ENSC380
Lecture 10

Objectives:

- Learn how to find the impulse response of a CT-LTI system ($h(t)$)
- Learn how to represent a system (DT or CT) in form of a block diagram

CT Impulse Response

- Any CT and LTI system can be shown with a linear constant coefficient differential equation of:

$$a_n y^{(n)}(t) + a_{n-1} y^{(n-1)}(t) + \ldots + a_1 y'(t) + a_0 y(t) =$$
$$b_m x^{(m)}(t) + b_{m-1} x^{(m-1)}(t) + \ldots + b_1 x'(t) + b_0 x(t)$$

- The impulse response of the system is the answer to the above equation when $x(t) = \delta(t)$.
- In Lecture 9 we assumed the impulse response is given, and we found that the response of the system to an arbitrary input (excitation) signal ($x(t)$ is:
- Today we want to step back and learn how to find the impulse response itself.
- We will see that depending on the values of m and n in the above equation, the form of the impulse response varies (slightly).
- We will see this through some examples.

Example 1

Find the impulse response of a system described by the following differential equation:

$$y'(t) + ay(t) = x(t)$$

This means find $h(t)$ such that:

$$h'(t) + ah(t) = \delta(t)$$

- For $t < 0$ we know that:
- for $t > 0$, the differential equation becomes and the general answer to it is:

Example 1 (Cont.)

- for $t = 0$ we need to find out if $h(t)$ includes a $\delta(t)$ or higher order singularity functions at $t = 0$ or not. The equation at $t = 0$ becomes:

$$h'(t) + ah(t) = \delta(t)$$

- If $h(t)$ includes a $\delta(t)$ function, then, $h'(t)$ includes the function. However, the right hand side of the equation ($\delta(t)$) does not have this function!
- So we conclude that $h(t)$ does not include $\delta(t)$ or higher order singularity functions and the complete solution is :

$$h(t) = Ke^{-at}u(t) \qquad K = ?$$

- To find K, we integrate both sides of the equation from 0^- to 0^+:

Example 2

Find the impulse response of:

$$y'(t) + ay(t) = x'(t)$$

- For $t < 0$ and $t > 0$ the results are as in the previous example:
  $$h(t) = 0 \;\; t < 0 \qquad \text{and} \qquad h(t) = K_0 e^{-at} \;\; t > 0$$
- However, at $t = 0$ the left hand side of equation needs to have a doublet function, which means $h(t)$ needs to have a function. The complete answer is thus:
  $$h(t) = K_0 e^{-at} u(t) + K_1 \delta(t)$$
- To find K_0 and K_1, we replace h(t) in the equation:
- Note: In the Text $\delta(t)$ is shown with $u_0(t)$, $\delta'(t)$ with $u_1(t)$, $\delta''(t)$ with $u_2(t)$, ...

Example 3

Find the impulse response of:

$$y'(t) + ay(t) = x''(t) + bx(t)$$

- Again, same situation for $t < 0$ and $t > 0$.
- To satisfy the equation at $t = 0$, $h(t)$ needs to have the $\delta(t)$ and its derivatives, such that $h'(t)$, will include a term $\delta''(t)$, thus:

$$h(t) = K_0 e^{-at} u(t) + K_1 \delta(t) + K_2 \delta'(t)$$

- Again, to find K_0, K_1, and K_2, we replace $h(t)$ in the differential equation:

General Rules

$$a_n y^{(n)}(t) + a_{n-1} y^{(n-1)}(t) + \ldots + a_1 y'(t) + a_0 y(t) =$$
$$b_m x^{(m)}(t) + b_{m-1} x^{(m-1)}(t) + \ldots + b_1 x'(t) + b_0 y(t)$$

- If $m < n$: The impulse response includes only the homogeneous response:

$$h(t) = y_h(t)u(t)$$

- If $m = n$: The impulse response includes the homogeneous response and an impulse function but no derivatives of the impulse function:

$$h(t) = K_0 y_h(t)u(t) + K_1 \delta(t)$$

- If $m > n$: The impulse response could contain an impulse function, plus derivatives of the impulse function up to the $(m-n)$th derivative.

$$h(t) = K_0 y_h(t)u(t) + K_1 \delta(t) + \ldots + K_{m-n} \delta^{m-n}(t)$$

DT Block Diagram

- We are already familiar with representing an DT-LTI system with its corresponding block diagram.
- Example:

$$y[n] + 3y[n-1] - 2y[n-2] = x[n]$$

CT Block Diagram

- CT block diagrams are similar to DT, with a little twist!
- Example:

$$2y''(t) + 5y'(t) + 4y(t) = x(t)$$

- Use blocks of differentiators to draw the block diagram:

- It is conventional to implement the differential equation using integrals instead of differentiators, because integrators are easier to implement in practice.

CT Block Diag. (Cont.)

- Write the differential equation with $y''(t)$ on the left hand side:

- Now, implement using blocks of integrators: