Lecture - 35+36

Note Title 11/22/2007

Robot Manipulator Control

free-space

✓ (1) position control: given desired traj. (joint-space), develop a "control law" that makes the robot move along it.

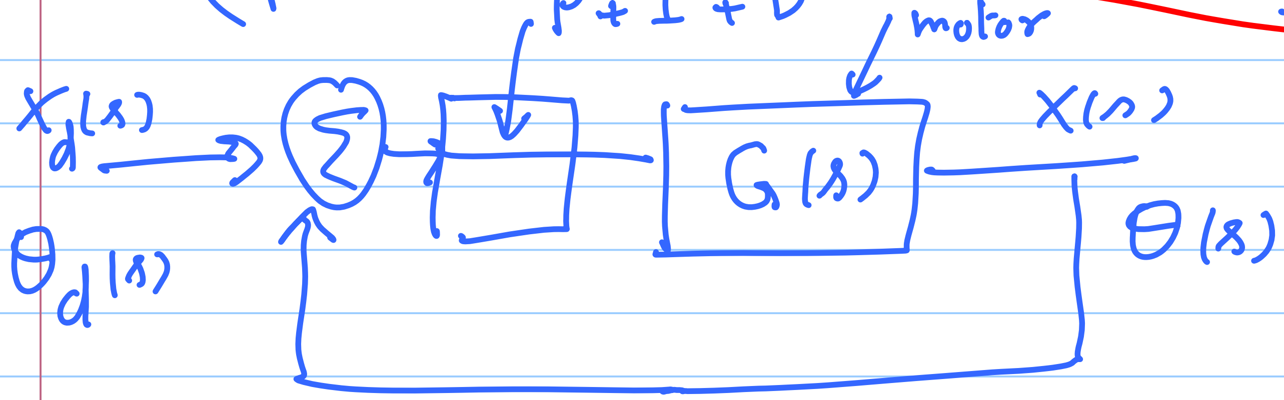
X (2) force-control: "need to sense force"

Position Control

MIMO

$$\underline{\tau} = M(\underline{\theta})\,\ddot{\underline{\theta}} + \underline{V}(\underline{\theta}, \dot{\underline{\theta}}) + \underline{G}(\underline{\theta}) + \underline{F}(\dot{\underline{\theta}})$$

$\left(K_p + sK_v + K_I/s\right)$



SISO

$\xrightarrow{\text{I}}$ Independent - JOINT - CONTROL

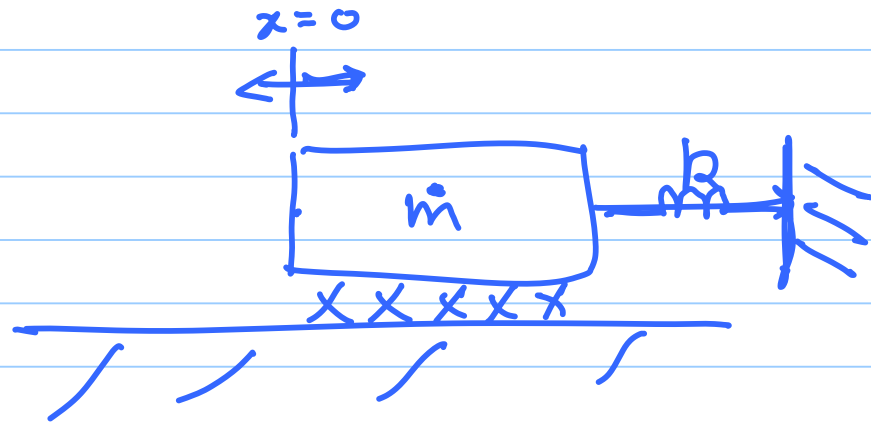
$\xrightarrow{\text{II}}$ Centralized "full-dynamics" based Control

$\xrightarrow{\text{III}}$ In-bet: I + II.

BRIEF REVIEW of SISO Control (383)

1) second-order mechanical system

REGULATION:
("control syst. to maintain $x = 0$)



Translational
or
Rotational
($m \Leftrightarrow J$
$b \Leftrightarrow b$
$k \Leftrightarrow k$

$$m\ddot{x} = -b\dot{x} - kx + f(t)$$

$$\Leftrightarrow \quad m\ddot{x} + b\dot{x} + kx = 0 \Leftrightarrow F(s)$$

$$\Leftrightarrow \quad (ms^2 + bs + k)\,X(s) = F(s)$$

$$F(s) \longrightarrow \boxed{\dfrac{1}{ms^2+bs+k}} \longrightarrow X(s)$$

char. poly.

roots of char. eqn : $\boxed{\text{char poly} = 0}$

$$s_{1,2} = \frac{-b \pm \sqrt{b^2-4mk}}{2m}$$

key char:

1. stability: BIBO stability
2. transient char: peak overshoot etc.
3. steady state error

① stability: $\overset{\text{nec.}}{\Leftarrow}$ all coeffs of char poly are +ve $\qquad e_{ss} = \lim_{t\to\infty} e(t) \quad [x_d - x(t)]$

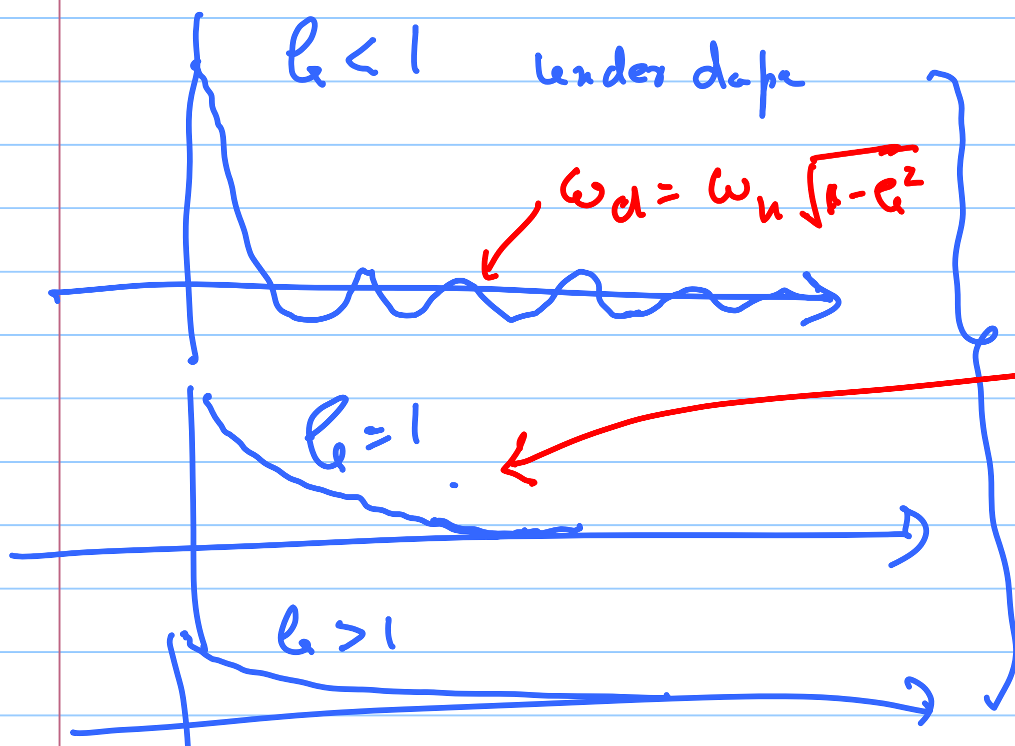
roots of char. poly

|| must lie in open LHP

Char. eqn:

$$\underbrace{s^2 + 2\boxed{\zeta}\,\omega_n s + \boxed{\omega_n}^2 = 0}$$

damping ratio $\qquad$ natural freq.



Critically damped closed loop syst.

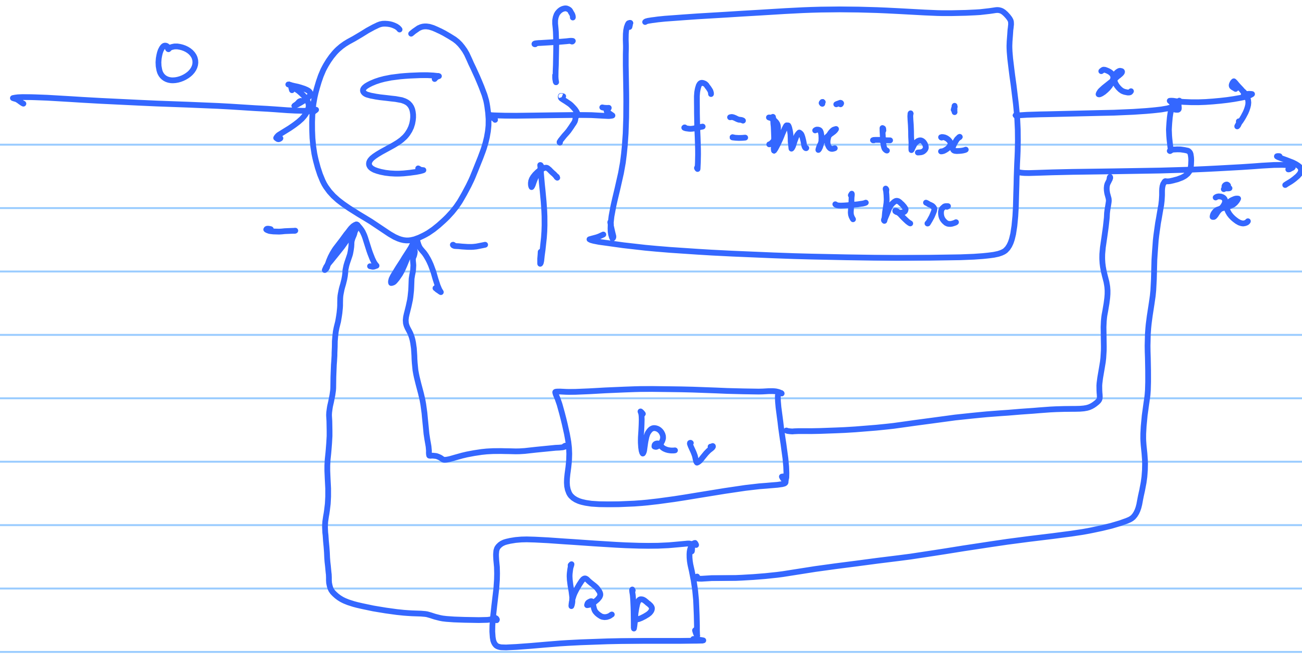
Syst. is stable

for Char. eqn $ms^2 + bs + k = 0$

$$\zeta = \frac{b}{2\sqrt{km}} \boxed{= 1}$$

$$\boxed{\omega_n = \sqrt{\frac{k}{m}}}$$

P+D $\rightarrow$ || Control law will essentially boil down to det. values for k_p, k_v

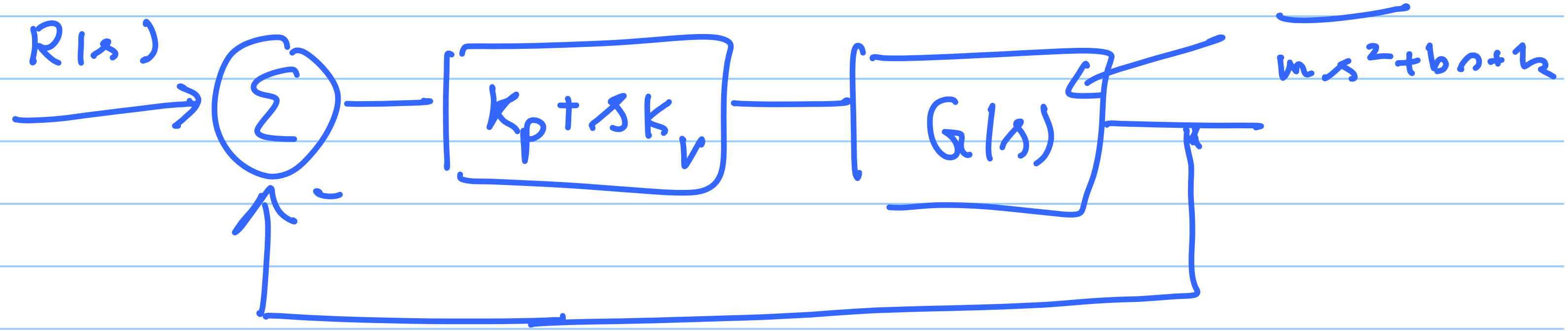


$$-k_p x - k_v \dot{x} = m\ddot{x} + b\dot{x} + k\cancel{x}$$

$$\Longleftrightarrow \quad m\ddot{x} + \underbrace{(b + k_v)}_{b'}\dot{x} + \underbrace{(k + k_p)}_{k'} x = 0$$

→ Choose k_p, k_v: $\boxed{b' = 2\sqrt{k' m}}$

Critically damped response



① → "Partitioned Control" }

$\xrightarrow{\text{general}}$ "feedback linearization"

"Inverse Dynamics Control"

"will be used later for full dynamics based controller"

$$\longrightarrow \boxed{f = m\ddot{x} + b\dot{x}} + kx \longrightarrow$$

"unit mass"

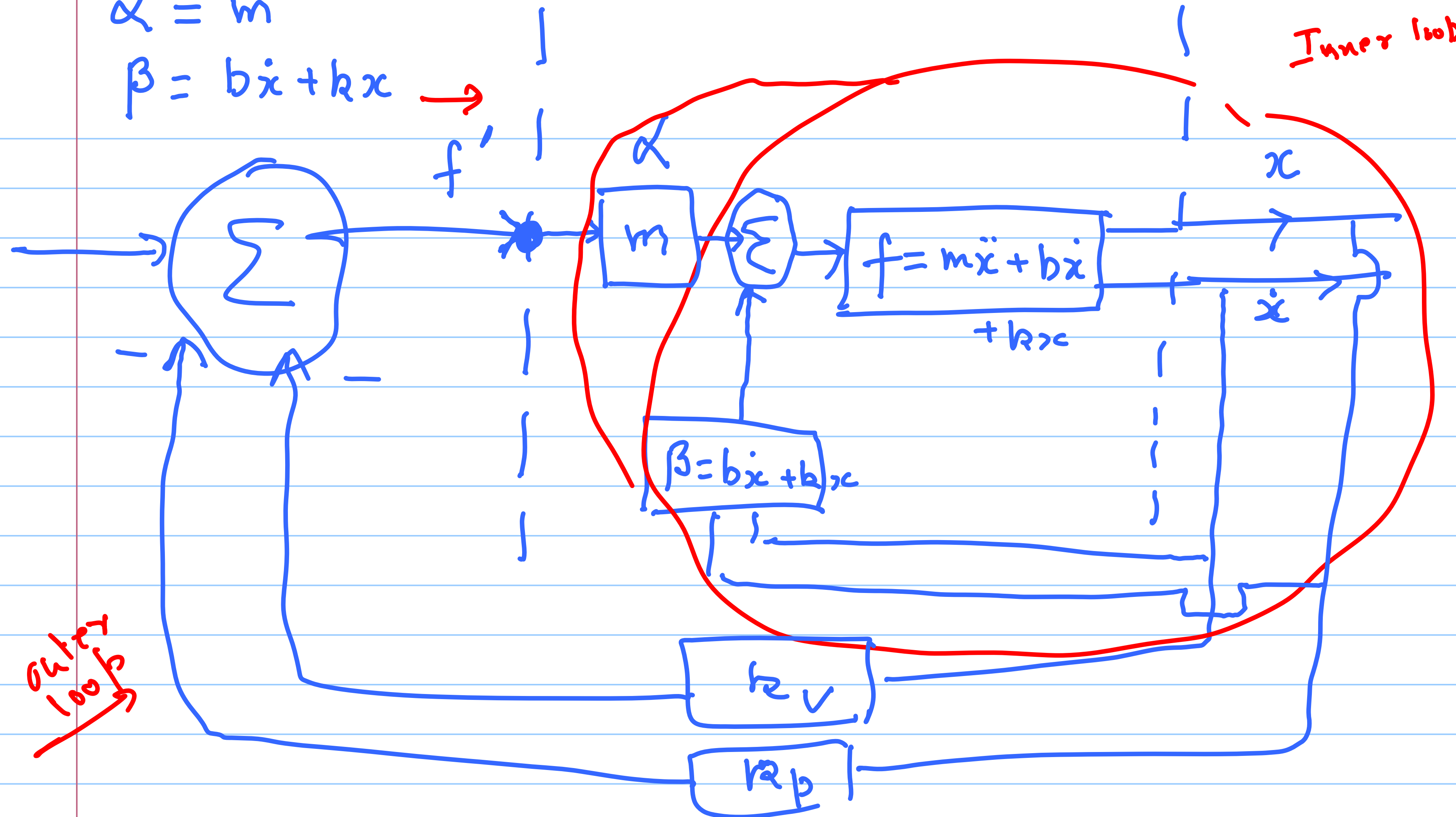
$$f = \alpha f' + \beta$$

$$= m\ddot{x} + b\dot{x} + kx$$

$$\boxed{f' = \ddot{x}}$$

$\alpha = m$

$\beta = b\dot{x} + kx$



Overall closed loop eqn :

$$f' = \ddot{x} = -k_p x - k_v \dot{x}$$

P+D control on unit mass

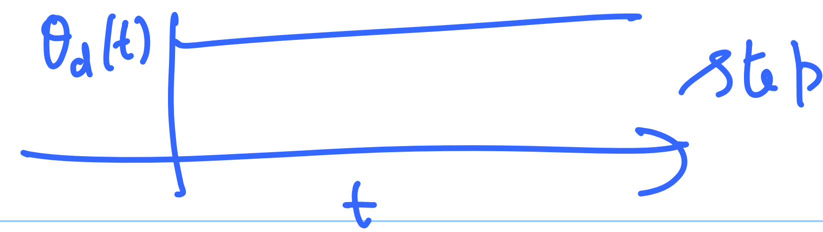
$$\Leftrightarrow \boxed{\ddot{x} + k_p x + k_v \dot{x} = 0}$$

$$\zeta = 1 \Rightarrow k_v = 2\sqrt{k_p}$$

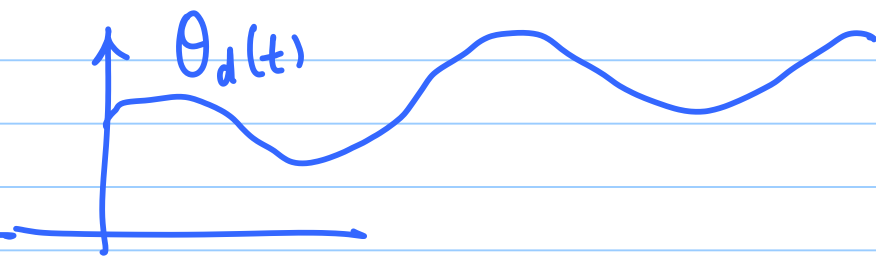
② Trajectory control

(feedforward control)

Set point :



Traj. Control



$$e(t) = x_d(t) - x(t)$$

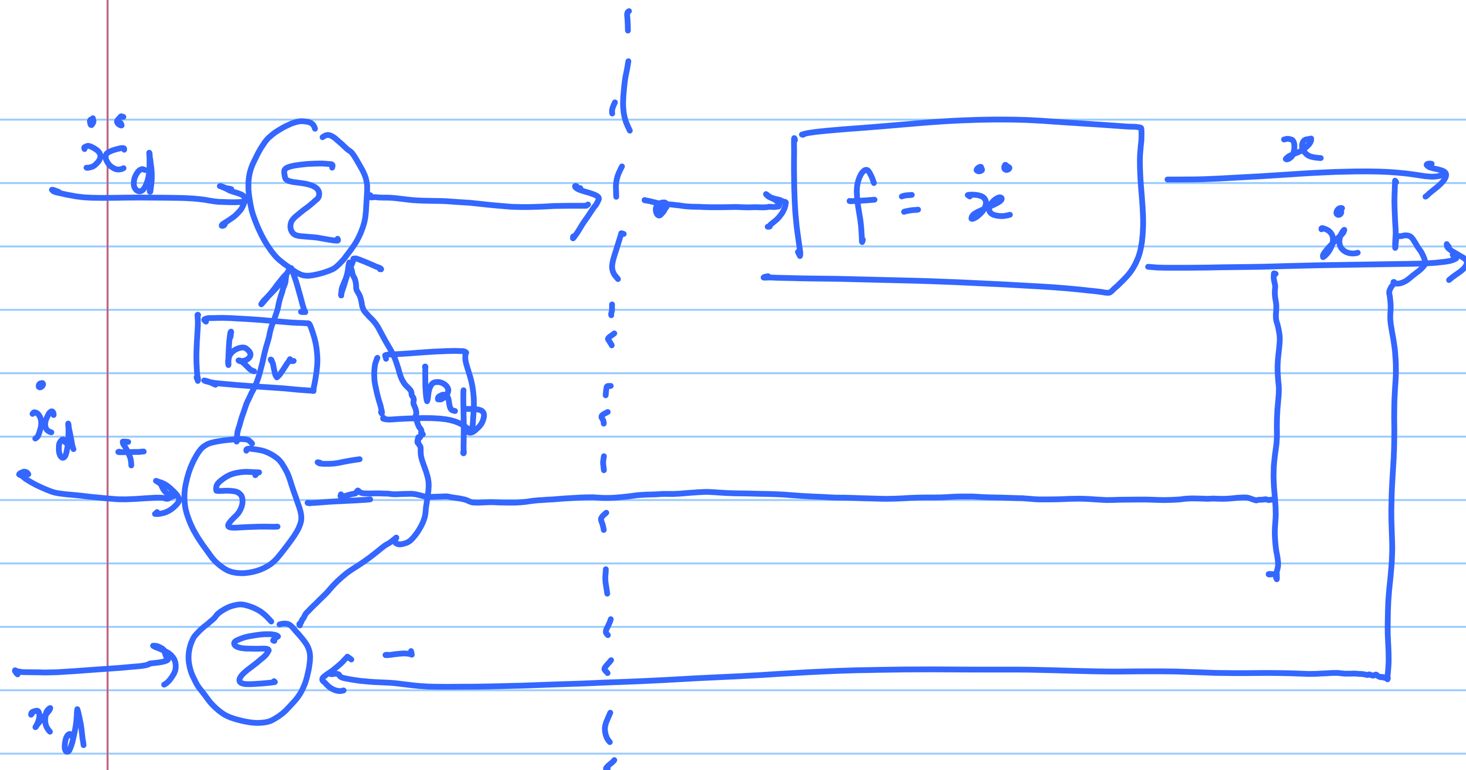
"regulation control" for $e(t)$

"make sure your closed loop control

eqn. in terms of $e(t)$ is nicely behaved"

traj control for unit mass

$$\boxed{f = \ddot{x}}$$



$e(t) = x_d - x$

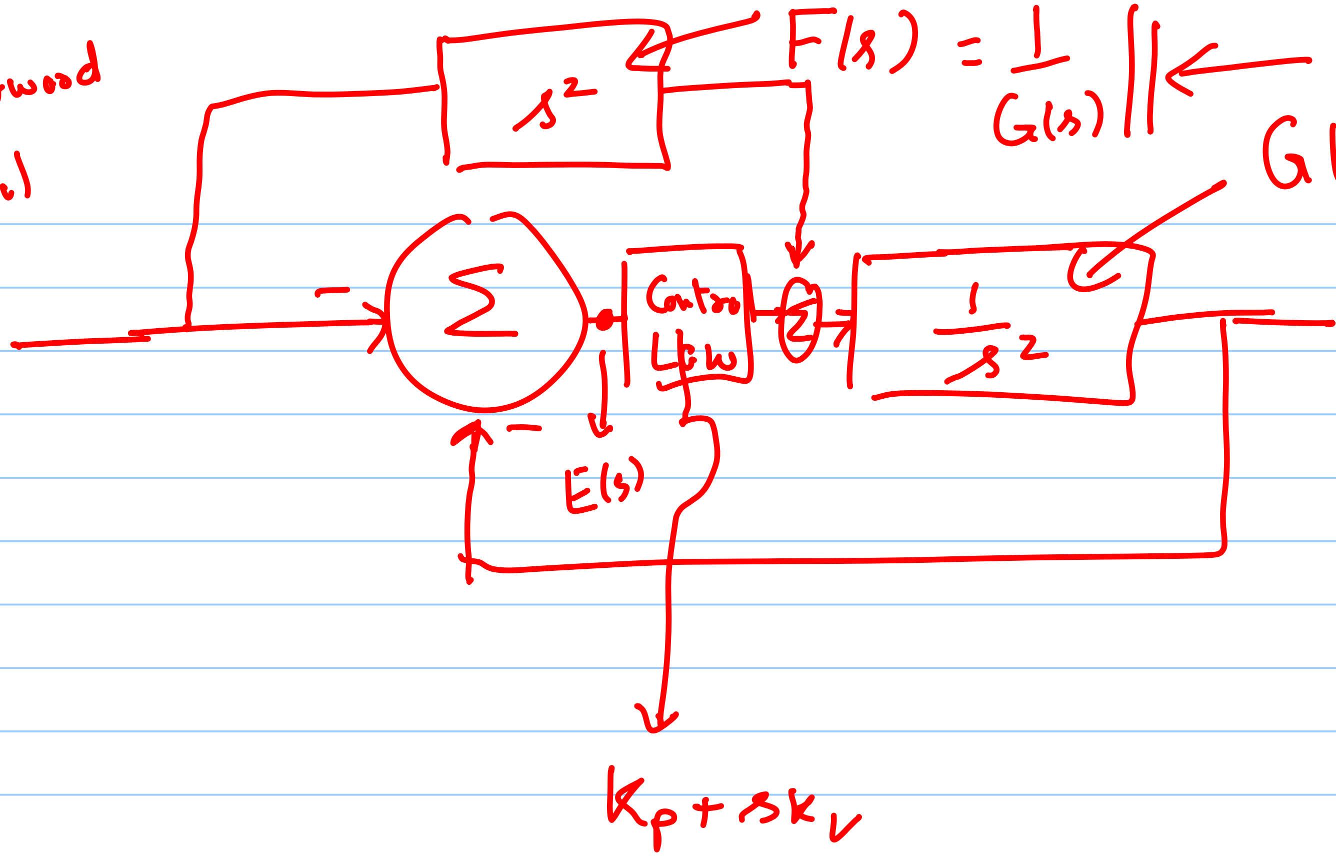
$$\ddot{x} = \ddot{x}_d + k_v(\dot{x}_d - \dot{x}) + k_p(x_d - x)$$

$$\boxed{\ddot{e} + k_v \dot{e} + k_p e = 0}$$

Choose k_p, k_v : $\zeta = 1$

$k_v = 2\sqrt{k_p}$

|| feedforward control



$F(s) = \frac{1}{G(s)}$ ||

$G(s)$ should not have zeros in RHP

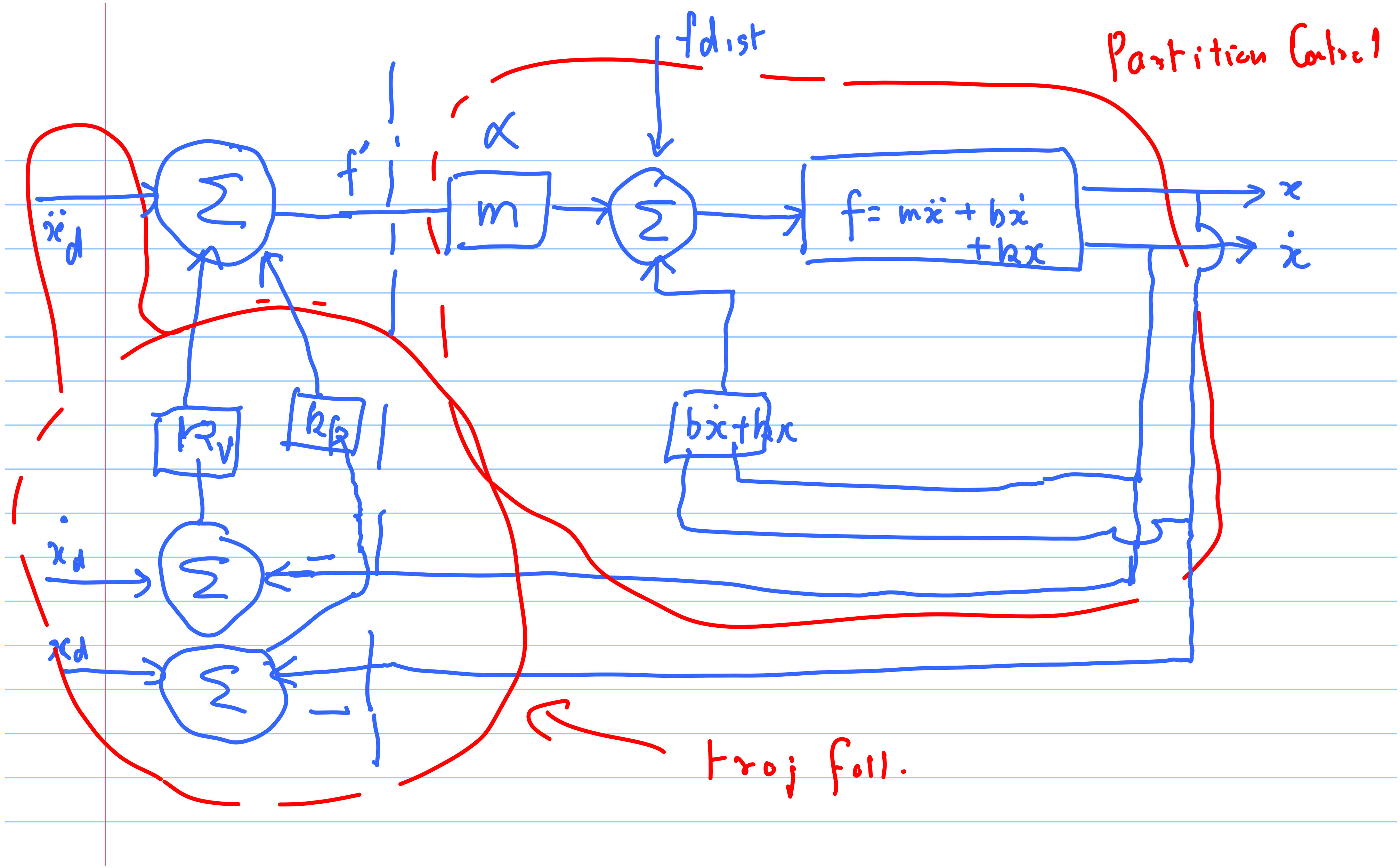
Control law positioning + traj foll.

Control

$$f = m\ddot{x} + b\dot{x} + kx$$

$$= \alpha f' + \beta$$

$$\alpha = m, \quad \beta = b\dot{x} + kx$$



Partition Control
f_{dist}
α
f'
$\ddot{x}_d$
$\dot{x}_d$
x_d
k_v
k_p
m
$f = m\ddot{x} + b\dot{x} + kx$
$b\dot{x} + kx$
x
$\dot{x}$
Traj foll.

$$\boxed{\ddot{e} + k_v \dot{e} + k_p e = 0}$$

$k_v = 2\sqrt{k_p}$ for critical damping

P+D control: e_{ss} due to a disturbance

$$\ddot{e} + k_v \dot{e} + k_p e = f_{dist}$$

$\ddot{e} \to 0$ $\qquad e_{ss} = \dfrac{f_{dist}}{k_p}$

$\dot{e} \to 0$

high $k_p \Rightarrow$ low e_{ss} but k_p can't be too high (will elab. later)

To reduce e_{ss}, will add an integral term $k_i \int e\,dt$ or $\frac{k_i}{s}$

k_i can't be too large (may lead to instability)

Actual Implementation :

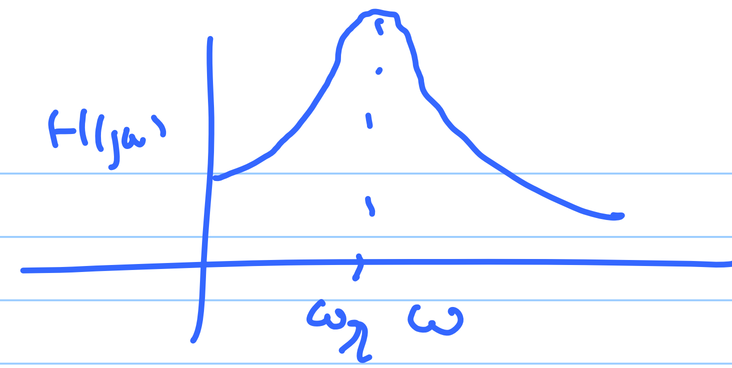
1. discrete control (sampling) :

   1. freq. content of your traj

      Sampling rate > 2. B.W. of the signal

2. structural resonance : due to the nature of mech. sys., you will have a resonance freq. of your

system.



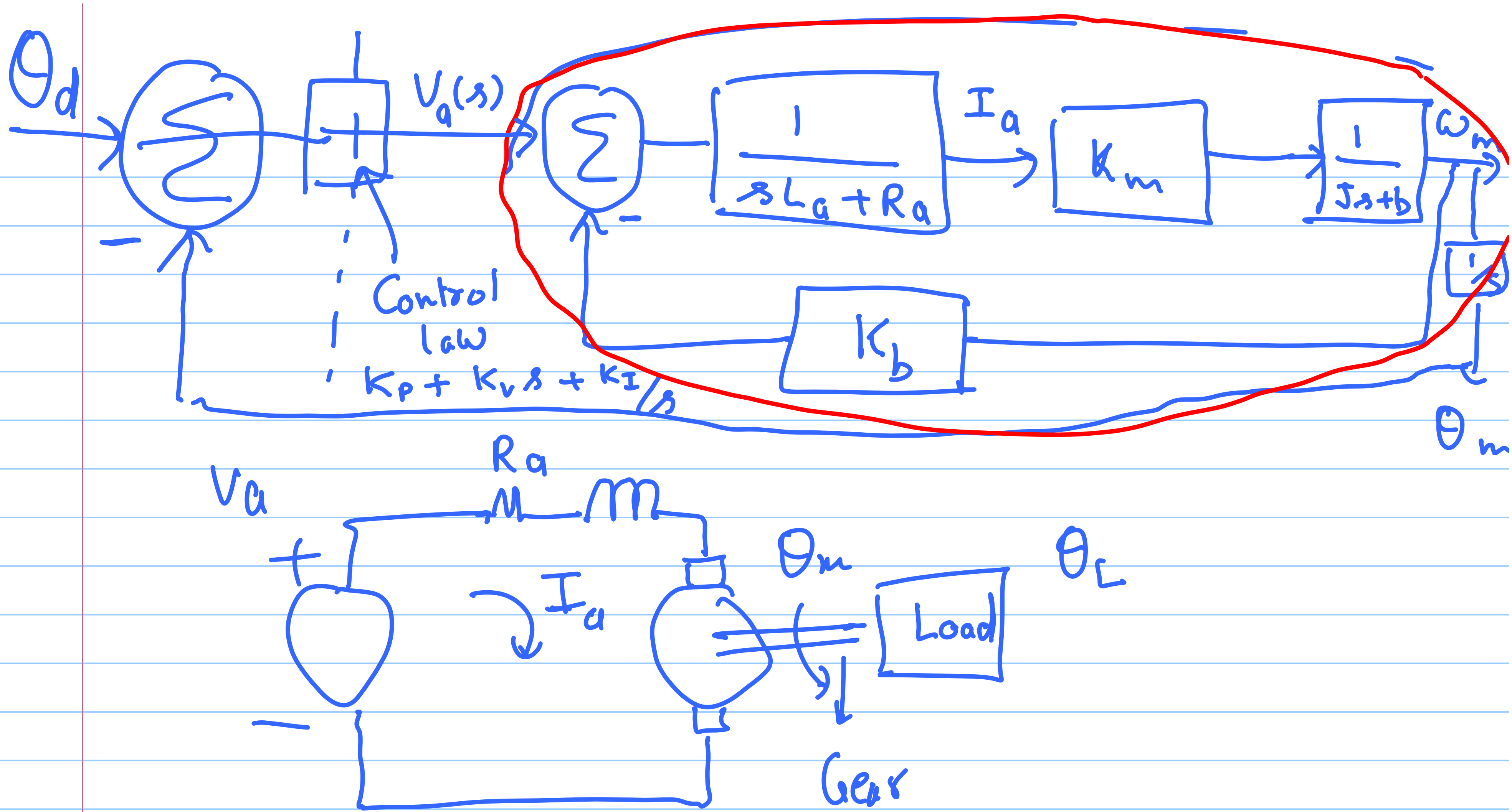
"lowest mode" ω_{res}

Sampling freq. $\geq 2\,\omega_{res}$

Independent Joint Control:

each joint of the robot is controlled independently, and the robot dynamics are essentially treated as disturbance.

revolute joint: (motor dc servomotor)



θd
Va(s)
Control law
Kp + Kv s + KI/s
1/(sLa + Ra)
Ia
Km
1/(Js+b)
ωm
1/s
Kb
θm
Va
Ra
Ia
θm
θL
Load
Gear

<u>assumptions:</u> neglect L_q, ~~we er~~ assuming "direct current control"

$$\boxed{\underline{\underline{\tau}} = k_m \, \underline{\underline{i_q}}}$$