

# Lecture - 8+9

## Manipulator arms

### "KINEMATICS":

$$\begin{pmatrix} f_1(\theta_1, \theta_2, \theta_3) \\ f_2(\theta_1, \theta_2, \theta_3) \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

Serial

Class of robots: open kinematic

↓  
Chains.

one end is free to

move. sequence of rigid bodies ("links")

Connected via "joints".

Link  $i-1$  & Link  $i$  are connected by joint  $i$ . joint  $i$  moves link  $i$ .

Example:

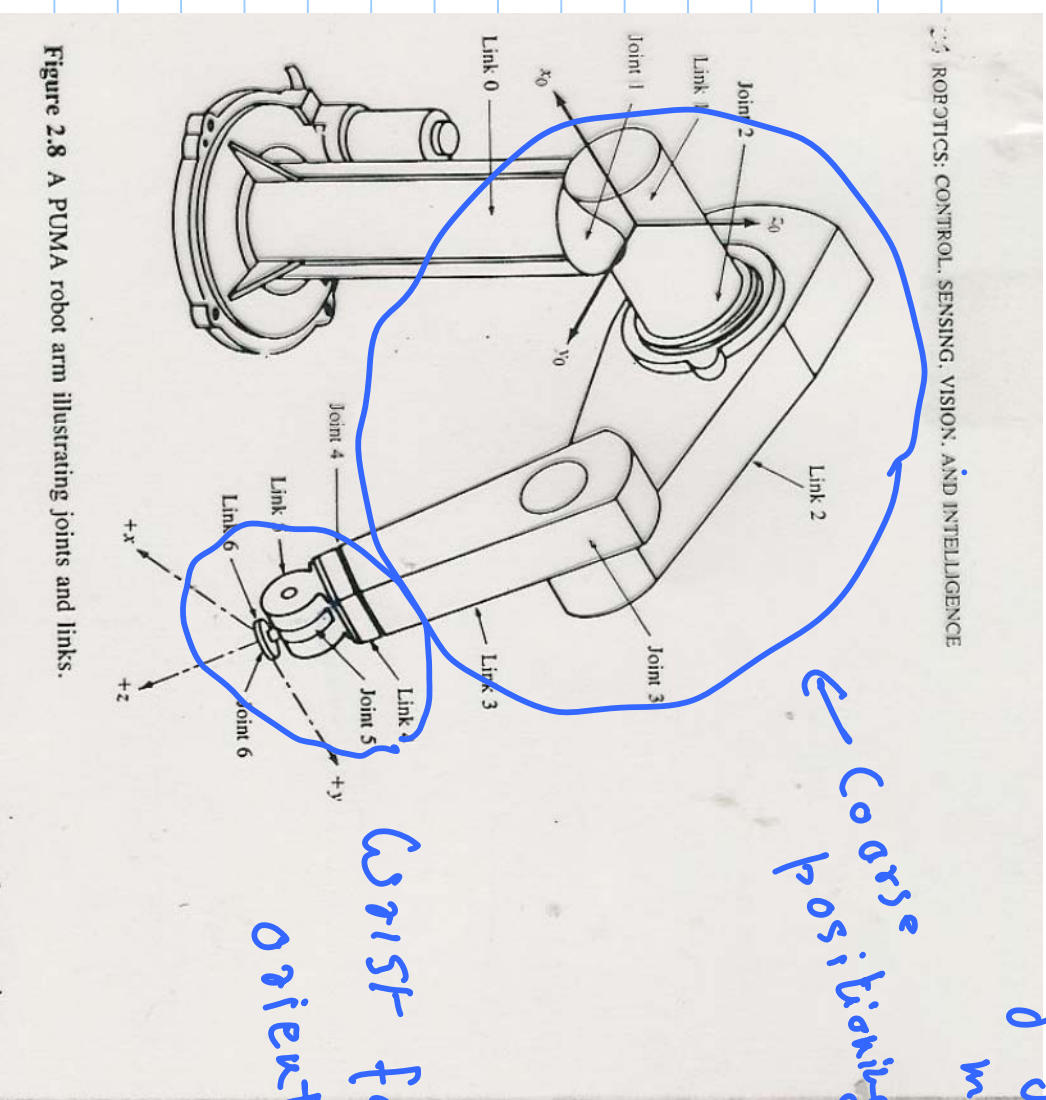


Figure 2.8 A PUMA robot arm illustrating joints and links.

← coarse positioning

assist for orienting

why? latex

dof "degrees of freedom" of a robot:  
≡ minimum # of independent  
parameters needed to specify  
the pos. + orient. of all components  
(links) of the robot.

A rigid body :  $3 + 3 = 6$  parameters  
pos ori.

hence 6-dofs

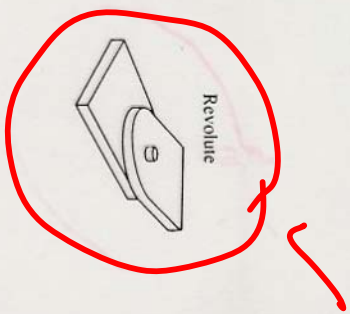
therefore, most robots for gen. pos.

fashs need at least 6-dof.

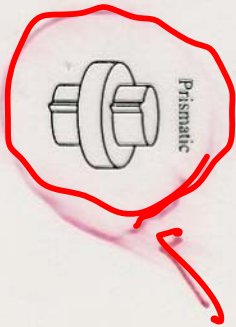
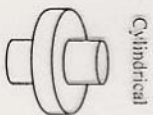
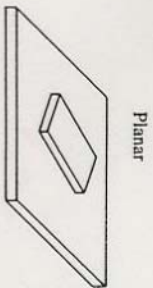
What are diff. types of joints:

"Lower pair" joints: two

surfaces are fully in contact.



2 of. dof  
1



trans. dof  
1

Spherical

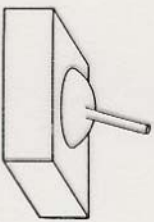
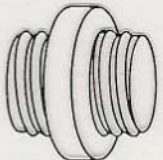


Figure 2.9 The lower pair.

Screw



Joint  $i + 1$

Label / classify  $\alpha$  hok based on

types of joints: R : revolute  
P : prismatic

KINEMATIC CONFIG:

RRP, PPP, RRR (3R)

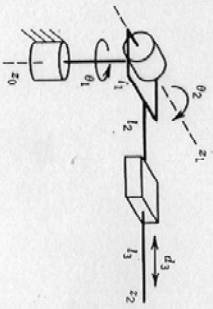


FIGURE 1-9  
The spherical manipulator configuration

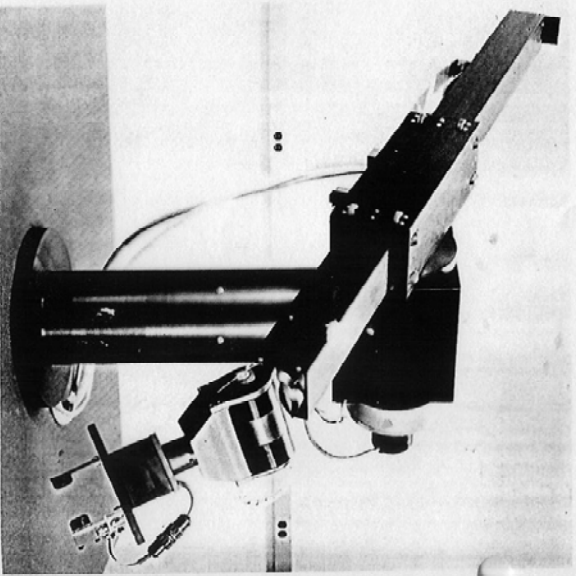
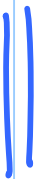
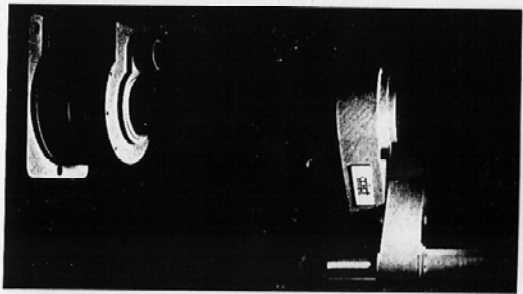


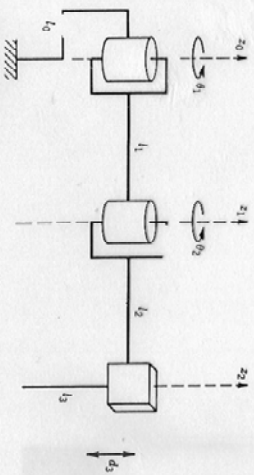
FIGURE 1-10  
The Stanford manipulator

R R P





**FIGURE I-13**  
The AdeptOne robot. Photo  
courtesy of Adept Techno-  
gies.



**FIGURE I-12**  
The SCARA [Selective Compliant Articulated Robot for  
Assembly]



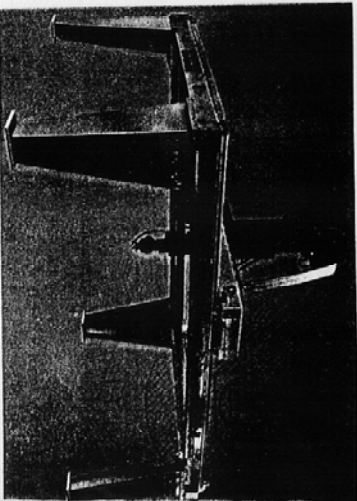


FIGURE 1-19  
A Gantry robot, the Cincinnati Milacron T-3 886. Photo courtesy Cincinnati Milacron.

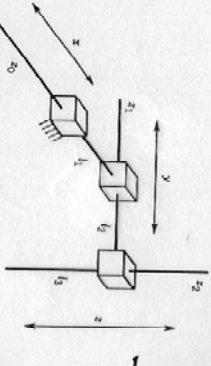


FIGURE 1-18  
The cartesian manipulator configuration.

PPP

Gantry Robots

generally speaking:  $\left\{ \begin{array}{l} P: \text{ more precise} \\ R: \text{ less precise} \end{array} \right.$  positioning

Alternative Classification DRIVE Technology

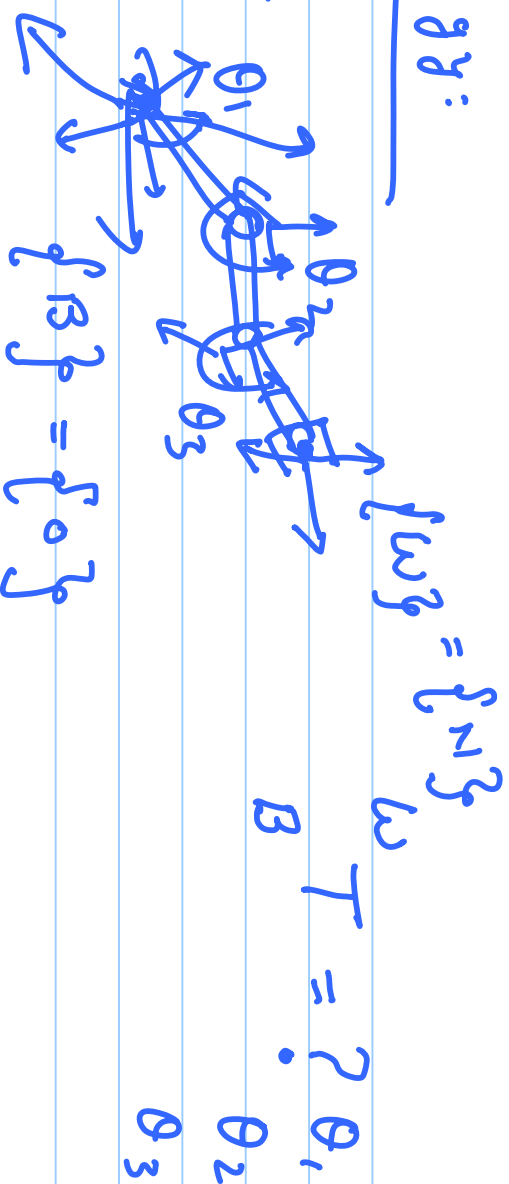
→ electric

Speed / precision / power → pneumatic  
→ hydraulic

# of dofs

Strategy:

$${}^0T_N = {}^0T_1 {}^1T_2 \dots {}^{N-1}T_N$$



1) assign frames to each link

2) D-H Denavit & Hartenberg

Notation (1955)

for assigning frames to various links.

Consider joint axes  $i-1$  &  $i$  common & bet.

them is

denoted

by  $q_{i-1}$ .

$\alpha_{i-1} =$

angle

bet.

( $i-1$ ) axis

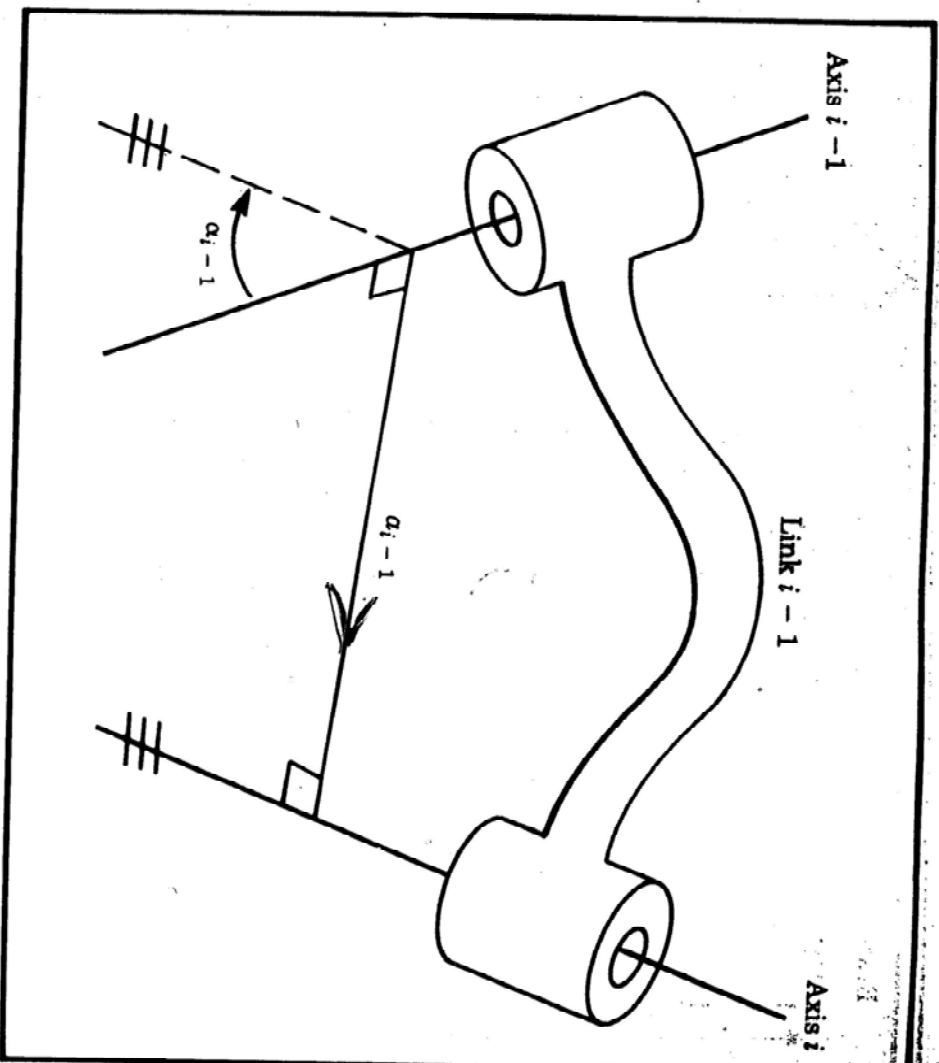
&

$i$  axis.

around

Common &

pointing from

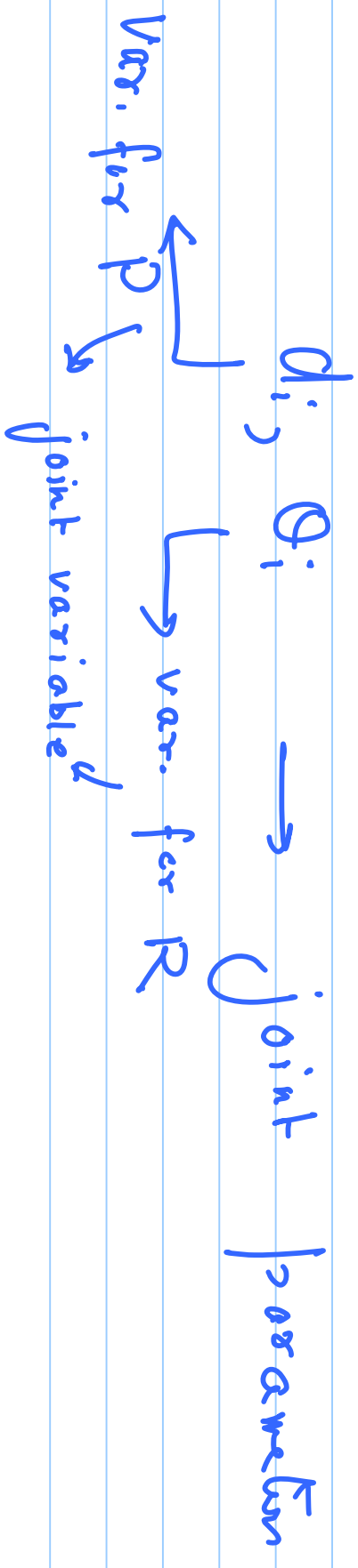


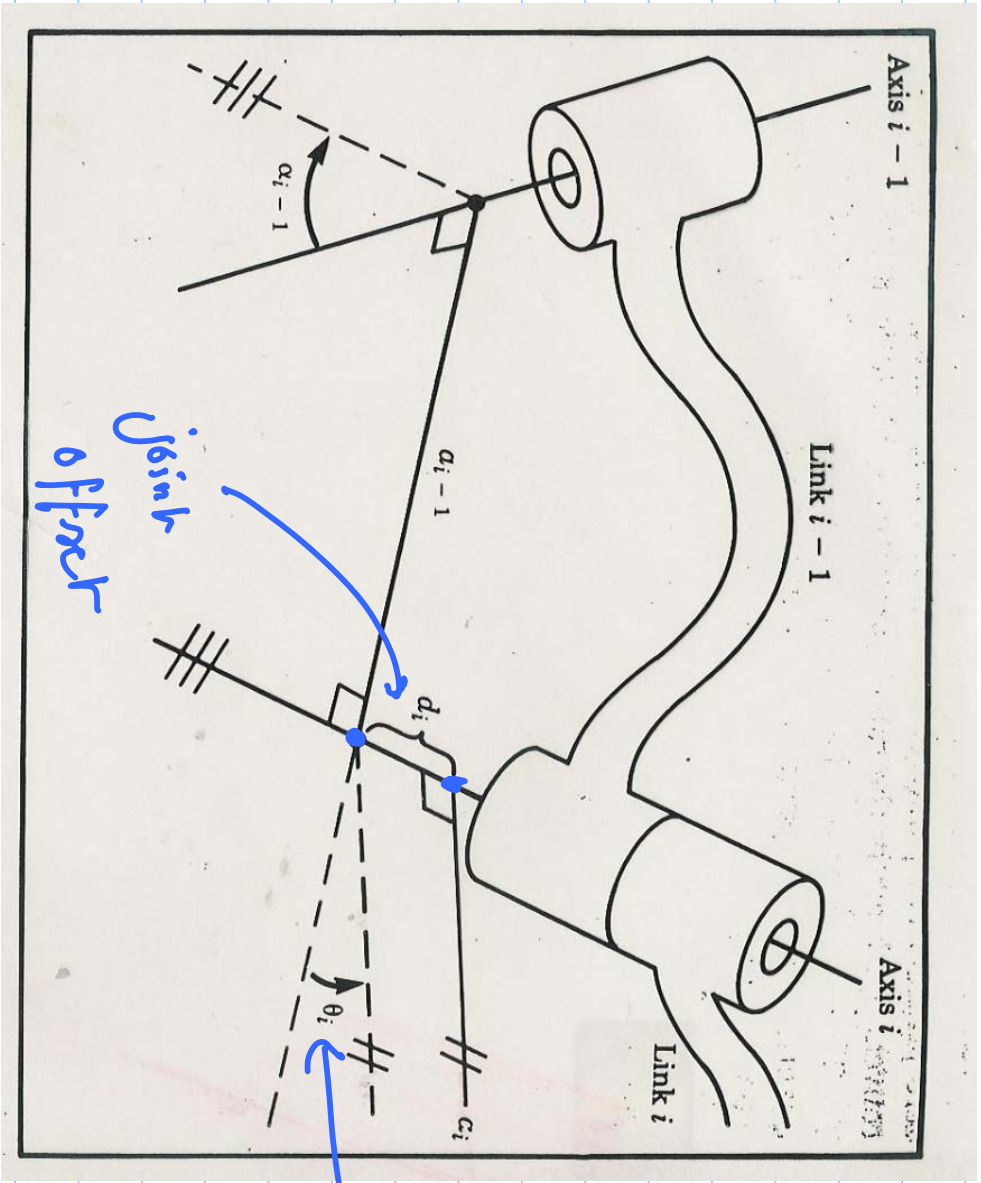
$a_{i-1} =$  link  
length

$\alpha_{i-1} =$  twist  
angle

$i-1$  to  $i$

$q_{i-1}, q_i \rightarrow$  link parameters





joint  
 angle  
 around  
 axis i

## D-H Notation for frame assignment;

~~#~~ assign  $\{i\}$  to joint axis  $i$ .

Recall joint  $i$  moves link  $i$ .

1) assign  $\hat{z}_{i-1}$  to axis  $i-1$ .

2) assign  $X_{i-1}^n$  to common  $\perp$  bet.

axis  $i-1$  &  $i$ . (along  $a_{i-1}$ )  
pointing from  $i-1$  to  $i$ .

3) Origin of  $\{i-1\}$  where the  
Common L. bet.  $\{i-1\}$  &  $i$  intersects  
 $\{i-1\}$  axis

Relationships bet. these two  $\{i-1\}$  &  $\{i\}$   
frames is given by:  $\alpha_{i-1}, a_{i-1}, \theta_i, d_i$

$${}^{i-1}T_i = (\alpha_{i-1}, a_{i-1}, \theta_i, d_i)$$



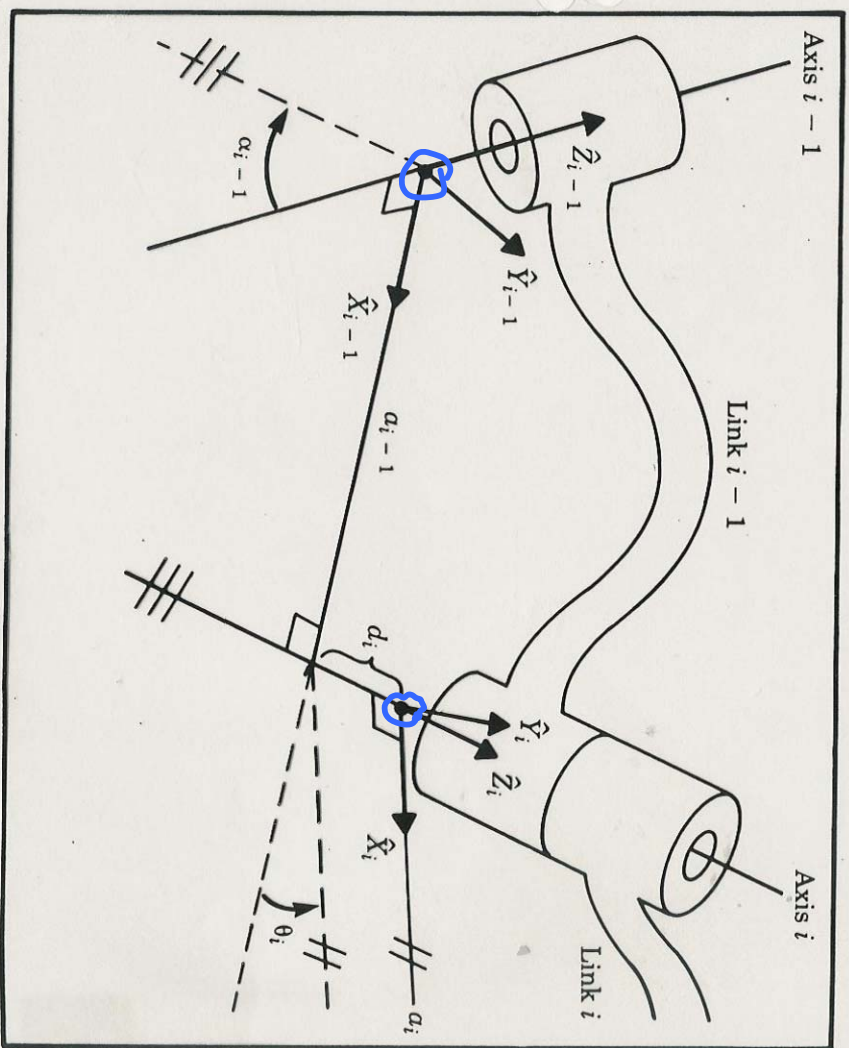


FIGURE 3.5 Link frames are attached so that frame  $\{i\}$  is attached rigidly to link  $i$ .

We can join intermediate frames  $\{P_i, \{Q_i, \{R_i\}$   
(see figure 3.15 below)

$${}_{i-1}T_i = {}_{i-1}R_Q T_P T_i$$

$$= R_X (R_{i-1})_{\text{Trans}}(a_{i-1}) R(\theta_i)_{\text{Trans}}(d_i)$$

We know the form of each of  
these matrices, combining, we  
get:

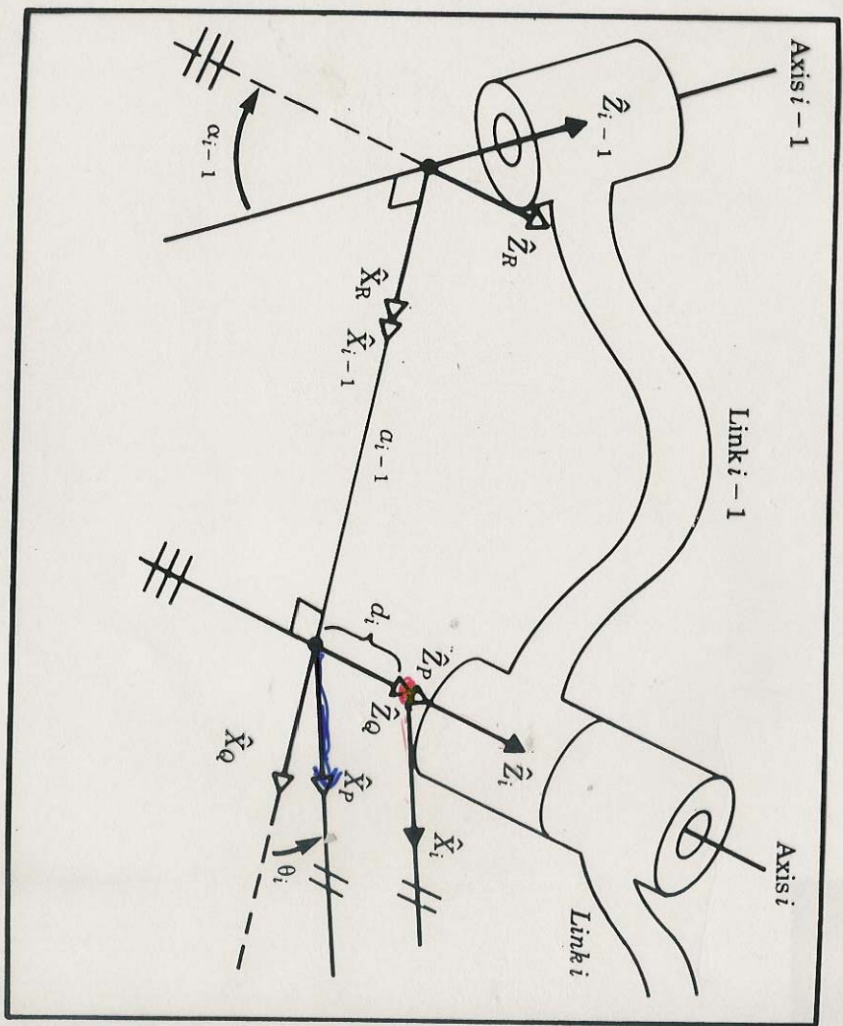


FIGURE 3.15 Location of intermediate frames  $\{P\}$ ,  $\{Q\}$ , and  $\{R\}$ .

we may write

$${}^{i-1}P = {}^{i-1}T {}^R T {}^Q T {}^P T {}^i P, \quad (3.1)$$

or

$${}^{i-1}P = {}^{i-1}T {}^i P, \quad (3.2)$$

where

$${}^{i-1}T = {}^{i-1}T {}^R T {}^Q T {}^P T. \quad (3.3)$$

Considering each of these transformations, we see that (3.3) may be written

$${}^{i-1}T = R_X(\alpha_{i-1}) D_X(a_{i-1}) R_Z(\theta_i) D_Z(d_i), \quad (3.4)$$

or

$${}^{i-1}T = \text{Screw}_X(a_{i-1}, \alpha_{i-1}) \text{Screw}_Z(d_i, \theta_i), \quad (3.5)$$

where the notation  $\text{Screw}_{\hat{Q}}(r, \phi)$  stands for the combination of a translation along an axis  $\hat{Q}$  by a distance  $r$  and a rotation about the same axis by an angle  $\phi$ . Multiplying out (3.4), we obtain the general form of  ${}^{i-1}T$ :

$${}^{i-1}T = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1} d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (3.6)$$