ENSC380
Lecture 23

Objectives:

- Sampling of CT signals
- Shannon's sampling Theorem and Nyquist rate
- Aliasing

Atousa Hajshirmohammadi, SFU

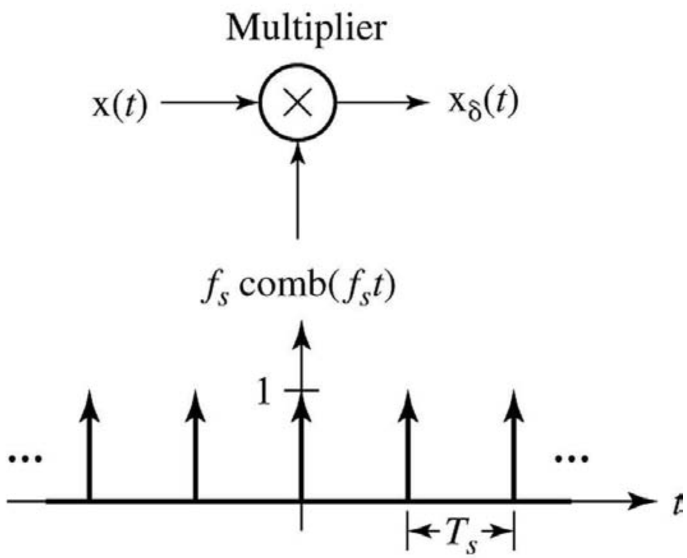
Sampling Methods

- For reasons mentioned before, most signal processing operations in the modern world are done with digital technology. This is often referred to as "Digital Signal Processing" or DSP.
- A CT signal should be "sampled" and "digitized" in order to be processed using DSP technology.
- The process of "digitization" of a signal usually consists of two stages: "Sampling" (and "Analog to Digital Conversion" (ADC).
- Here we will focus on the first part,i.e., sampling.

Ideal Sampling

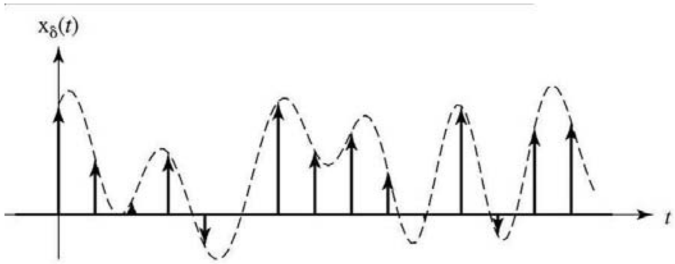
- Consider a CT signal, with a bandwidth of f_m:

- An "ideal sampler" is a system that multiplies the CT signal by a train of impulses, repeating every T_s seconds. T_s is called the "sampling period" and $f_s = 1/T_s$ is the "sampling rate".

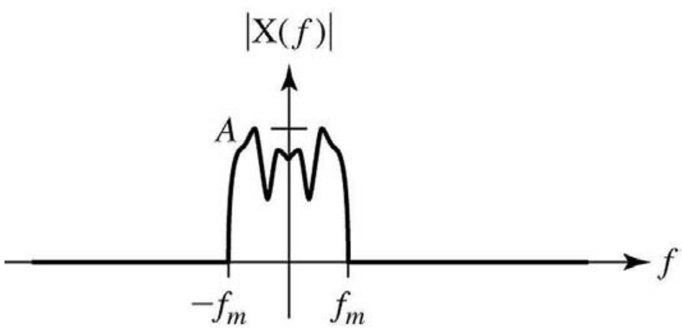


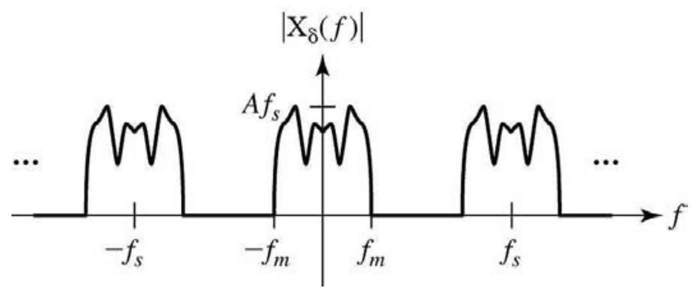
Sampling (Cont.)

- The output signal of the ideal sampler:



- Let's find the CTFT of $x_\delta(t)$:





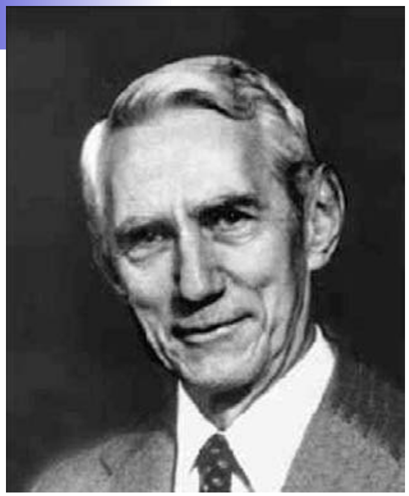
Sampling Rate and Aliasing

- The important question now is: How fast should we sample the CT signal in order to be able to recover the original signal, $x(t)$ from $x_\delta(t)$?

- As long as , we can retrieve the replica of $X(f)$ from $X_\delta f$, by using a .

- If sampling rate (f_s) drops to less than $2f_m$, the replicas of $X(f)$ will overlap with each other and we cannot recover $X(f)$ anymore. This is referred to as “aliasing”.

Shannon's Sampling Theorem



- Shannon's Sampling Theorem: If a signal is sampled for all time at a rate more than twice the highest frequency at which its CTFT is non-zero it can be exactly reconstructed from the samples.
- The sampling rate $2f_m$ is called "Nyquist rate".
- A signal sampled at a rate higher than the Nyquist rate, it is called "over sampled".
- A signal sampled at a rate lower than the Nyquist rate, it is called "under sampled".
- Note: A signal that is "time limited" cannot be "band limited". This means that there is no sampling rate high enough for sampling this signal such that it is recoverable!

Example 1

Sample the signal $x(t) = A\text{sinc}(\frac{t}{3})$ using an "ideal sampler" at twice its Nyquist rate. Find and sketch the CTFT of the sampled signal. Can you retrieve the original signal? How?

Example 2

Sample the signal $x(t)$ of Example 1, but at half its Nyquist rate. Find and sketch the CTFT of the sampled signal. Can you retrieve the original signal? How?

Practical Perspective

- Shannon's theorem tells us that sampling at Nyquist rate (twice the bandwidth of the signal) is enough to retrieve the original signal.
- The above theorem however, assumes the use of an ideal LPF for original signal retrieval. But in practice our filters are non-ideal (Causal).
- Thus in practice signals are almost always "over sampled" in order to compensate as much as possible for the non-ideality of the filters.