ENSC380
Lecture 18

Objectives:

- Summary of different Fourier Transform methods
- Learn the relationship between different DT methods

Summary of CT-FTs

- CTFS: Applies to CT and periodic signals:

$$x(t) = \sum_{k=-\infty}^{\infty} X[k] e^{j2\pi(kf_0)t}$$

$$X[k] = \frac{1}{T_0} \int_{t_0}^{t_0+T_0} x(t) e^{j2\pi(kf_0)t} dt$$

- CTFT: Applies to CT signals (periodic or aperiodic):

$$X(f) = \mathcal{F}(x(t)) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt$$

$$x(t) = \mathcal{F}^{-1}(X(f)) = \int_{-\infty}^{\infty} X(f) e^{j2\pi ft} df$$

Summary of DT-FTs

- DTFS: Applies to DT and periodic signals:

$$x[n] = \sum_{k=k_0}^{k_0+N_0-1} X[k]e^{j2\pi(kF_0)n} = \sum_{<N_0>} X[k]e^{j2\pi(kF_0)n}$$

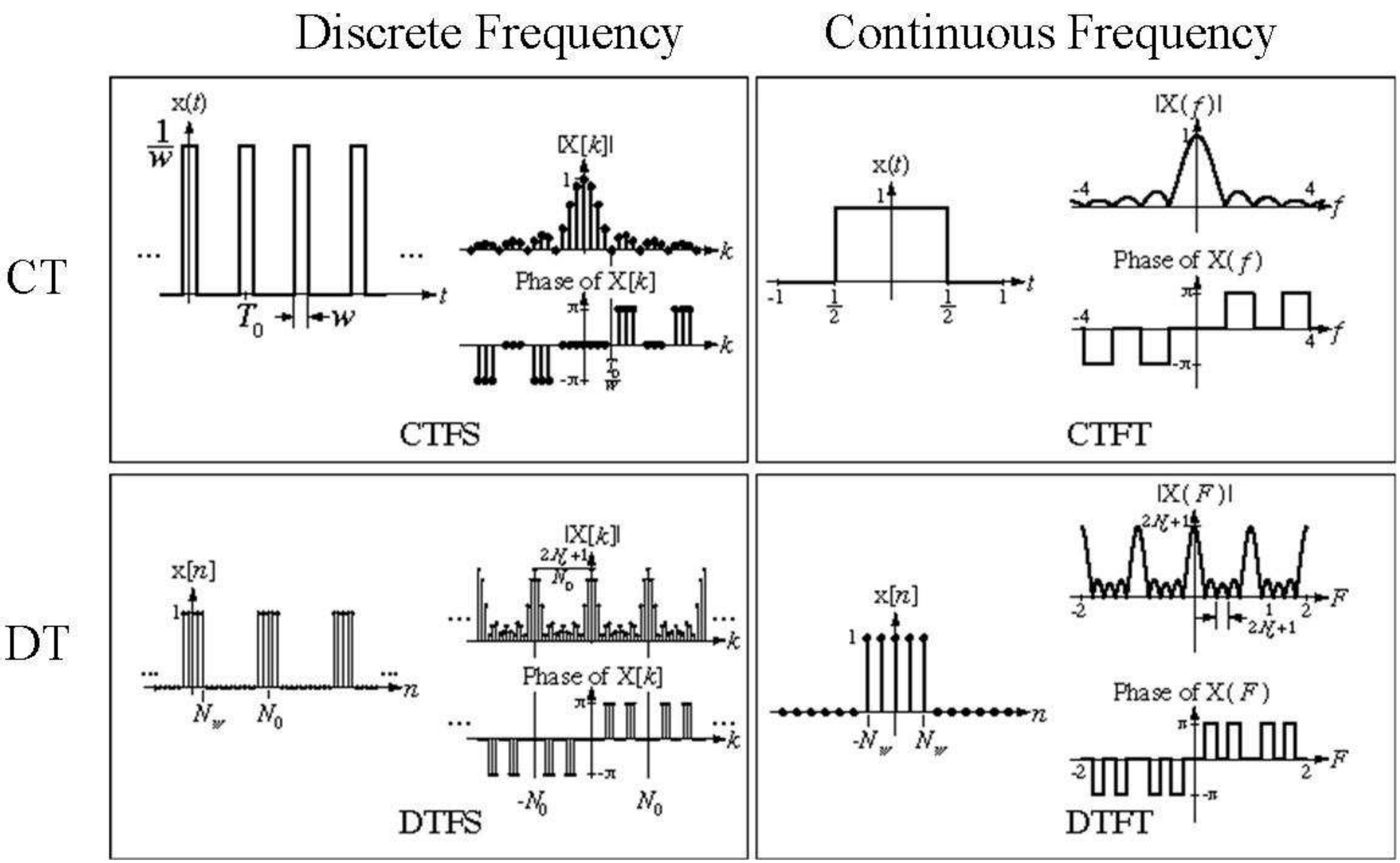
$$X[k] = \frac{1}{N_0}\sum_{n=n_0}^{n_0+N_0-1} x[n]e^{-j2\pi(kF_0)n} = \sum_{n=<N_0>} x[n]e^{-j2\pi(kF_0)n}$$

- DTFT: Applies to DT signals (periodic or aperiodic):

$$x[n] = \int_1 X(F)e^{j2\pi Fn}dF$$

$$X(F) = \sum_{n=-\infty}^{\infty} x[n]e^{-j2\pi Fn}$$

FT Methods


Discrete Frequency
Continuous Frequency
CT
DT
x(t)
|X[k]|
Phase of X[k]
CTFS
|X(f)|
Phase of X(f)
CTFT
x[n]
DTFS
|X(F)|
Phase of X(F)
DTFT

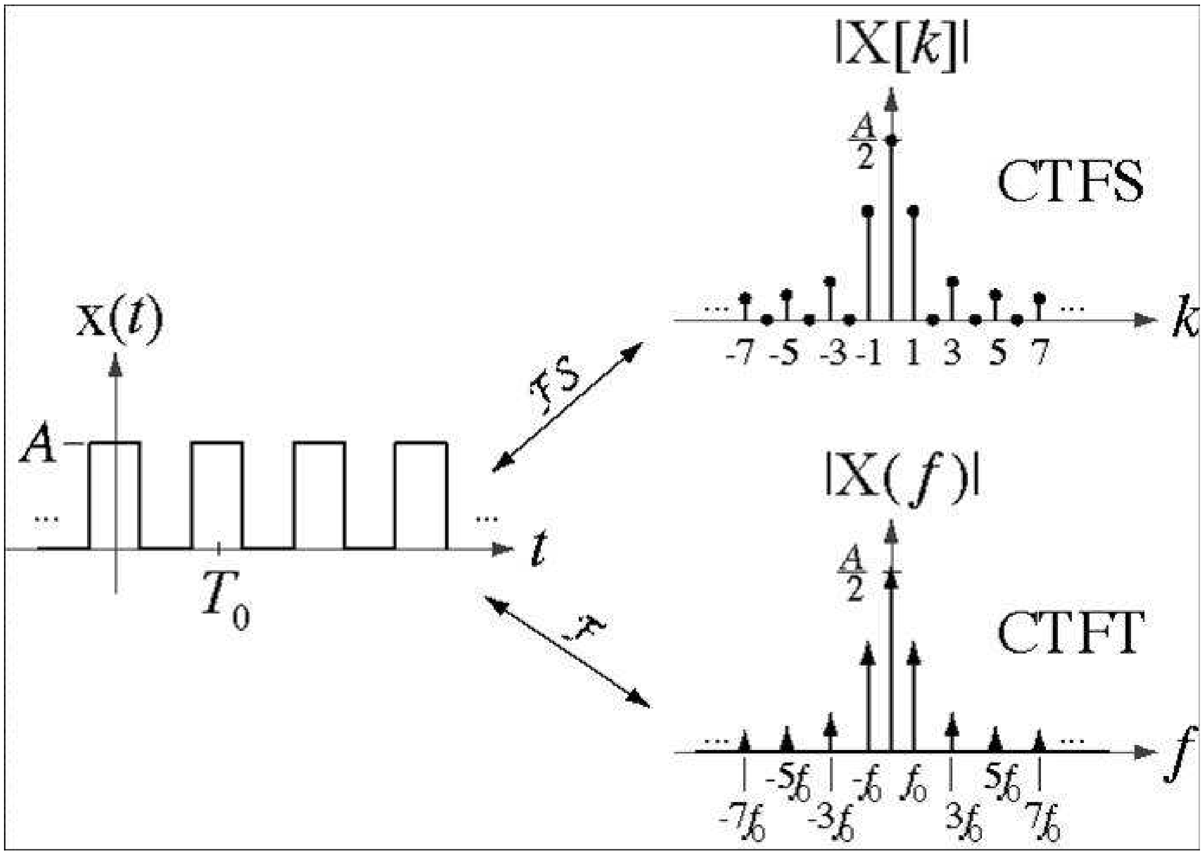
CTFT-CTFS Relationship (1)

If a periodic signal, $x(t)$, has the CTFS coefficients (harmonic function), $X[k]$, we can write its CTFS representation as:

$$x(t) =$$

Then the CTFT for $x(t)$ is:

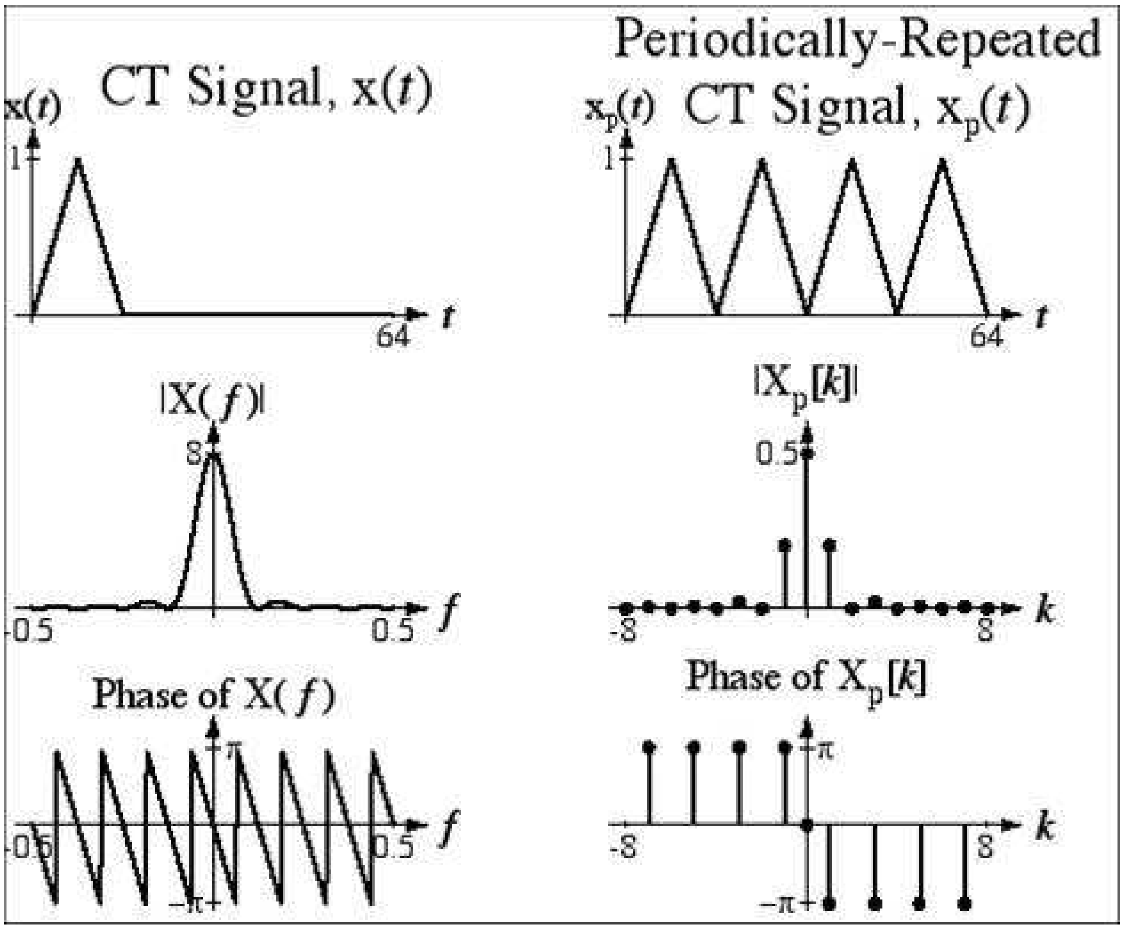
$$X(f) = \mathcal{F}(x(t)) =$$



CTFT-CTFS Relationship (2)

An aperiodic signal, $x(t)$, has CTFT $X(F)$. Create a periodic signal, $x_p(t)$, from $x(t)$ by repeating it every $T_p = 1/f_p$ seconds. $x_p(t)$ then has a CTFS representation and its harmonic function, $X_p[k]$ relates to $X(f)$:

$$X_p[k] =$$



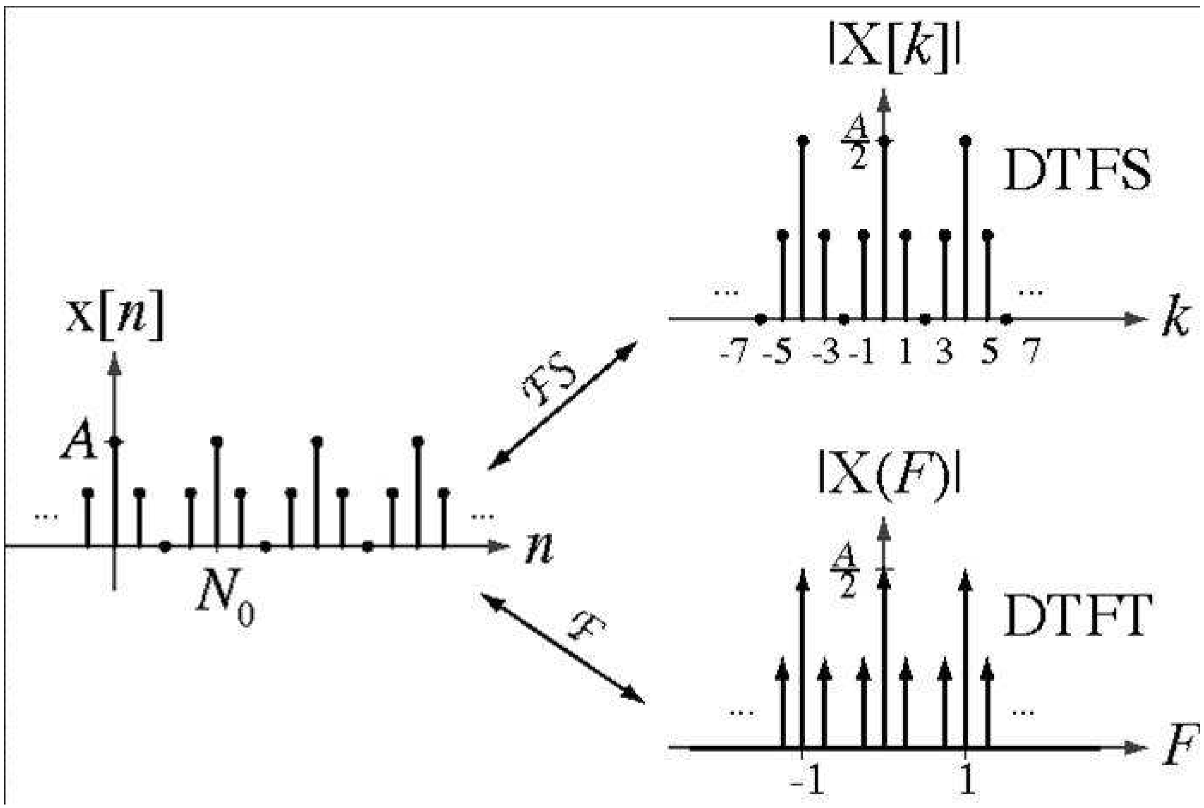
DTFT-DTFS Relationship (1)

If a periodic signal, $x[n]$, has the DTFS coefficients (harmonic function), $X[k]$, we can write its DTFS representation as:

$$x[n] =$$

Then the DTFT for $x[n]$ is:

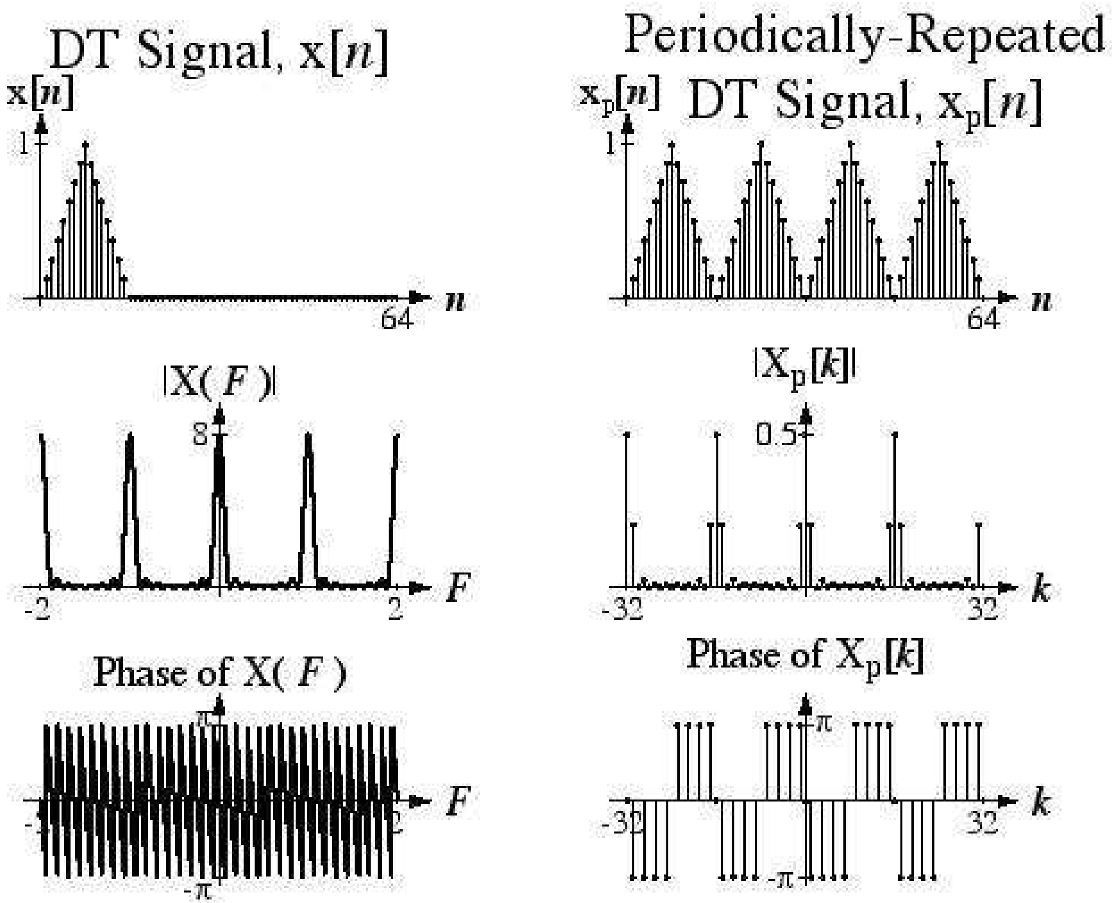
$$X(F) = \mathcal{F}(x[n]) =$$



DTFT-DTFS Relationship (2)

An aperiodic signal, $x[n]$, has DTFT $X(F)$. Create a periodic signal, $x_p[n]$, from $x[n]$ by repeating it every $N_p = 1/F_p$ samples. $x_p[n]$ then has a DTFS representation and its harmonic function, $X_p[k]$ relates to $X(f)$:

$$X_p[k] = \frac{1}{N_p} X(KF_p)$$



CTFT-DTFT Relationship

- Finally we study the relationship between the FT of CT signal, and the FT of the DT signal which is a sampled from the CT signal every T_s seconds.
- Consider the signal $x(t)$. Let

$$x_\sigma(t) = x(t). \sum_{n=-\infty}^{\infty} \delta(t - nT_s) = \sum_{n=-\infty}^{\infty} x[n]\delta(t - nT_s) \quad \text{where} \quad x[n] = x(nT_s)$$

- Find the CTFT of $x_\sigma(t)$, in terms of $X(f)$, using the multiplication-convolution property:

CTFT-DTFT Relationship (Cont.)

- Also find the CTFT of $x_\sigma(t)$, in terms of the DTFT of $x[n]$:

- Finally find the relationship between the DTFT of $x[n]$ and the CTFT of $x(t)$:

CTFT-DTFT Relationship (Cont.)