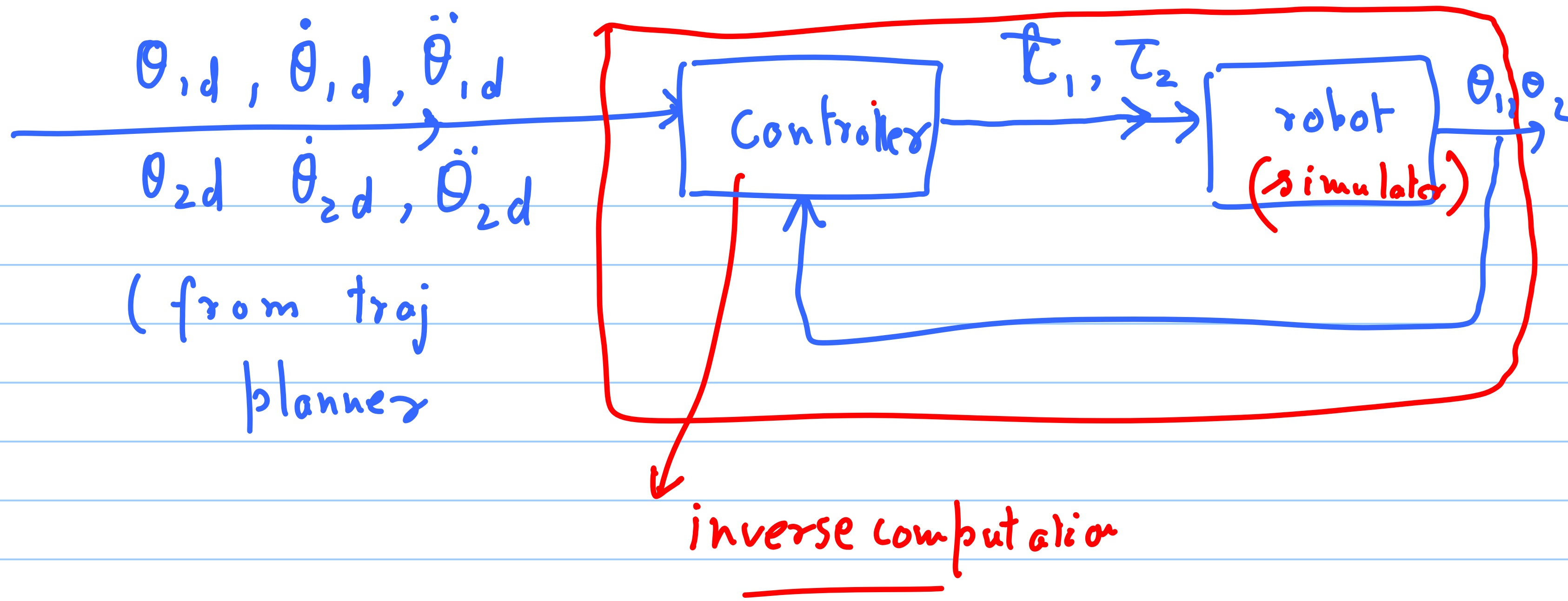

$$R_D(t=0) = I$$

$$R_D(t) = R_{\hat{k}}\left[\theta_f(t)\right]$$

# Dynamics

$$\underline{\tau} \xrightarrow{\text{torques}} \boxed{\text{robot}} \longrightarrow \underline{\theta}, \dot{\underline{\theta}}, \ddot{\underline{\theta}} \qquad \underline{\theta}(t)$$

$$\begin{tikzcd}
\underline{\tau} \arrow[r, bend left, "\text{forward}"] & \underline{\theta}, \dot{\underline{\theta}}, \ddot{\underline{\theta}} \arrow[l, bend left, "\text{inverse}"]
\end{tikzcd}$$


$\theta_{1d}, \dot{\theta}_{1d}, \ddot{\theta}_{1d}$
$\theta_{2d}, \dot{\theta}_{2d}, \ddot{\theta}_{2d}$
(from traj planner
Controller
$\tau_1, \tau_2$
robot
(simulator)
$\theta_1, \theta_2$
inverse computation

# Dynamic Eqns. for a manipulator

1. Rigid body in 3-D

2. apply it to robot, iteratively

1a. velocities, acc. across diff. frames.

1b. forces/torques for a rigid body

(Newton's 2nd law: general)

Newton - Euler Eqns.

1a : Vel & acc. across frames:

$$ {}^{A}Q = {}^{A}P_{BORG} + {}^{A}_{B}R \, {}^{B}Q $$

$$ = {}^{A}\dot{P}_{BORG} + {}^{A}_{B}R \, {}^{B}\dot{Q} + {}^{A}_{B}\dot{R} \, {}^{B}Q $$

$$ {}^{A}V_{Q} = \frac{d}{dt}\left[ {}^{A}Q \right] = {}^{A}V_{BORG} + \underbrace{{}^{A}_{B}R \, {}^{B}V_{Q}}_{\mathrm{I}} $$

$$ + \ {}^{A}\Omega_{B} \times \left( {}^{A}_{B}R \, {}^{B}Q \right) $$

$\mathrm{II} \rightarrow {}^{A}\Omega_{B} \times \left( {}^{A}_{B}R \, {}^{B}Q \right)$

$$ {}^{A}\dot{V}_{Q} = {}^{A}\dot{V}_{BORG} + \frac{d}{dt}(\mathrm{I}) + \frac{d}{dt}(\mathrm{II}) $$

$$ \frac{d}{dt}\left[ {}^{A}_{B}R\, {}^{B}V_{Q} \right] = {}^{A}_{\underline{\underline{B}}}\dot{R}\, {}^{B}V_{Q} + {}^{A}_{B}R\, {}^{B}\dot{V}_{Q} $$

$$ = \boxed{ {}^{A}\Omega_{B} \times \left( {}^{A}_{B}R\, {}^{B}V_{Q} \right) + {}^{A}_{B}R\, {}^{B}\dot{V}_{Q} } $$

$$ \frac{d}{dt}\left[ {}^{A}\Omega_{B} \times \left( {}^{A}_{B}R\, {}^{B}Q \right) \right] = {}^{A}\dot{\Omega}_{B} \times \left( {}^{A}_{B}R\, {}^{B}Q \right) $$

$$ {}^{A}\Omega_{B} \times \left[ \frac{d}{dt} \left( {}^{A}_{B}R\, {}^{B}Q \right) \right] $$

$$= \boxed{{}^{A}\dot{\Omega}_{B} \times \left({}^{A}_{B}R\,{}^{B}Q\right) + {}^{A}\Omega_{B} \times \left[{}^{A}\Omega_{B} \times {}^{A}_{B}R\,{}^{B}Q + {}^{A}_{B}R\,{}^{B}V_{Q}\right]}$$

<u>Collating all terms:</u>

$$\underline{\underline{{}^{A}\dot{V}_{Q}}} = \underline{\underline{{}^{A}\dot{V}_{BORG}}} + \underbrace{2\,{}^{A}\Omega_{B} \times \left({}^{A}_{B}R\,{}^{B}V_{Q}\right)}_{\text{Coriolis}} + \underline{{}^{A}\dot{\Omega}_{B} \times {}^{A}_{B}R\,{}^{B}Q} + \underbrace{{}^{A}\Omega_{B} \times \left({}^{A}\Omega_{B} \times {}^{A}_{B}R\,{}^{B}Q\right)}_{\text{Centripetal}}$$

lin. acc. of orig. of {B} rel

ang. acc. of ~~{B} rel to {A}~~

to {A}

$$+ {}^{A}_{B}R \, {}^{B}\dot{v}_{Q} \quad // \leftarrow$$ lin. acc. of Q in {B}

2) Ang. Acc:

$${}^{A}\Omega_{C} = {}^{A}\Omega_{B} + \underbrace{{}^{A}_{B}R \, {}^{B}\Omega_{C}}_{\text{co-ord xfer.}}$$

$${}^{A}\dot{\Omega}_{C} = {}^{A}\dot{\Omega}_{B} + {}^{A}_{B}\dot{R} \, {}^{B}\Omega_{C} + {}^{A}_{B}R \, {}^{B}\dot{\Omega}_{C}$$

$$= \underline{\underline{{}^{A}\dot{\Omega}_{B}}} + \underline{\underline{{}^{A}\Omega_{B} \times \left({}^{A}_{B}R \, {}^{B}\Omega_{C}\right)}} + {}^{A}_{B}R \, \underline{\underline{{}^{B}\dot{\Omega}_{C}}}$$

$\uparrow$ ang. acc.

due to "interaction"

of two ang. vel.