ENSC380
Lecture 17

Objectives:

- Learn the definition of the **discrete-time Fourier Transform** (DTFT)
- Learn the properties of DTFT

DTFT

- DTFS is defined for DT signals which are periodic.
- DTFT is defined for all DT signals (periodic or aperiodic).
- The approach to reach from DTFS to DTFT is very similar to the CT case.
- Here we only state the final result:

$$x[n] = \int_1 X(F)e^{j2\pi Fn}dF$$

Note: The integral is over any time interval of length 1 !

$$X(F) = \sum_{n=-\infty}^{\infty} x[n]e^{-j2\pi Fn}$$

Note: $X(F)$ is periodic with period .

- Notations:

$$x[n] \stackrel{\mathcal{F}}{\longleftrightarrow} X(F) \qquad X(F) = \mathcal{F}(x[n]) \qquad x[n] = \mathcal{F}^{-1}(X(F))$$

Examples

- **Example 1:** Find the DTFT of $\delta[n-1] + \delta[n+1]$

- **Example 2:** Find DTFT of $(\frac{1}{2})^{n-1}u[n-1]$

Properties of DTFT

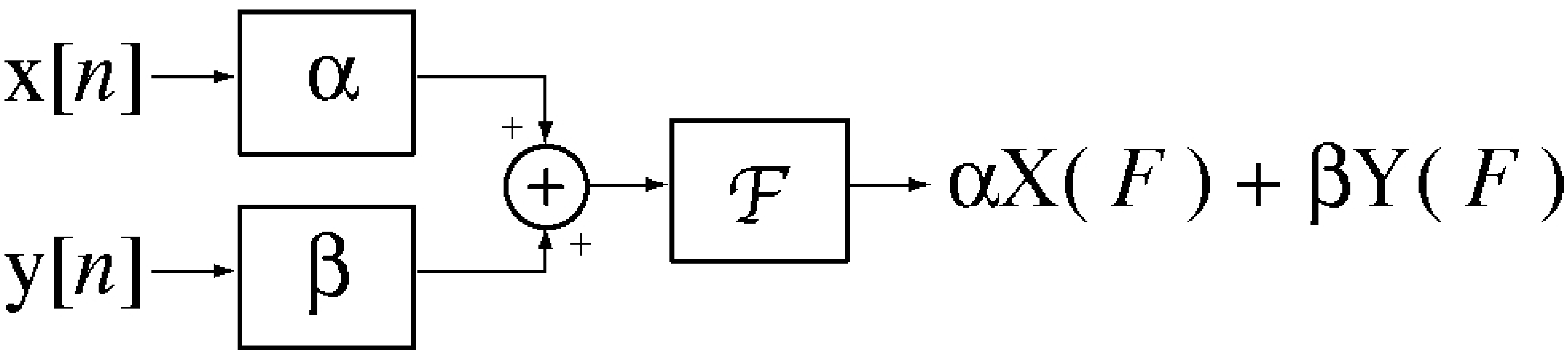
- Here we list the properties of DTFT.
- Using these properties and the DTFT pairs listed in Appendix E, we can find the DTFT of many signals.

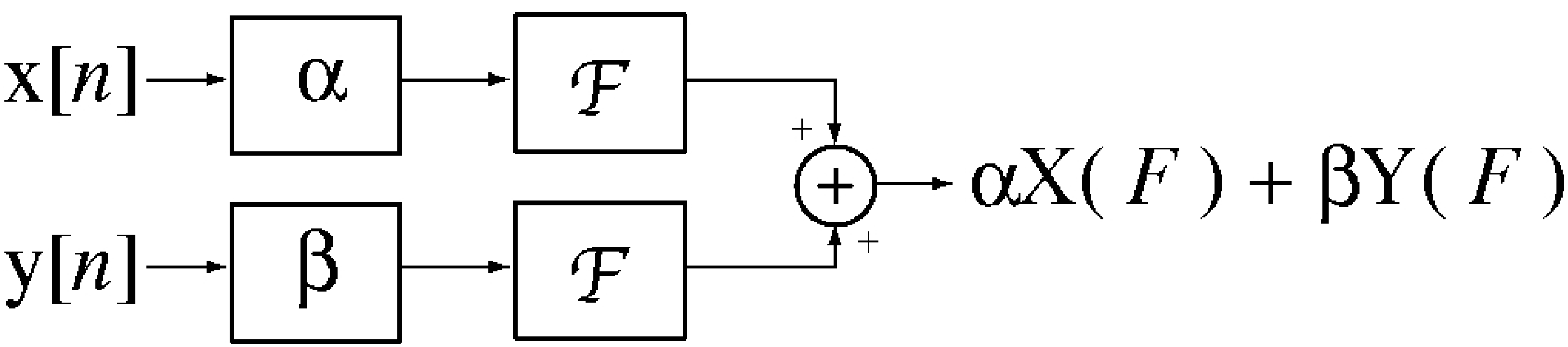
DTFT Properties

Linearity

$$\alpha\,\mathrm{x}[n]+\beta\,\mathrm{y}[n] \xleftrightarrow{\mathrm{F}} \alpha\,\mathrm{X}(F)+\beta\,\mathrm{Y}(F)$$

$$\alpha\,\mathrm{x}[n]+\beta\,\mathrm{y}[n] \xleftrightarrow{\mathrm{F}} \alpha\,\mathrm{X}(j\Omega)+\beta\,\mathrm{Y}(j\Omega)$$



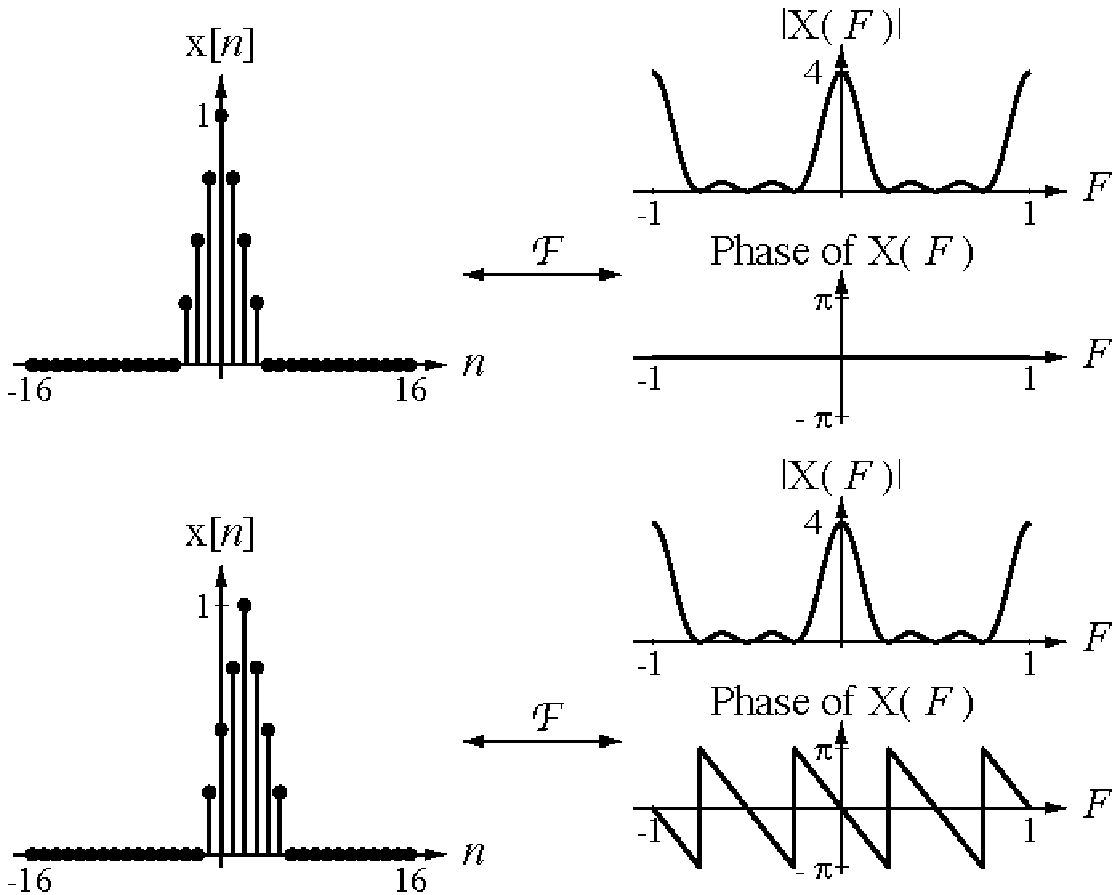


DTFT Properties

Time Shifting

$$x[n-n_0] \xleftrightarrow{\mathcal{F}} e^{-j2\pi F n_0}\,X(F)$$

$$x[n-n_0] \xleftrightarrow{\mathcal{F}} e^{-j\Omega n_0}\,X(j\Omega)$$



DTFT Properties

Frequency Shifting

$$e^{j2\pi F_0 n}\,\mathrm{x}[n] \xleftrightarrow{\mathcal{F}} \mathrm{X}(F - F_0)$$

$$e^{j\Omega_0 n}\,\mathrm{x}[n] \xleftrightarrow{\mathcal{F}} \mathrm{X}(j(\Omega - \Omega_0))$$

Time Reversal

$$\mathrm{x}[-n] \xleftrightarrow{\mathcal{F}} \mathrm{X}(-F)$$

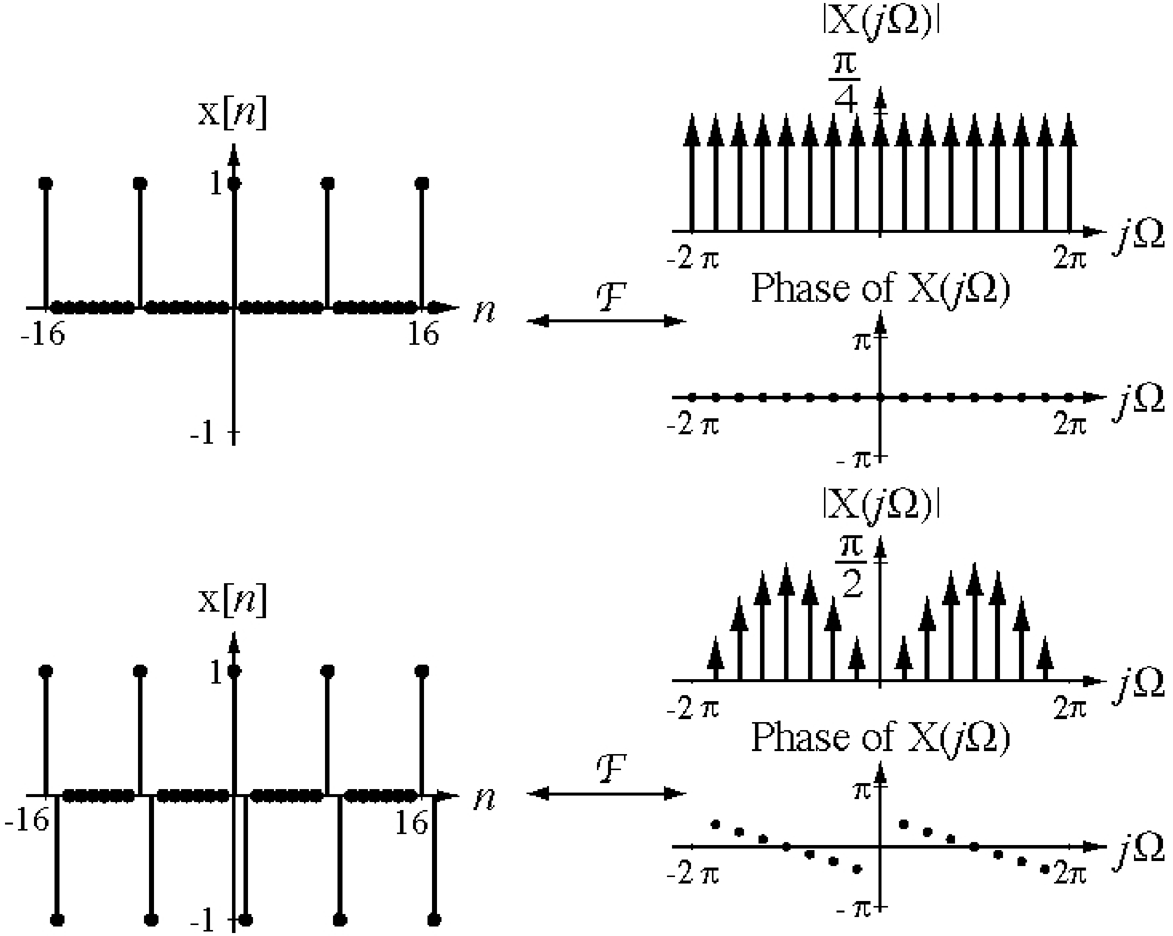
$$\mathrm{x}[-n] \xleftrightarrow{\mathcal{F}} \mathrm{X}(-j\Omega)$$

DTFT Properties

Differencing

$$x[n]-x[n-1]\xleftrightarrow{\mathcal{F}}\left(1-e^{-j2\pi F}\right)X(F)$$

$$x[n]-x[n-1]\xleftrightarrow{\mathcal{F}}\left(1-e^{-j\Omega}\right)X(j\Omega)$$

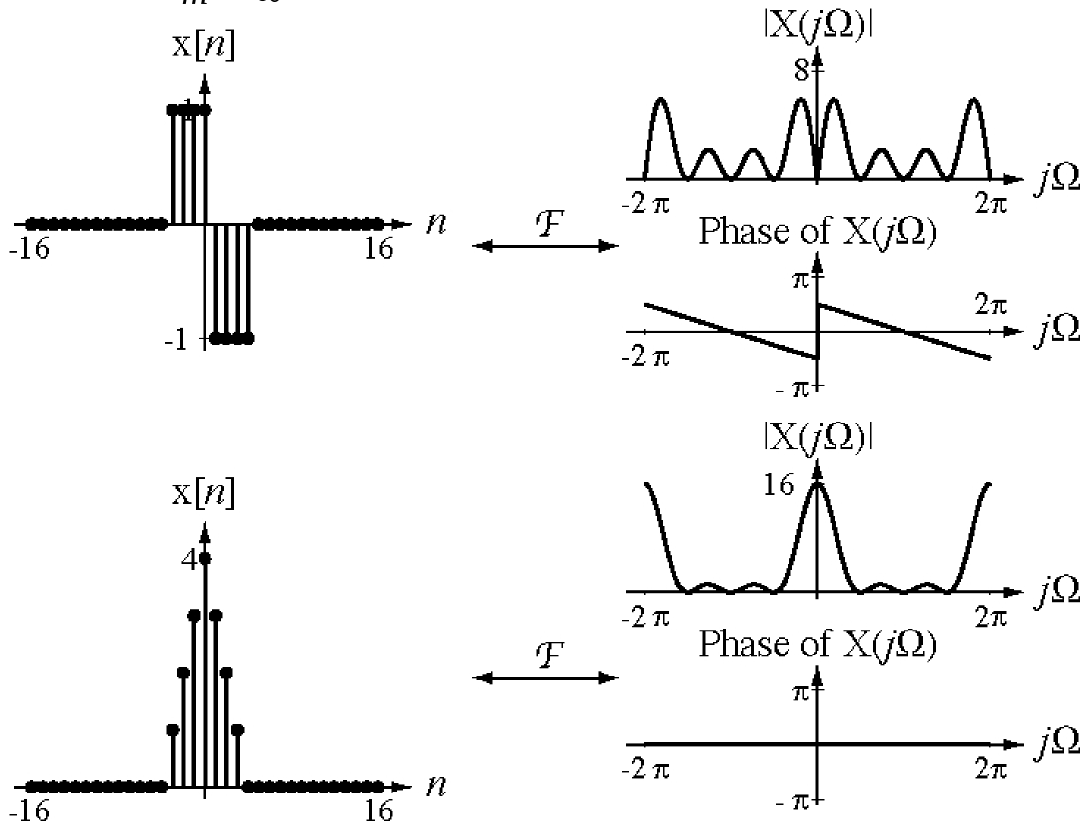


DTFT Properties

Accumulation

$$\sum_{m=-\infty}^{n} \mathrm{x}[m] \xleftrightarrow{\mathrm{F}} \frac{\mathrm{X}(F)}{1-e^{-j2\pi F}} + \frac{1}{2}\mathrm{X}(0)\mathrm{comb}(F)$$

$$\sum_{m=-\infty}^{n} \mathrm{x}[m] \xleftrightarrow{\mathrm{F}} \frac{\mathrm{X}(j\Omega)}{1-e^{-j\Omega}} + \frac{1}{2}\mathrm{X}(0)\mathrm{comb}\left(\frac{\Omega}{2\pi}\right)$$



DTFT Properties

Multiplication-Convolution Duality

$$\mathrm{x}[n] * \mathrm{y}[n] \xleftrightarrow{\mathcal{F}} \mathrm{X}(F)\mathrm{Y}(F)$$

$$\mathrm{x}[n] * \mathrm{y}[n] \xleftrightarrow{\mathcal{F}} \mathrm{X}(j\Omega)\mathrm{Y}(j\Omega)$$

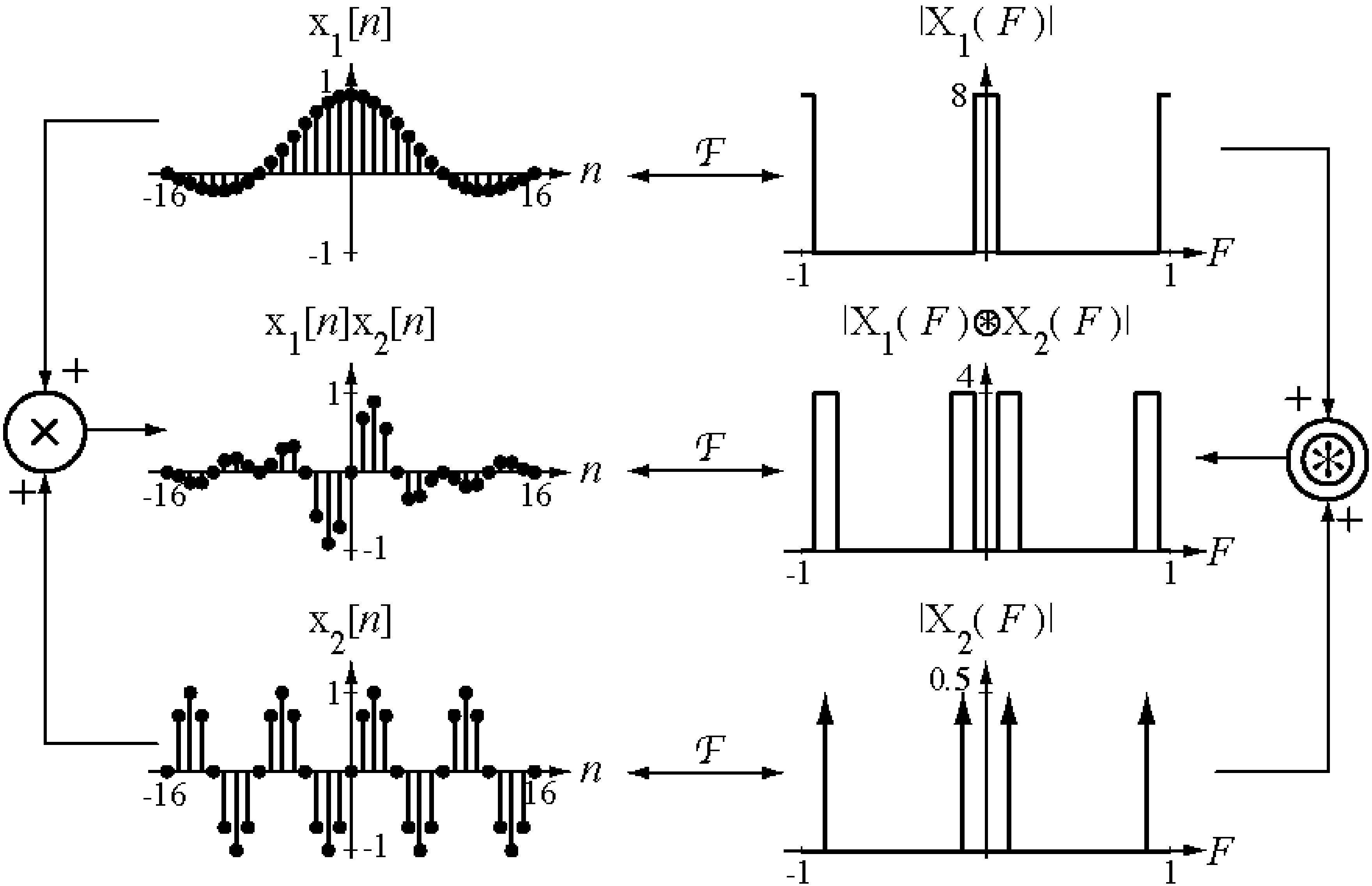
$$\mathrm{x}[n]\mathrm{y}[n] \xleftrightarrow{\mathcal{F}} \mathrm{X}(F) \circledast \mathrm{Y}(F)$$

$$\mathrm{x}[n]\mathrm{y}[n] \xleftrightarrow{\mathcal{F}} \frac{1}{2\pi}\mathrm{X}(j\Omega) \circledast \mathrm{Y}(j\Omega)$$

As in other transforms, convolution in the time domain is equivalent to multiplication in the frequency domain

$$\mathrm{x}[n] \rightarrow \boxed{\mathrm{h}[n]} \rightarrow \mathrm{y}[n] = \mathrm{h}[n] * \mathrm{x}[n] \qquad \mathrm{X}(F) \rightarrow \boxed{\mathrm{H}(F)} \rightarrow \mathrm{Y}(F) = \mathrm{H}(F)\mathrm{X}(F)$$

DTFT Properties



DTFT Properties

Accumulation Definition of a Comb Function

$$\sum_{n=-\infty}^{\infty} e^{j2\pi Fn} = \mathrm{comb}(F)$$

Parseval's Theorem

$$\sum_{n=-\infty}^{\infty} \left|\mathrm{x}[n]\right|^2 = \int_1 \left|\mathrm{X}(F)\right|^2 dF$$

$$\sum_{n=-\infty}^{\infty} \left|\mathrm{x}[n]\right|^2 = \frac{1}{2\pi}\int_{2\pi} \left|\mathrm{X}(j\Omega)\right|^2 d\Omega$$

The signal energy is proportional to the integral of the squared magnitude of the DTFT of the signal over one period.

Example 3

Find the signal energy of

$$x[n] = \frac{1}{5}\text{sinc}(\frac{n}{100})$$

Example 4

Assuming the DTFT of $x[n]$ is $X(F)$, find the DTFT of

$$\frac{x^*[-n] + x[n]}{2}$$