ENSC380
Lecture 2

Objectives:

- Understanding the difference between **continuous** and **discontinuous** signals
- Understanding the difference between **continuous** signals and **continuous time** signals
- Reviewing some important signals with discontinuity (Emphasis: **Unit Step, Unit Impulse**)
- Understanding signal transformations; Amplitude scaling, Time shifting, Time scaling
- Applying multiple transformations to signals and understanding that the **order** of the transformations **matters**.

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Continuous vs Discontinuous

- In Lecture 1 we saw some categorizations of signals.
- Another categorization for **CT signals** is: **Continuous** signals vs **signals with discontinuity**.
- **Continuous** signals are signals which do not have sudden changes in their value. In other words

$$\lim_{\epsilon \longrightarrow 0} g(t+\epsilon) = \lim_{\epsilon \longrightarrow 0} \qquad \forall t$$

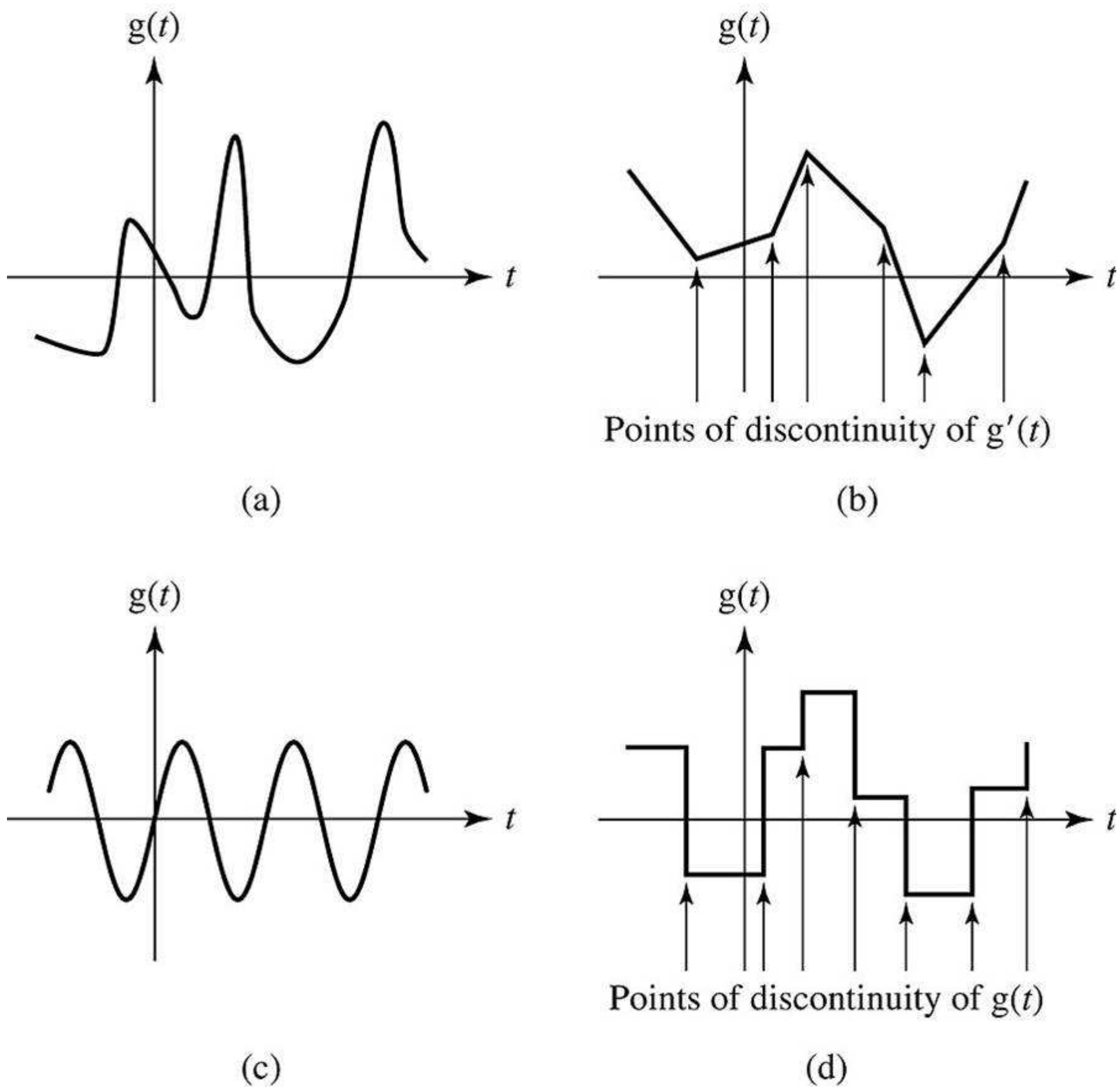
- **Discontinuous** signals are those which at some points of time have sudden changes in their value:

$$\exists t \quad \lim_{\epsilon \longrightarrow 0} g(t+\epsilon) \neq \lim_{\epsilon \longrightarrow 0}$$

- Note that a Continuous Time (CT) signal is not necessarily “Continuous”

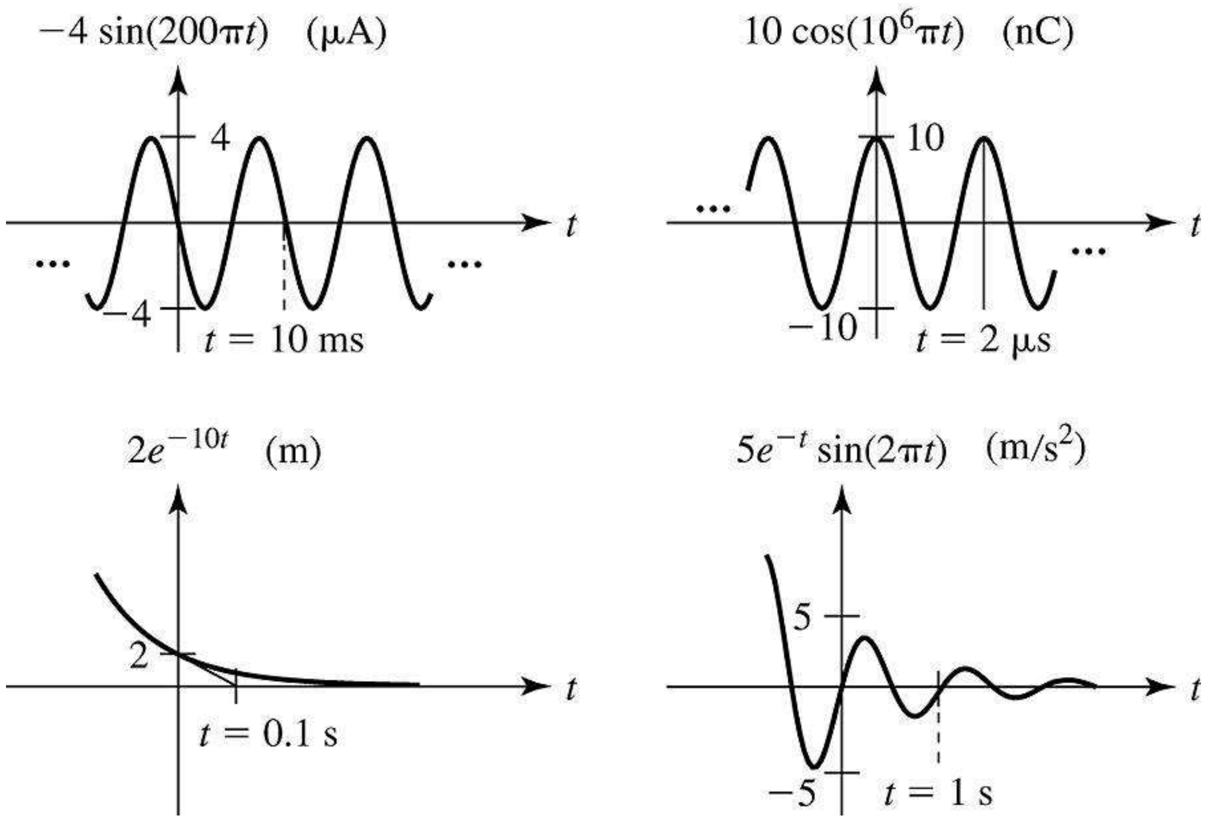
Examples

- Identify the continuous and discontinuous signals:



Important Continuous Signals

Exponential functions, Sinusoidal function, and their combination



Singularity Functions

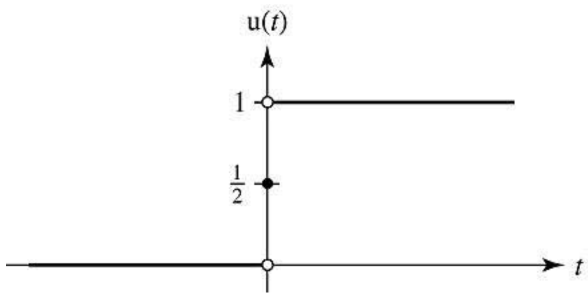
- In real life most signals are continuous, however a group of signals with discontinuity or discontinuous derivatives are important from a theoretical point of view. These signals are called **singularity functions**.

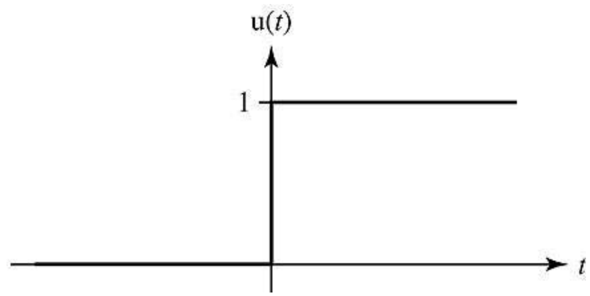
- Singularity Functions are important in signals and systems because they are often used to mathematically describe other signals with discontinuities, as well as some common system operations.

- Some important singularity functions and functions related to them are:
  Unit Step, Unit Impulse, Unit Comb, Signum, Unit Ramp, Unit Triangle, Unit Rectangle, . . .

Unit Step

$$u(t) = \begin{cases} 0 & t < 0 \\ \frac{1}{2} & t = 0 \\ 1 & t > 0 \end{cases}$$

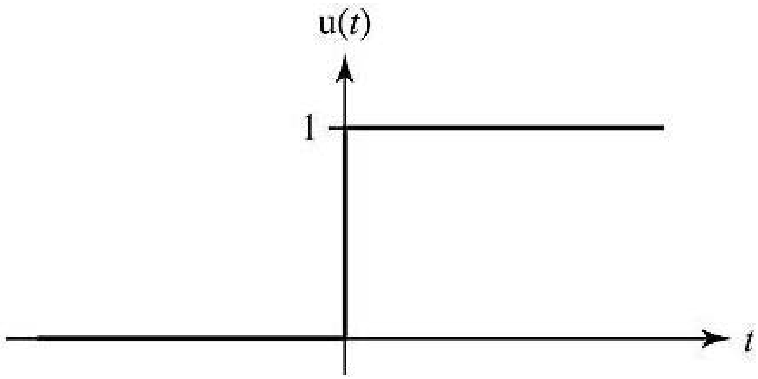




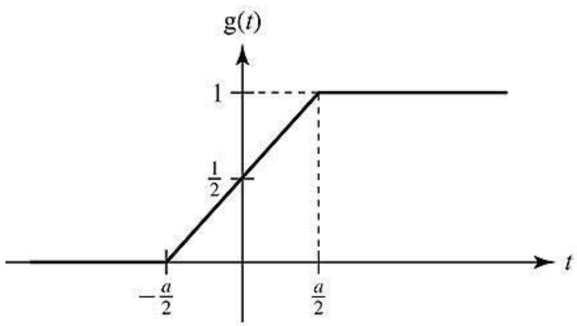
Note: Sometimes the value of Step function at $t = 0$ is defined differently, e.g., equal to 0, equal to 1, or “undefined”.

Unit Impulse

- **Unit Impulse** is the derivative of the unit step function. Find the derivative of u(t) given below:



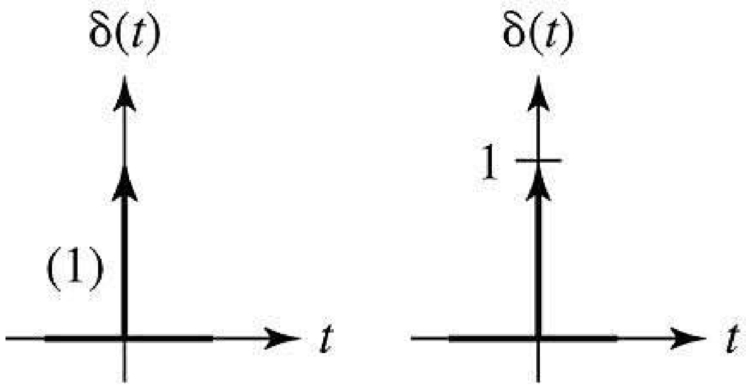
- To better understand the unit impulse function, consider $g(t)$ given below. Find the derivative of $g(t)$ (call it $\delta_a(t)$):



- What are the limits of $g(t)$ and $\delta_a(t) = g'(t)$ as a approaches 0?

Unit Impulse (Cont.)

- Unit Impulse is a function with value **zero for all** $t \neq 0$ and **undefined** at $t = 0$. The **area** under the unit impulse function is equal to **1**.
- Graphical representation:



- Important properties:

$$\delta(t) = u'(t) \qquad\qquad u(t) =$$

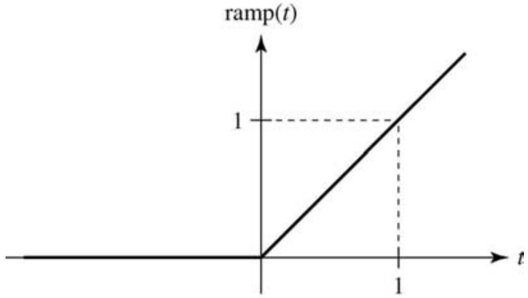
$$\int_{-t_1}^{+t_2} \delta(t)dt = \qquad\qquad \forall t_1, t_2 > 0$$

$$\int_{-t_1}^{+t_2} \delta(t)g(t)dt = \qquad\qquad \forall t_1, t_2 > 0$$

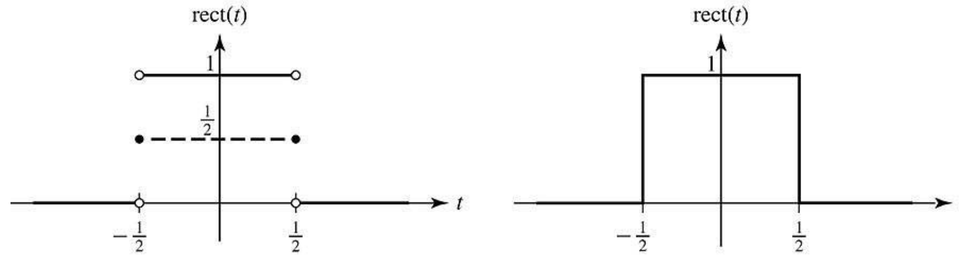
- Show that the last equality is true.

Other Functions (Text pp 24-36)

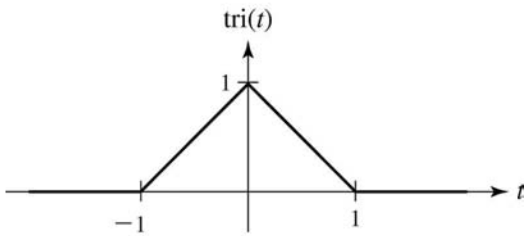
- Unit Ramp :



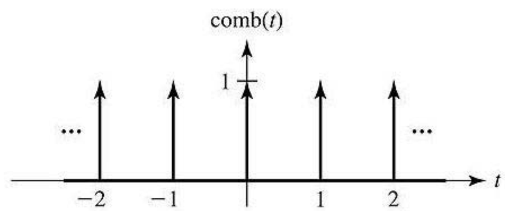
- Unit Rectangle :



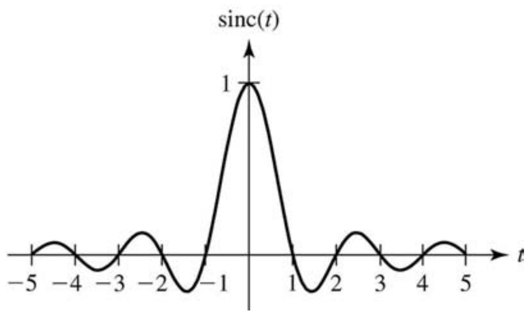
- Unit Triangle :



- Unit Comb :
  comb$(t) = \sum_{n=-\infty}^{\infty} \delta(t-n)$



- Unit Sinc :

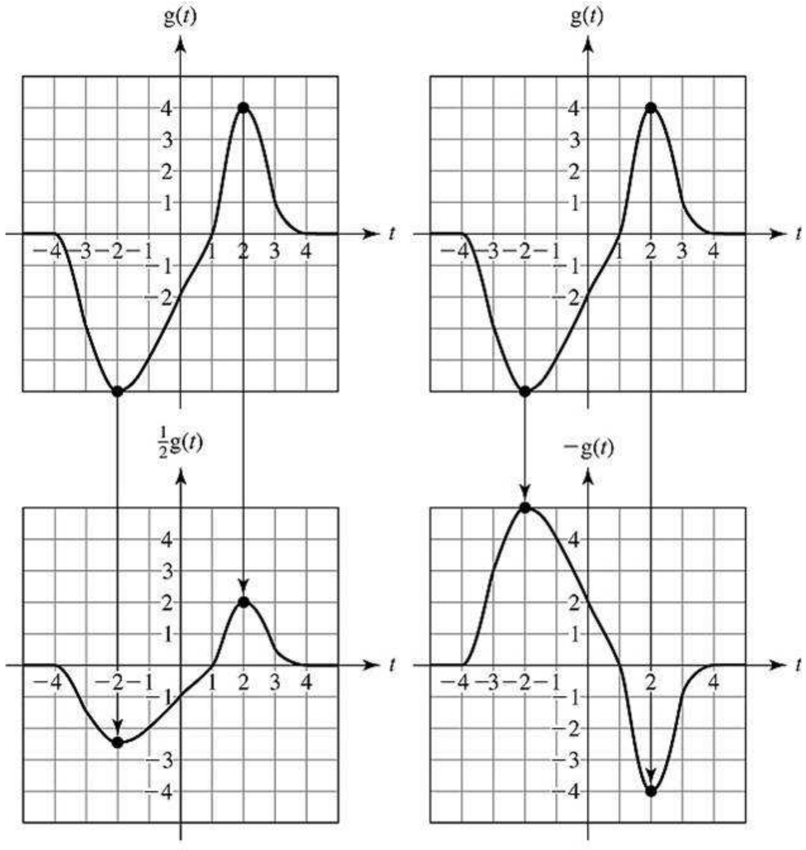


sinc$(t) =$

sinc$(0) =$

Transformation: Amplitude scaling

- Simply multiplying the signal by a scalar



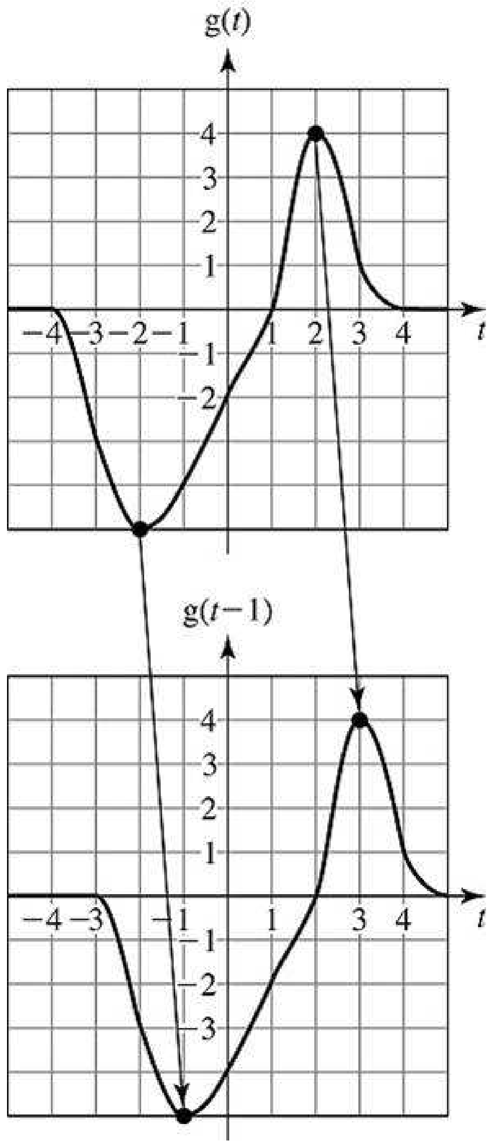
(a) (b)

- Example: Show $4\delta(t)$

- Real life example:

Transformation: Time shifting

- Results from replacing t with $t - t_0$, where t_0 is a real-valued constant.

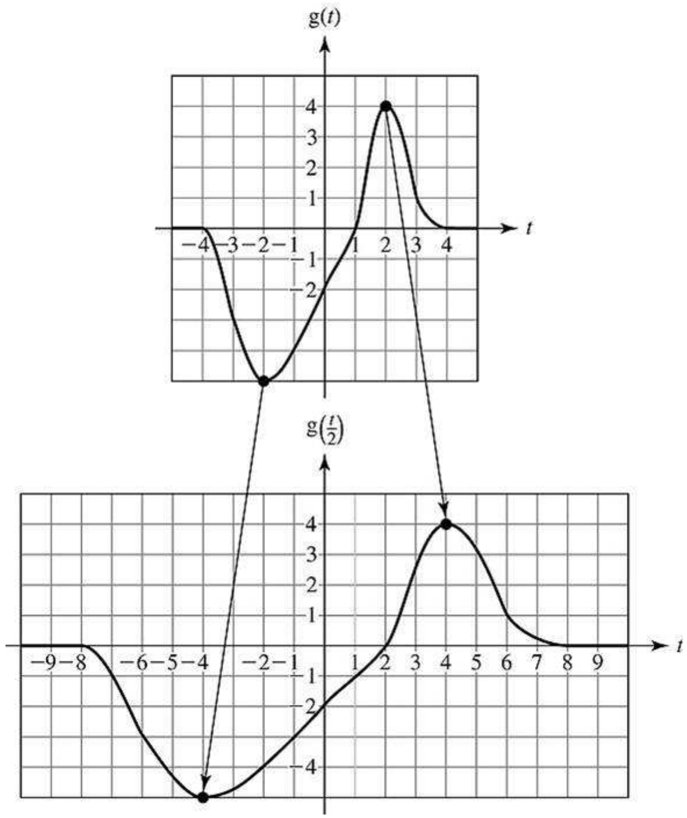


- Example Show $u(t-1)$

- Example: Show $\delta(t+1.5)$

Transformation: Time scaling

- Results from replacing t with at, where a is a real-valued constant.

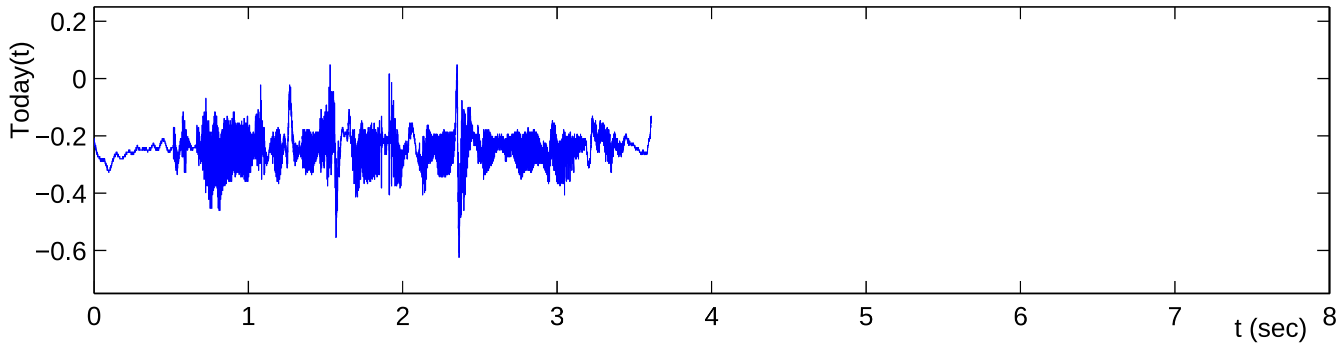


- Example: Show $tri(2t)$

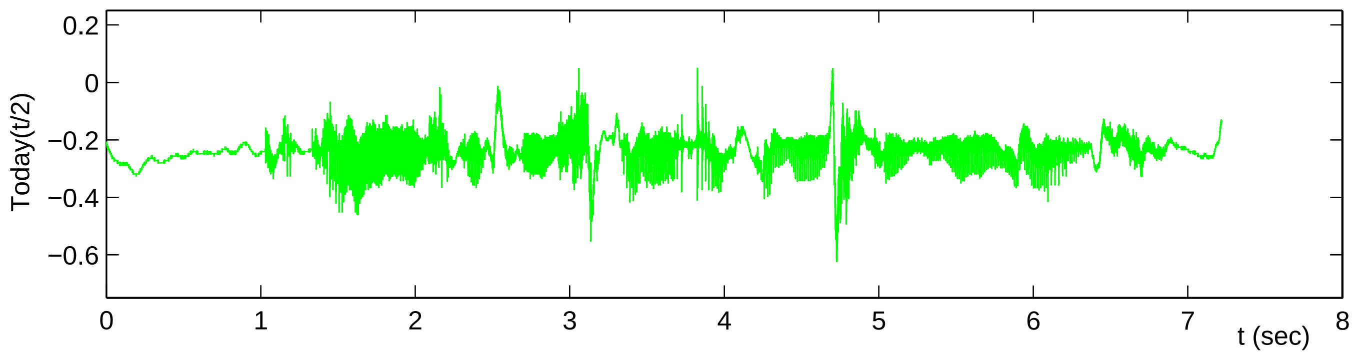
- Example: Show $\delta(t/5)$

Time Scaling (Cont.)

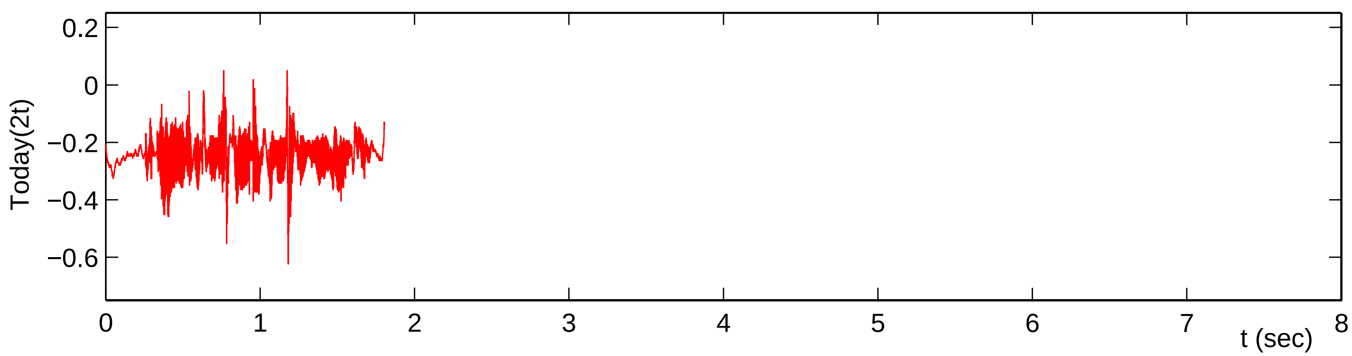
Here is an example of the sentence “Today is the first day of your life”, scaled in time by factors of 2 and 1/2. Listen to the audio clips and note the effects.



Original audio clip (Today_original_....wav)



Stretched in time by a factor of 2 (Today_stretched.wav)



Squeezed in time by a factor of 2 (Today_squeezed.wav)

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Multiple Transformations

- We can obviously transform a signal using a series of the different transformations given above.
- Important Note: The order of transformations can result in a different signals.
- Let's look at some examples:
- Example: The signal below is a transformation of the unit step function, $u(t)$. Break down the transformation of $u(t)$ into a series of single transformations that result in $g(t)$. Sketch $g(t)$.

$$g(t) = 5\,u(\frac{t-2}{3})$$

- Reverse the order of the last two transformations in the above example. What is the resulting signal?

Examples

Sketch the following transformed signals:

$34u(3-t)$, $\text{rect}(\frac{t+1}{4})$, $-5\text{ramp}(0.1t)$, $-7\text{tri}(\frac{t-4}{8})$