2nd Feb, 2010

Potential function approach

We saw C-space difficult to represent obstacles.

potential function → an alternative approach "similar to penalty function approach" in optimization

Consider pt. robot: first

rather than view an obstacle as a "binary" region, think of them as "repelling" the robot. robot is then essentially a +ve charged "particle" moving under the influence of these forces. [Goal is a -ve charged location]

Lots of different ~~ways here. We will see some.~~

Not in both plans "the field" you impose depends on you. No underlying physics per se!

for now assume point robot $U(q)$ so $C = R^m$

any differentiable real valued function. $U: R^m \to R$

$$\nabla U(q) = \left[\frac{\partial U}{\partial q_1}, \frac{\partial U}{\partial q_2} \cdots \frac{\partial U}{\partial q_m} \right]^T \leftarrow \text{col. vector}$$

[point in the steepest ascent direction of U]

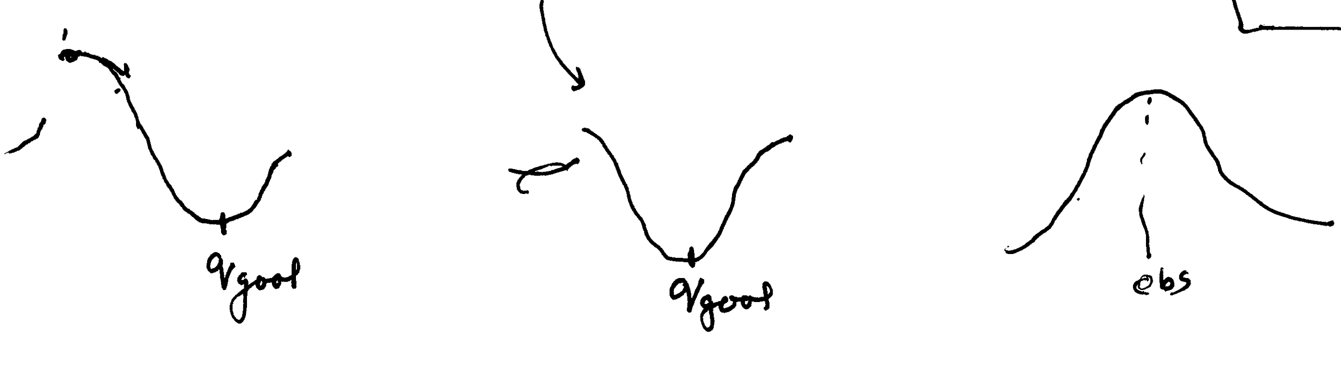
$$q = \begin{bmatrix} q_1 \\ \vdots \\ q_m \end{bmatrix}$$ m-dim config space

$$F(q) = -\nabla U(q)$$

Not nec. → Fig 4.2 chosen as an example

$$U(q) = U_{att}(q) + U_{rep}(q)$$

Fig 1 ch 7. Latombe



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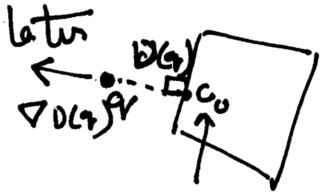
Examples: $U_{att}(q) = \frac{1}{2} \zeta\, d^2(q, q_{goal})$ ← quadratic has nice numerical prop.

$\nabla U_{att}(q) = \zeta\,(q - q_{goal})$

∇U decreases as $q \to q_{goal}$

$$U_{rep}(q) = \begin{cases} \frac{1}{2}\eta \left(\frac{1}{D(q)} - \frac{1}{D^*} \right) & D(q) \le D^* \\ 0 & D(q) > D^* \end{cases}$$

$D(q)$ = "closest distance to obstacle" ⟶ later



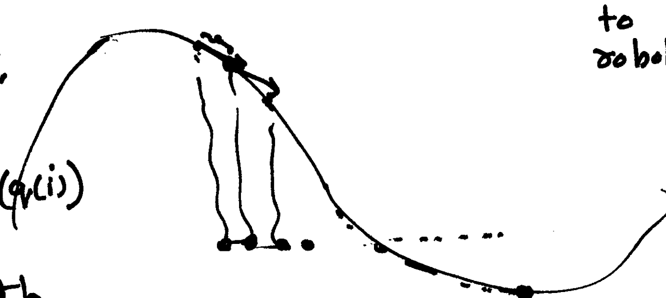
$-\boxed{\nabla U_{att} + \nabla U_{rep}}$ ← robot moves under this

robot particle moves under this

Gradient Descent:

$q(i+1) = q(i) + \underline{\underline{\alpha(i)}} \cdot \nabla U(q(i))$

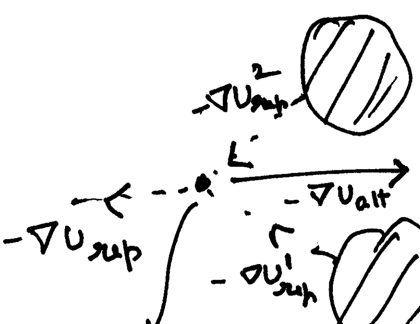
↑ small step



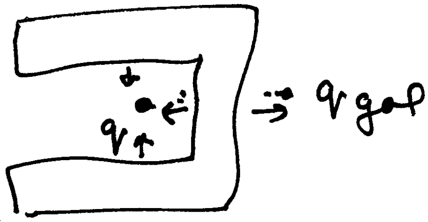
Local minima problem: variety of possibilities

U is not convex!

multiple minima



. q_{goal}



OK for low dim

evaluate not gradient

Lay a grid "Best First" planning like Grad. search until local min

fills the well of local min until you can remove the BF

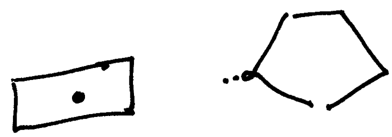
Extension of potential functions to

non-point robots:

1) Translation : $C = R^N$

straight forward.

"only need to figure out closest distance" between ~~two~~ polygons



rest follows.

2) Trans + rotation

C = manifold. $R^N \times SO(N)$

recall we have an atlas, collection of overlapping charts.

~~Once you can~~

1) Use a metric defined ~~in~~ over C, say d

define U_{att}, ~~U goal~~ U_{rep} same way as ~~before~~ based on d.

Computationally difficult

2) ~~$[\nabla U(q)]$~~, Jacobians ~~gradients~~ of "charts" can be used to ~~xfer~~ move from one chart to another.

A practical approach for non-point robots

1) define attractive & repulsive potential function in $W = R^N$ ~~and~~ at several workspace "points" of robot $\mathcal{A}$ and combine their effect.

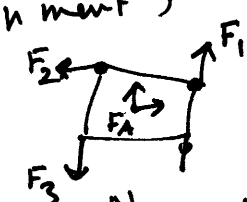
Let $a_j, j = 1, \dots N$ be the # of pts selected on the robot. N = dim. of robot.

workspace pot. → directly V

(no degeneracy: eg. alignment)

$x \in W$

$$V^j_{att}(x) = \frac{1}{2} G \,\|x - a_j(q_{goal})\|$$



$$\vec{F}^j_{att} = - \nabla V^j_{att}(x)$$

→ exerted on a_j

$$F_{att}(q) = \sum_{j=1}^{N} \cancel{\vec{F}^j_{att}(q)} \; J^T \vec{F}^j_{att}(q)$$

↑ 6-d entity

~~sim. U~~

c-space pot → $U^j_{att}(q) = V^j_{att}(a_j(q))$

needs coll. det. → i.e. this configs when coll. occurs are not allowed

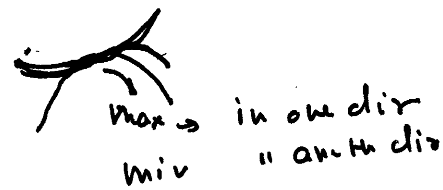
similar for repulsive

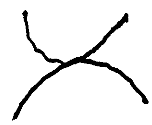
this defines a pot. field over C-space

Navigation functions

a pot. func. with a unique minima

Result: at least some "saddle" points will exist.





horse saddle
curves up in "one" dir
" down " "oth" "

Not critical. slight perturbation, and can resume Grad. search.

Grid Navigation functions

→ no minima due to workspace potentials

Latombe Fig 3.1, 3.2, 4, 5

fast to compute

Finally: Use these as workspace pot.

Use control points on robot.

Gives ⟹ C-space pot. func. + coll. detec.

Discuss RPP briefly.