School of Engineering Science
Simon Fraser University

ENSC-380, Spring 2009
Midterm Test
February 19, 2009

Last Name:

First Name: Solutions

Student No.: _ _ _ _ _ – _ _ _ _ _

- **Aids allowed:** One double sided letter size formula sheet only. **No other aids allowed.**
- Write your answers neatly in the space provided after each question. Show the steps of your solutions. Use the back side of exam sheet for your scrap work.
- **Do not detach the exam sheets.**
- There are **six** questions in this exam for a total of 25 points.
- **Time: 1 Hour and 45 Minutes**

Caution : In accordance with the Academic Honesty Policy (T10.02), academic dishonesty in any form will not be tolerated. Prohibited acts include, but are not limited to, the following:

- *making use of any books, papers, electronic devices or memoranda, other than those authorized by the examiners.*
- *speaking or communicating with other students who are writing the examination.*
- *copying from the work of other candidates or purposely exposing written papers to the view of other candidates.*

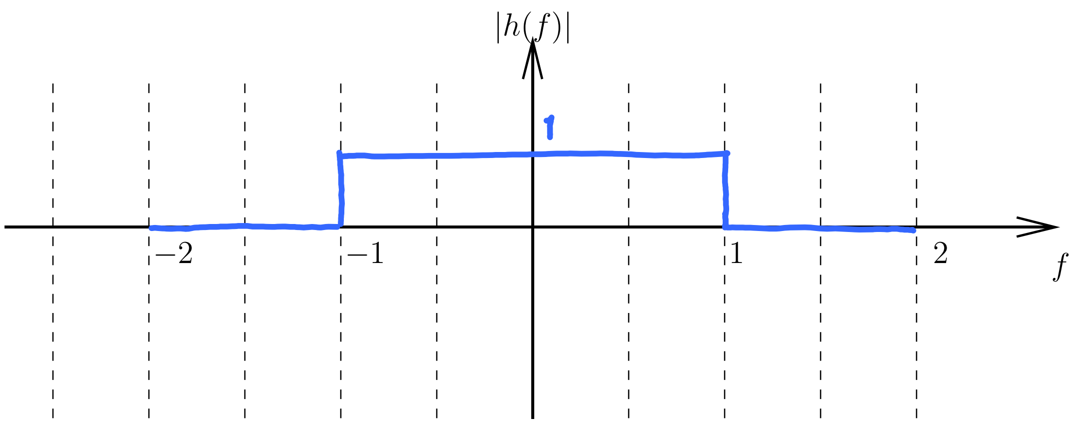
Question	Score
1	/3
2	/3
3	/6
4	/3
5	/5
6	/5
Total	/25

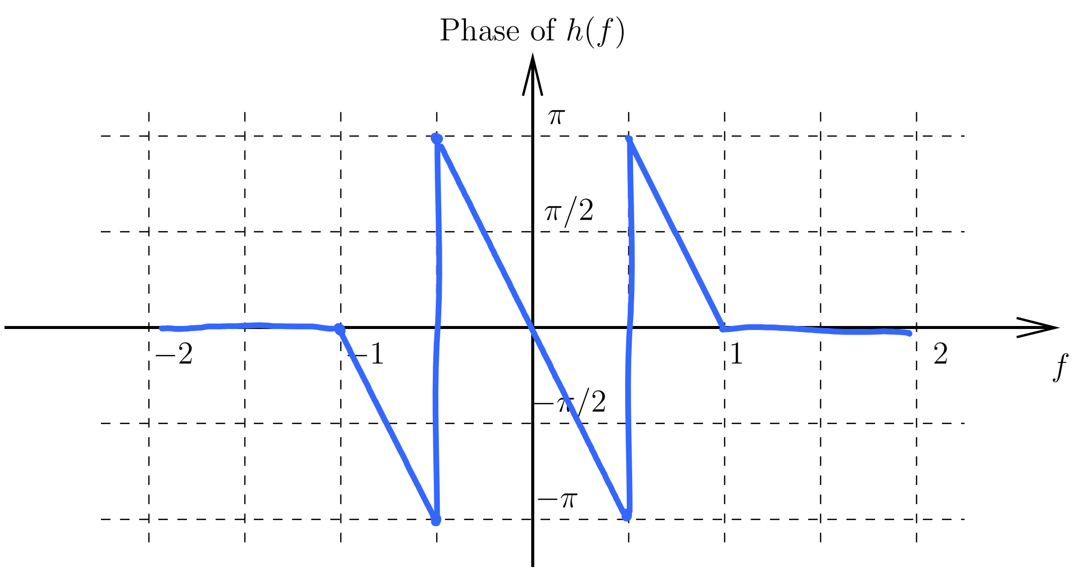
1. **(3 Points)** Sketch the magnitude and phase of the following signal, for $-2 \leq f \leq 2$:

$$h(f) = e^{-j2\pi f} \ \mathrm{rect}(\frac{f}{2})$$

Use the space below for your intermediate work and show your final answer on the provided graphs. Show all phase values in the range of $[-\pi, \pi]$ only.

$|h(f)| = \mathrm{rect}(\frac{f}{2})$
$\angle h(f) = -2\pi f$





2. **(3 Points)** Find the analytical answer to the following DT convolution and show the result on the graph provided. Indicate the values of n and $y[n]$ clearly on the graph.

$$y[n] = \mathrm{rect}[n] * \mathrm{comb}_{10}[2n]$$

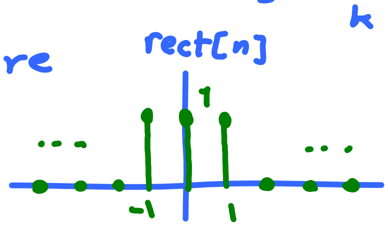
$$\mathrm{comb}_{10}[n] = \sum_{k=-\infty}^{\infty} \delta[n-10k]$$

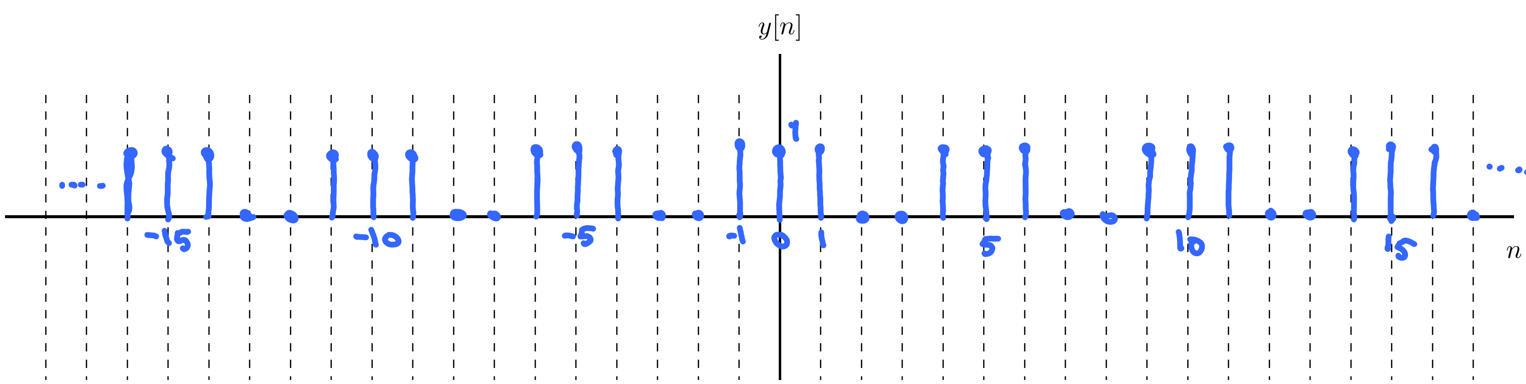
$$\mathrm{comb}_{10}[2n] = \sum_{k} \delta[2n-10k] = \sum_{k} \delta[2(n-5k)]$$

$$= \sum_{k} \delta[n-5k] = \mathrm{comb}_5[n]$$

$$\Rightarrow y[n] = \mathrm{rect}[n] * \sum_{k} \delta[n-5k] = \sum_{k} \mathrm{rect}[n-5k]$$

where





3. The input-output relationship of a CT system is given: ($x(t)$:Input, $y(t)$: Output)

$$y(t) = \int_{-\infty}^{t/3} x(\lambda)\, d\lambda$$

- **a)(2 Points)** Is this system "time invariant"? Why?

If $g(t) = x(t-t_0)$ & $y_g(t)$ is the response to $g(t)$, we have:

$y_g(t) = \int_{-\infty}^{t/3} g(\lambda)\, d\lambda = \int_{-\infty}^{t/3} x(\lambda - t_0)\, d\lambda$

Let $\lambda - t_0 = \gamma \Rightarrow \frac{t}{3} \rightarrow \frac{t}{3} - t_0$, $d\lambda = d\gamma$

$y_g(t) = \int_{-\infty}^{\frac{t}{3}-t_0} x(\gamma)\, d\gamma \neq \int_{-\infty}^{\frac{t-t_0}{3}} x(\gamma) d\gamma = y(t-t_0)$

$\Rightarrow$ Time Variant

- **b)(2 Points)** Is this system "stable"? Why?

Assume $x(t) = k < \infty$ bounded

$y(t) = \int_{-\infty}^{\frac{t}{3}} x(\lambda) d\lambda = \int_{-\infty}^{\frac{t}{3}} k\, d\lambda = \infty$ unbounded

$\Rightarrow$ Unstable

- **c)(2 Points)** Is this system "invertible"? Why?

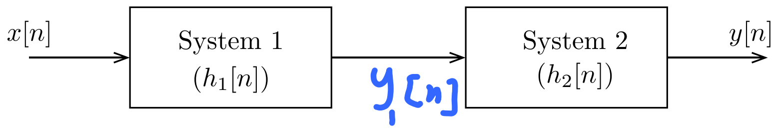
$y(t) = \int_{-\infty}^{\frac{t}{3}} x(\lambda) d\lambda \Rightarrow y'(t) = x(\frac{t}{3})$

$\Rightarrow x(t) = y'(3t)$

$\Rightarrow$ if we have $y(t)$ we can find $x(t)$ uniquely

$\Rightarrow$ Invertible

4. **(3 Points)** Consider the following cascade connection of two LTI systems, where the impulse response of each sub-system is given:



$$h_1[n] = \alpha^n \, u[n] \qquad\qquad h_2[n] = sin(8n)$$

What is the response of the system to $x[n] = \delta[n] - \alpha\delta[n-1]$?

$$y[n] = x[n] * (h_1[n] * h_2[n]) = (x[n] * h_1[n]) * h_2[n]$$

$$= y_1[n] * h_2[n]$$

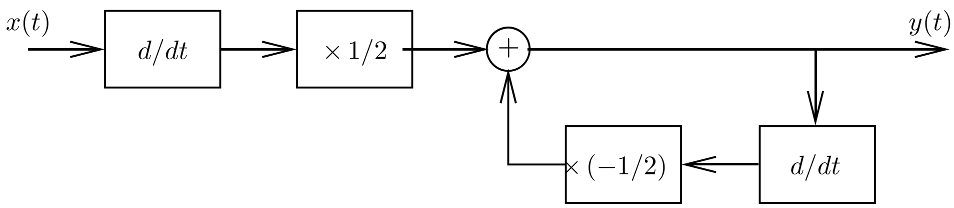
$$y_1[n] = (\delta[n] - \alpha\,\delta[n-1]) * \alpha^n u[n]$$

$$= \alpha^n u[n] - \alpha(\alpha^{n-1} u[n-1]) = \alpha^n (u[n] - u[n-1])$$

$$= \alpha^n \delta[n] = \delta[n]$$

$$y[n] = y_1[n] * h_2[n] = \delta[n] * \text{Sin}(8n) = \text{Sin}(8n)$$

5. The block diagram of a CT-LTI system is given below, where d/dt represents a differentiation module.



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- **a)(2 Points)** What is the differential equation corresponding to this systems?

$$y(t) = \frac{1}{2}x'(t) - \frac{1}{2}y'(t)$$

$$\Rightarrow y'(t) + 2y(t) = x'(t)$$

b)(3 Points) What is the impulse response of the system?

$$h'(t) + 2h(t) = \delta'(t)$$

for $t<0 \Rightarrow h(t) = 0$

for $t>0 \Rightarrow h'(t) + 2h(t) = 0 \Rightarrow h(t) = Ae^{-2t}u(t)$

Since we have $\delta'(t)$ on the R.H.S. of the equation, we need to have $\delta(t)$ in $h(t)$, thus:

$$h(t) = Ae^{-2t}u(t) + B\delta(t)$$

at $t=0$ we have:

$$-\cancel{2Ae^{-2t}u(t)} + A\underbrace{e^{-2t}}_{=1}\delta(t) + B\delta'(t)$$
$$+\cancel{2Ae^{-2t}u(t)} + 2B\delta(t) = \delta'(t)$$

$$\Rightarrow \begin{cases} A + 2B = 0 \\ B = 1 \end{cases} \Rightarrow \begin{cases} A = -2 \\ B = 1 \end{cases} \Rightarrow h(t) = -2e^{-2t}u(t) + \delta(t)$$

6. **(5 Points)** Use the properties of CTFS and the formula given below:

$$\text{tri}(\frac{t}{w}) * \frac{1}{T_0}\text{comb}(\frac{t}{T_0}) \overset{FS}{\longleftrightarrow} \frac{w}{T_0}\text{sinc}^2(\frac{w}{T_0}k) \qquad T_F = T_0$$

to find the CTFS representation of:

$$x(t) = 5[\text{tri}(t-1) - \text{tri}(t+1)] * \frac{1}{4}\text{comb}(\frac{t}{4}) \qquad T_F = 4$$

Try to simplify your answer as much as possible.

Using the formula: $\text{tri}(t) * \frac{1}{4}\text{comb}(\frac{t}{4}) \underset{T_F = T_0 = 4}{\overset{FS}{\longleftrightarrow}} \frac{1}{4}\text{Sinc}^2(\frac{k}{4})$

$w = 1$

Time shifting:

$$\text{tri}(t-1) * \frac{1}{4}\text{comb}(\frac{t}{4}) \underset{T_F = 4}{\overset{FS}{\longleftrightarrow}} e^{-j2\pi k f_0 \times 1} \times \frac{1}{4}\text{Sinc}^2(\frac{k}{4})$$

$$= \frac{1}{4} e^{-j\frac{k\pi}{2}} \text{Sinc}^2(\frac{k}{4})$$

$$\text{tri}(t+1) * \frac{1}{4}\text{comb}(\frac{t}{4}) \underset{T_F = 4}{\overset{FS}{\longleftrightarrow}} \frac{1}{4} e^{j\frac{k\pi}{2}} \text{Sinc}^2(\frac{k}{4})$$

$$\Rightarrow x(t) \underset{T_F = 4}{\overset{FS}{\longleftrightarrow}} \frac{5}{4}(e^{-j\frac{k\pi}{2}} - e^{j\frac{k\pi}{2}}) \text{Sinc}^2(\frac{k}{4})$$

$$= \frac{-j5}{2} \text{Sin}(\frac{k\pi}{2}) \text{Sinc}^2(\frac{k}{4})$$