Note Title 9/11/2007

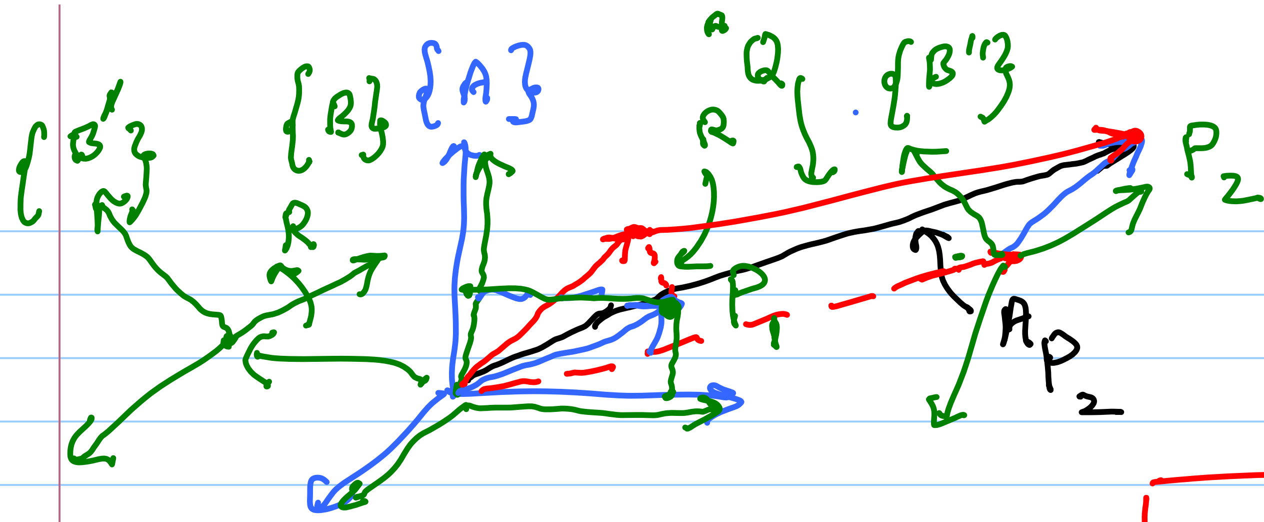
Lecture - 4

RIGID BODY MOTIONS (CONTD.)

${}^{A}_{B}T \rightarrow$ 1) Represents {B} w.r.t. {A}

2) used as a mapping that maps pts. (vectors) expressed in {B} to that in {A}

3) operator : represent xformation (motion) of rigid body pts. in the same co-ord sys



$${}^{B}_{B''}R\,{}^{B''}P_2$$

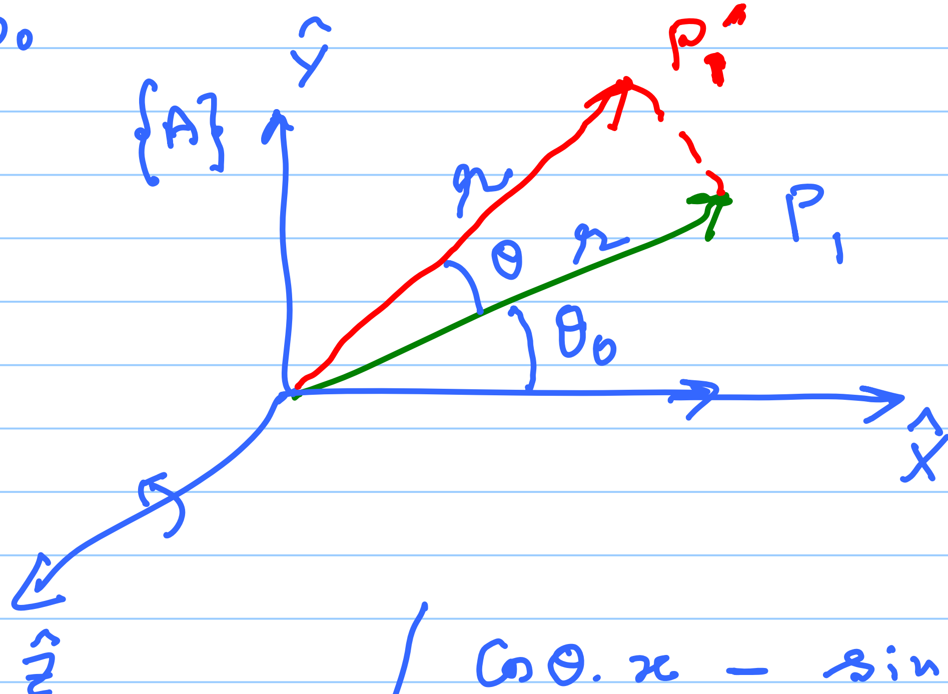
$${}^{A}P_2 = {}^{B}P_{B'ORG} + {}^{B}_{B''}R\,{}^{B''}P_2$$

$${}^{A}P_1 = {}^{A}Q + R\,P_1$$

DIRECT Computation of R

$r \sin\theta \cos\theta_0$

$r \cos\theta \sin\theta_0$



$$^{A}P_1 = \begin{pmatrix} r\cos\theta_0 \\ r\sin\theta_0 \\ 0 \end{pmatrix} = \begin{pmatrix} x \\ y \\ 0 \end{pmatrix}$$

$$^{A}P_1' = \begin{pmatrix} r\cos(\theta+\theta_0) \\ r\sin(\theta+\theta_0) \\ 0 \end{pmatrix}$$

$$^{A}P_1' = \begin{pmatrix} \cos\theta . x - \sin\theta\, y \\ \sin\theta\, x + \cos\theta\, y \\ 0 \end{pmatrix}$$

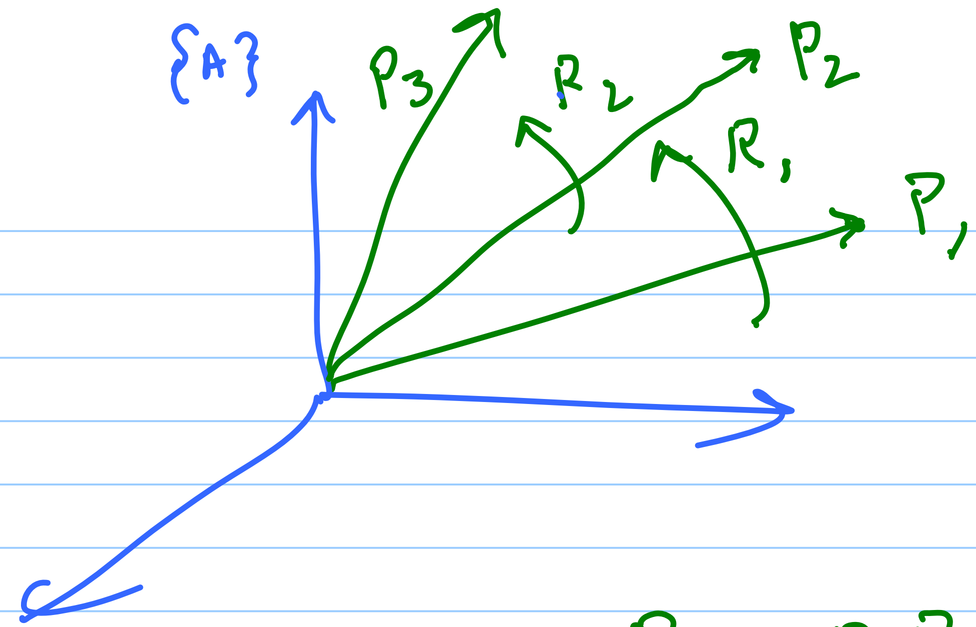
$$= \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 0 \end{pmatrix}$$

$R_z(\theta)$

$R_K(\theta) \longrightarrow$ rot. of θ around $\hat{K}$

$R_y(\theta)$ $\leftarrow$ Principal Rotations

$R_x(\theta)$



$$P_3 \overset{?}{=} P_1$$

$$P_2 = R_1 P_1$$

$$P_3 = R_2 P_2$$

$$P_3 = \underbrace{R_2 R_1}_{R} P_1$$

Operator interpretation:

1. successive rotations corr. to pre. multiplication by corr. matrices.

2. finite Rotations do not commute

TRANSFORM ARITHMATICS

1) Compounding Transforms:

Given $\{A\}$, $\{B\}$, $\{C\}$,

and ${}^{A}_{B}T$, ${}^{B}_{C}T$ Take point P.

${}^{A}_{C}T = ?$ ${}^{A}P = {}^{A}_{B}T \; {}^{B}P$ ${}^{B}P = {}^{B}_{C}T \; {}^{C}P$

[0 0 0 | 1]

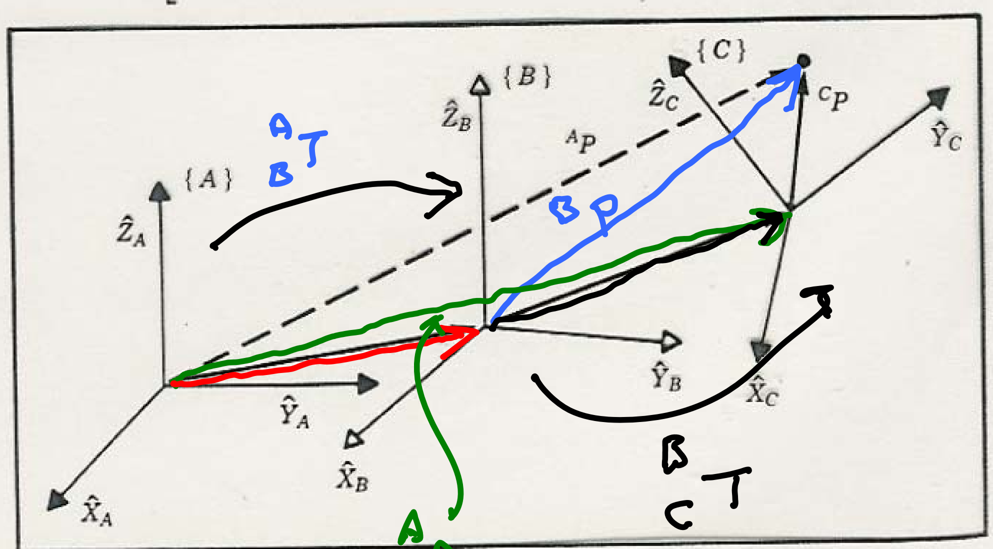


FIGURE 2.12 Compound frames: each is known relative to previous.

$$\boxed{{}^{A}P = {}^{A}_{C}T \; {}^{C}P}$$

$$\therefore \; {}^{A}_{C}T = {}^{A}_{\cancel{B}}T \; {}^{\cancel{B}}_{C}T$$

$$ {}^{A}_{C}T = \left[\begin{array}{c|c} {}^{A}_{C}R & {}^{A}P_{CORG} \\ \hline 0 & 1 \end{array}\right] $$

$$ {}^{A}_{C}R = {}^{A}_{B}R \; {}^{B}_{C}R $$

$$ {}^{A}P_{CORG} = {}^{A}P_{BORG} + {}^{A}_{B}R \; {}^{B}P_{CORG} $$

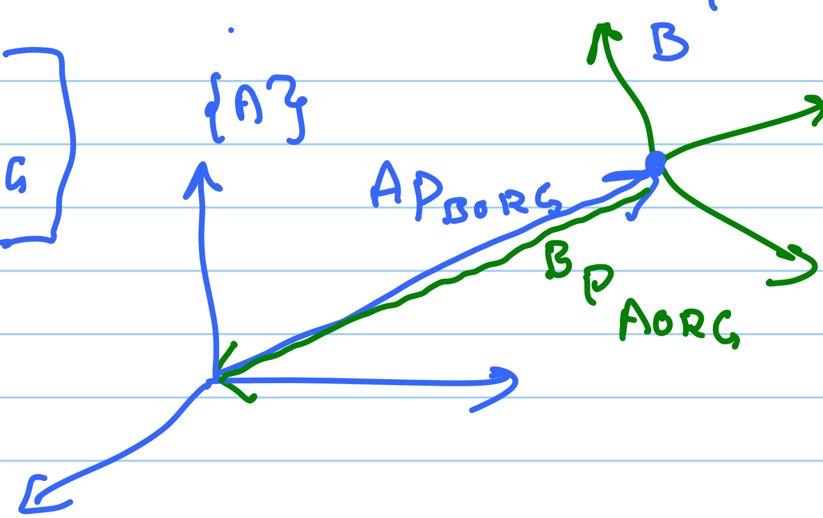
$$ {}^{A}_{B}T = \left[\begin{array}{c|c} {}^{A}_{B}R & {}^{A}P_{BORG} \\ \hline 0 & 1 \end{array}\right] $$

$$ {}^{B}_{C}T = \left[\begin{array}{c|c} {}^{B}_{C}R & {}^{B}P_{CORG} \\ \hline 0 & 1 \end{array}\right] $$

Lecture 5

2) Inverse of a xfrom: ${}^{A}_{B}T$

$${}^{A}_{B}T = \left[\begin{array}{c|c} {}^{A}_{B}R & {}^{A}P_{BORG} \\ \hline \underline{0} & 1 \end{array}\right]$$



we know $R^{-1} = R^{T}$

$$\left[{}^{A}_{B}T\right]^{-1} = {}^{B}_{A}T = \left[\begin{array}{c|c} {}^{B}_{A}R & {}^{B}P_{AORG} \\ \hline \underline{0} & 1 \end{array}\right]$$

$${}^{B}P_{AORG} = -\,{}^{A}_{B}R^{T}\,{}^{A}P_{BORG}$$

$$= \left[\begin{array}{c|c} {}^{A}_{B}R^{T} & \underbrace{{}^{A}_{B}R^{T}}\,\underbrace{-{}^{A}P_{BORG}} \\ \hline \underline{0} & 1 \end{array}\right]$$

3) TRANSFORM EQNS. : Determine ${}^{T}_{G}T$

KNOWN:

${}^{B}_{T}T$, ${}^{B}_{S}T$

${}^{S}_{G}T$

2.8 More on representation of orientation 43

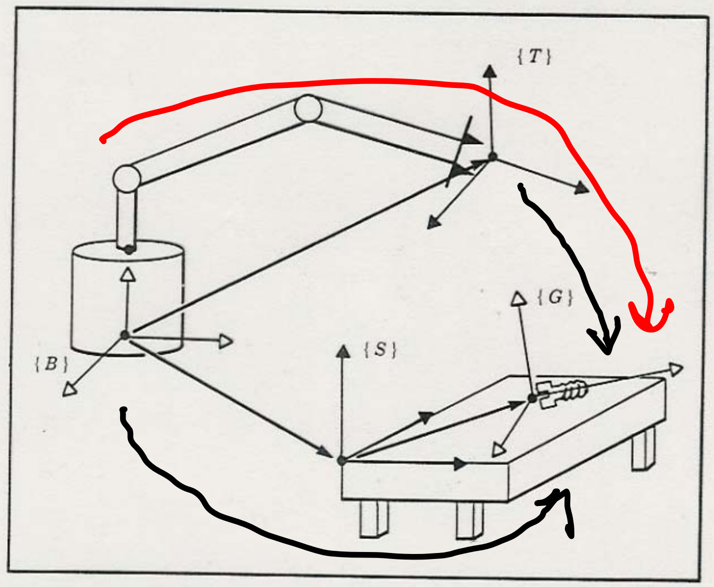


FIGURE 2.16 Manipulator reaching for a bolt.

Lower path : $${}^{B}_{G}T = {}^{B}_{S}T \; {}^{S}_{G}T$$

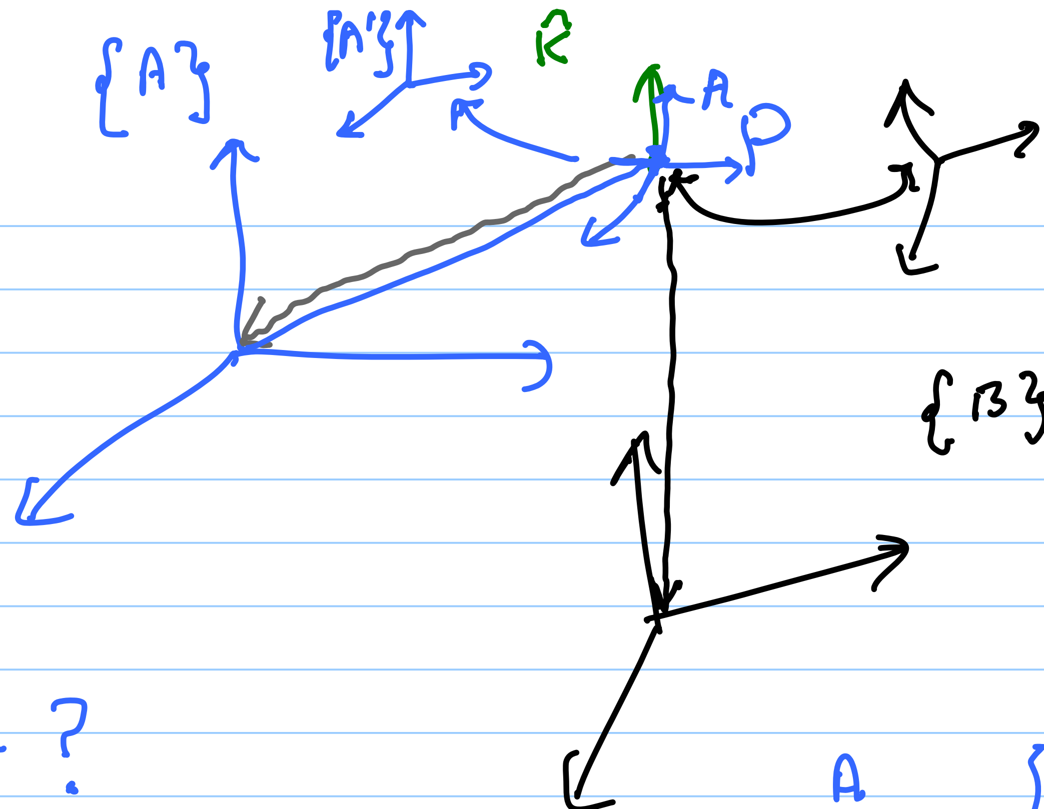
Upper path : $${}^{B}_{G}T = {}^{B}_{T}T \; {}^{T}_{G}T$$

$$\Rightarrow \; {}^{B}_{S}T \; {}^{S}_{G}T = {}^{B}_{T}T \; {}^{T}_{G}T$$

$$\Rightarrow \; {}^{T}_{G}T = {}^{B}_{T}T^{-1} \left[{}^{B}_{S}T \; {}^{S}_{G}T \right]$$

$$= {}^{T}_{\cancel{B}}T \; {}^{\cancel{B}}_{\cancel{S}}T \; {}^{\cancel{S}}_{G}T$$

So far all rotations, are around axes (vectors) that pass through the origin. Same "transform arithmatic" can be used to det. effects of rotation around axes that do no pass through origin.



$\hat{K} = \hat{Z}$, $\theta = 30^\circ$

$R_{\hat{Z}}(30^\circ)$

${}^{A}_{A'}R = I$

${}^{B}_{B'}R = I$

${}^{A}_{B}T = ?$

${}^{A}_{B}T = {}^{A}_{A'}T \; {}^{A'}_{B'}T \; {}^{B'}_{B}T$

$${}^{B'}_{B}T = \left[\begin{array}{c|c} I & -{}^{A}P \\ \hline 0 & 1 \end{array}\right]$$

$${}^{A}_{A'}T = \left[\begin{array}{c|c} I & {}^{A}P \\ \hline 0 & 1 \end{array}\right]$$

$${}^{A'}_{B'}T = \left[\begin{array}{c|c} R_{\hat{Z}}(30^\circ) & 0 \\ \hline 0 & 1 \end{array}\right]$$