ENSC-887: Computational Robotics
Assignment 1: Background + C-space

Part I Due February 14, 2012; Part II due February 21

Part I: Exercises

1. `Complexity of Motion Planning`.

   In the paper by Canny and Reif (distributed in class), the authors show NP-hardness of shortest path planning (for a point robot) in 3 dimensions. Briefly summarize the result in your own word, giving the idea behind the proof (you need not explain the detailed proof). What is the key distinguishing characteristic of this lower bound as opposed to that for the ruler folding problem we covered in lectures.

2. `Topology of C-space`.

   Consider a simplified 2-link (in plane) robot: the first joint is prismatic with $[-d, d]$ the range of motion and the second joint is revolute with no joint limits. Completely characterize the configuration space of this manipulator. Comment on its local and global topology.

3. Define clearly $\mathcal{CB}_{overlap}$, $\mathcal{CB}_{contact}$, $\partial\mathcal{CB}$ and show an example (other than the one in the book) where $\partial\mathcal{CB} \subset \mathcal{CB}_{contact}$.

4. `Distances in C-space`.

   Problem 6 from Chapter 1, Latombe.

5. `Jacobian`.

6. The position of origin of frame $\{2\}$ of 2-link R-P manipulator (R: revolute joint, P: prismatic joint)is given by:

$$^0P_{2PRG} = \begin{pmatrix} l_1c_1 - d_2s_1 \\ l_1s_1 + d_2c_1 \end{pmatrix}$$

   Give the 2×2 Jacobian matrix that relates the two joint rates to the linear velocity of the origin of frame $\{2\}$. Determine the joint configuration(s) for which there is a singularity. What subspace of velocities is not achievable at these singular configurations.

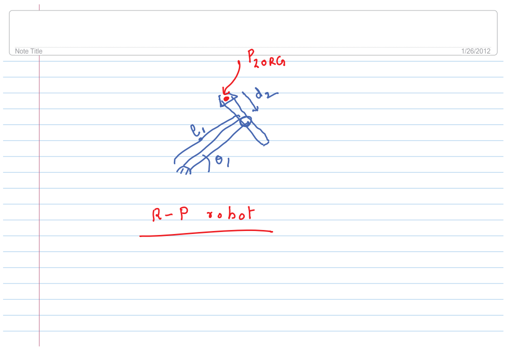

Figure 1: A 2-dof R-P manipulator.

7. `Minkowski Sum`.

   Problem 17 from Chapter 2, Latombe.

8. `Structure of Cspace obstacles: translation and rotation`.

   Consider a simple example where the robot $\mathcal{A}$ is a line segment of unit length and the polygonal obstacle $\mathcal{B}$ is an equilateral triangle with each side of length 2 units. Write the exact expression for the configuration space obstacle $\mathcal{CB}$ in terms of predicate `CB` with each constraint clearly specified with all its parameters and applicability condition.

`Part II: Programming Assignment`

1. Write a program to compute the Minkowski sum of two convex polygons. Your program should accept any convex polygon(s) as its input and should display the input polygons and the output polygon. A brief description of the algorithm is given in Latombe, Chapter 3, Section 1.6. Justify that the complexity of the algorithm is $O(n)$. N.B. Write this nicely. you will subsequently be using this piece of code in the next assignment.