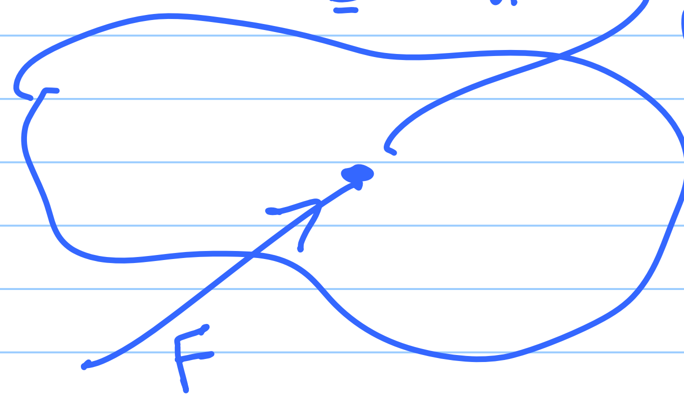
Lecture - 31

Note Title — 11/13/2007

Eqns. of motion for a rigid body { forces / torques and acc. for a rigid body

1) Linear Acc. : $\underline{F} = \frac{d}{dt}(m\underline{v}_c)$ ← Centre of mass



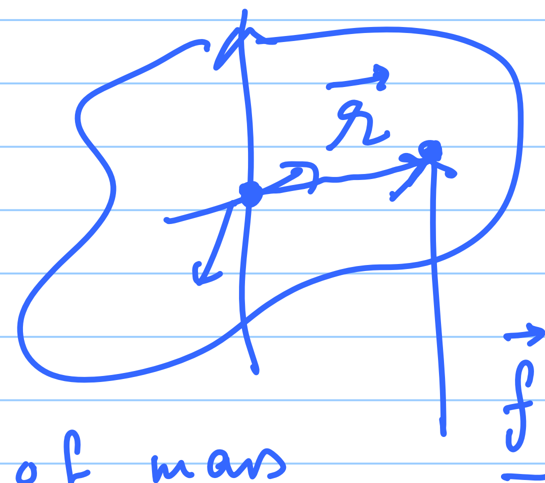
(net force)

$$\boxed{\underline{F} = m \cdot \dot{\underline{v}}_c}$$

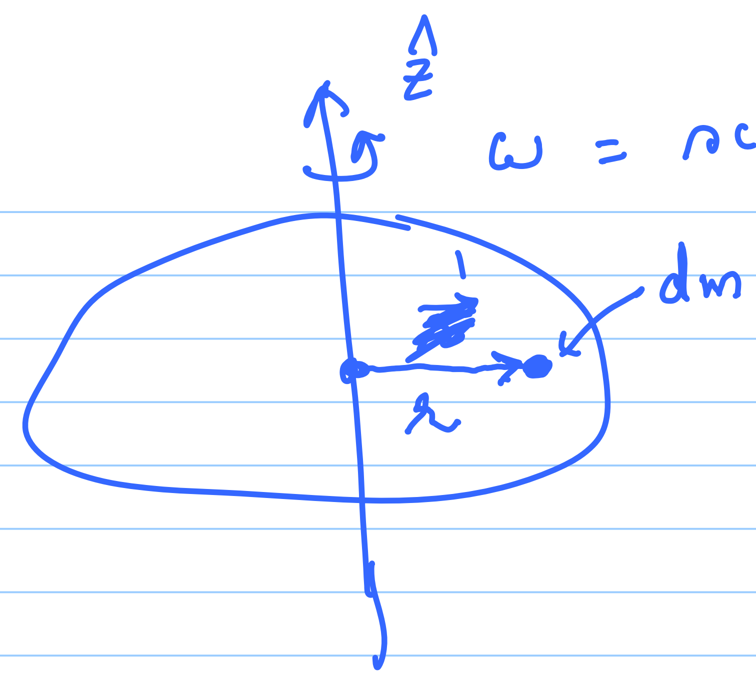
2) Rotation : Moment ? ang. acc.

w.r.t. a frame

$N = \vec{r} \times \vec{f}$



"Inertia" analog of mass for rotational motion



ω = scalar

K.E. :

$$dE = \frac{1}{2}(dm)\underline{v}^2$$

$$v = \omega \cdot r$$

$$dE = \frac{1}{2}\, dm\, (\omega \cdot r)^2$$

$$\int_{\text{Body}} dE = \int_{\text{Body}} \frac{1}{2}\, \omega^2 r^2\, dm$$

$$= \frac{1}{2}\,\omega^2 \int_{\text{Body}} r^2\,dm$$

$$= \frac{1}{2}\,\omega^2\, I_{zz}$$

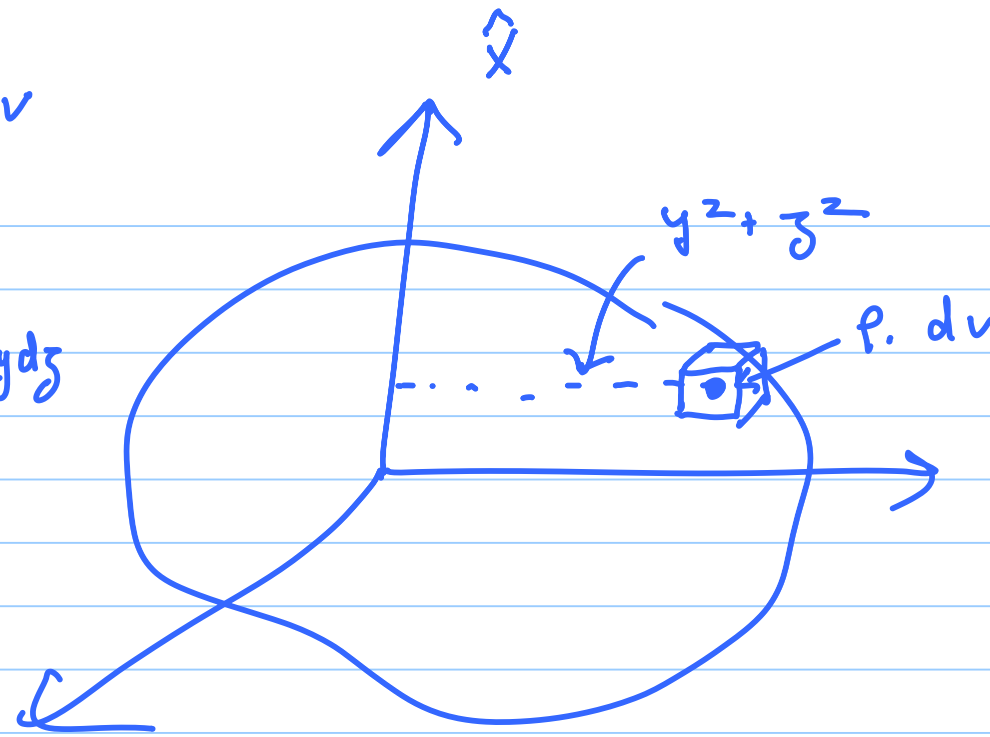
$$I = \begin{pmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{xy} & I_{yy} & -I_{yz} \\ -I_{xz} & -I_{yz} & I_{zz} \end{pmatrix}$$

$$I_{xx} = \int_V (y^2+z^2)\,\underbrace{\rho \cdot dV}_{dm}$$

I_{yy}, I_{zz} similar

$$I_{xy} = \int_V x\,y\,\rho\,dv$$

$$= \rho \iiint_{x\,y\,z} x\,y\,dx\,dy\,dz$$

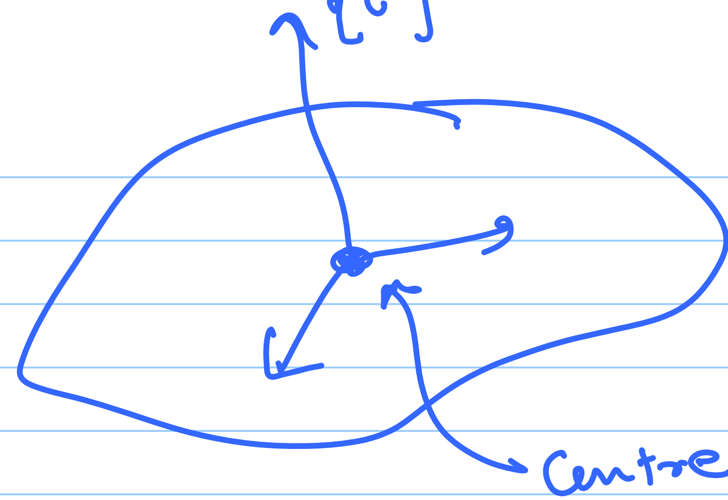


$$H = I\omega$$

$$\boxed{N = \frac{d}{dt}\Big|_c H = {}^{c}I\dot{\omega} + \omega \times \left({}^{c}I\,\omega\right)}$$

Euler's eqn. of motion

Newton-Euler
Eqn. of motion
for a rigid body



Bit more on inertia:

1. I dep. on the frame w.r.t.

$\{A\}$: ref. frame which it is computed

$\{B\}$: fixed on body

A + B ~~___~~ have co-inc. origins.

a) pure rot.

$$ {}^{B}I = \underbrace{{}^{B}_{A}R \, {}^{A}I \left({}^{B}_{A}R\right)^{T}} $$

Rot. (kinetic) Energy:

$$ = \frac{1}{2} \omega^{T} I \omega $$

ind. of fram

$$ {}^{A}\omega = {}^{A}_{B}R \, {}^{B}\omega $$

$$ \frac{1}{2} \left({}^{A}\omega\right)^{T} {}^{A}I \, {}^{A}\omega = \frac{1}{2} \left({}^{B}\omega\right)^{T} {}^{B}I \, {}^{B}\omega $$

b) pure trans:

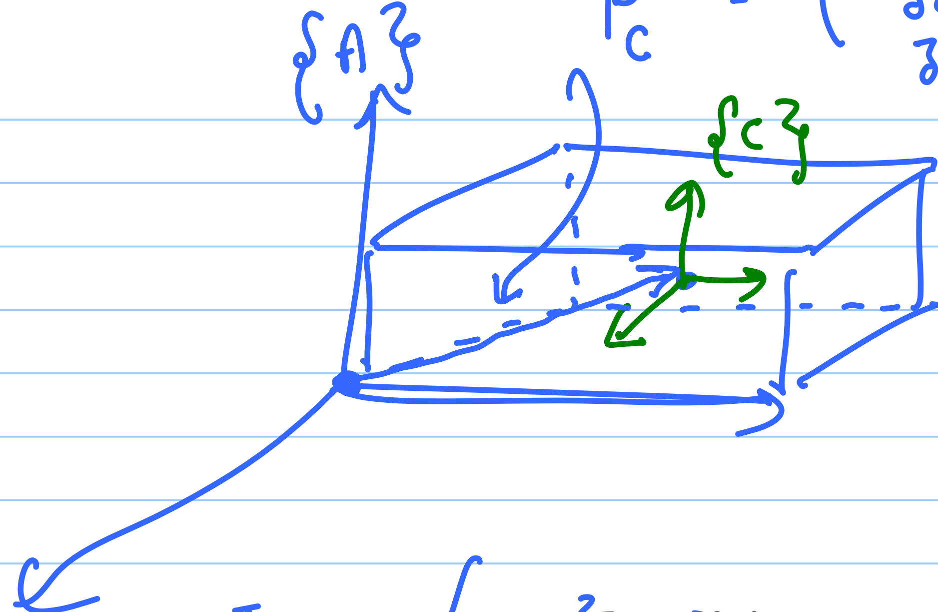
$\{C\}$ = frame with orig. at c.o.m. of the body

$\{A\}$ = same orient as $\{C\}$, but origin in diff.

$$ {}^{A}\underline{\underline{I}} = {}^{C}\underline{\underline{I}} + m\left[\underline{\underline{P_C^T P_C}}\, \underline{\underline{I}}_{3\times3} - P_C P_C^T \right] $$

identity

$$P_c = \begin{pmatrix} x_c \\ y_c \\ z_c \end{pmatrix}$$



$$P_c P_c^T = \begin{pmatrix} x_c^2 & x_c y_c & x_c z_c \\ \cdot & y_c^2 & \cdot \\ \cdot & \cdot & z_c^2 \end{pmatrix}$$

example for a cube carried out in text. Read it.

Now we are ready to apply

N-E Eqn. of motion + Linear Acc. Eqn + Ang. Acc. Eqn.

1. apply Linear Acc / Ang. Acc. Eqns. to each link i, to get v_i, $\dot{v}_i$, ω_i, $\dot{\omega}_i$

2. derive $\dot{v}_{c_i}$ from 1

(3) use N-E eqns. of motion to calculate forces/moments for each link.

(4) <u>joint-torque:</u> moments applied by joint i in $\underline{\hat{z}}_i$