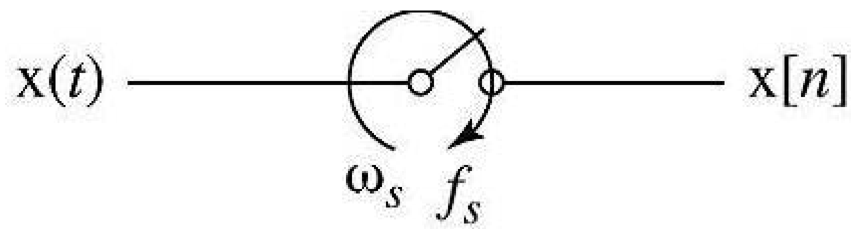
ENSC380
Lecture 4

Objectives:

- Introduction to discrete-time (DT) signals
- DT **exponential** and **sinusoidal** signals
- DT **unit step** and **unit impulse** functions
- Transformation of DT signals: amplitude scaling, time shifting, **time scaling**, differencing, accumulating
- Energy and power signals (DT or CT)

DT Signals

- Discrete time signals are signals which are defined only at integer values of their argument.
- Consider a CT signal which is sampled every T_s seconds. This means that the switch in the circuit below, closes every T_s seconds. The frequency of sampling is then $f_s =$



- If $x(t)$ is sampled every T_s seconds, the signal at the output of the switch is

$$x(\qquad)$$

- This signal is instead shown as $x[n]$, where n is an integer.
- Some of the more important DT signals are "exponentials" and "sinusoids". These are analogous to their CT counterparts, with slight but important differences!

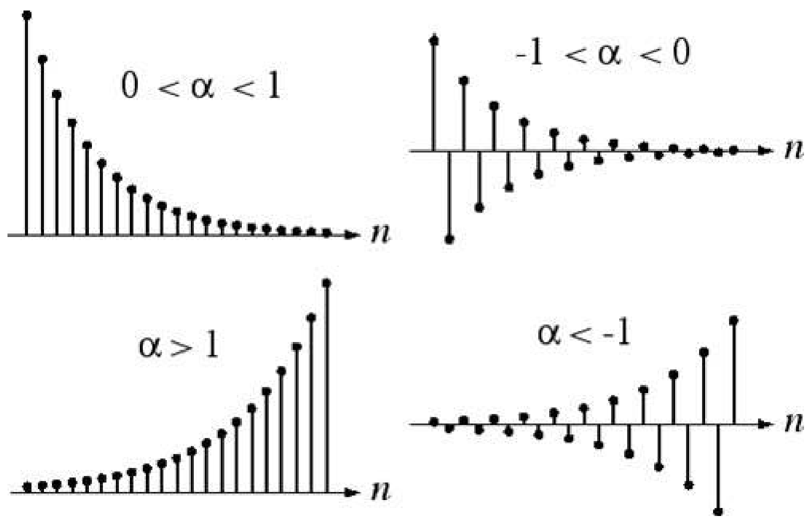
DT Exponential

Mathematical formula:

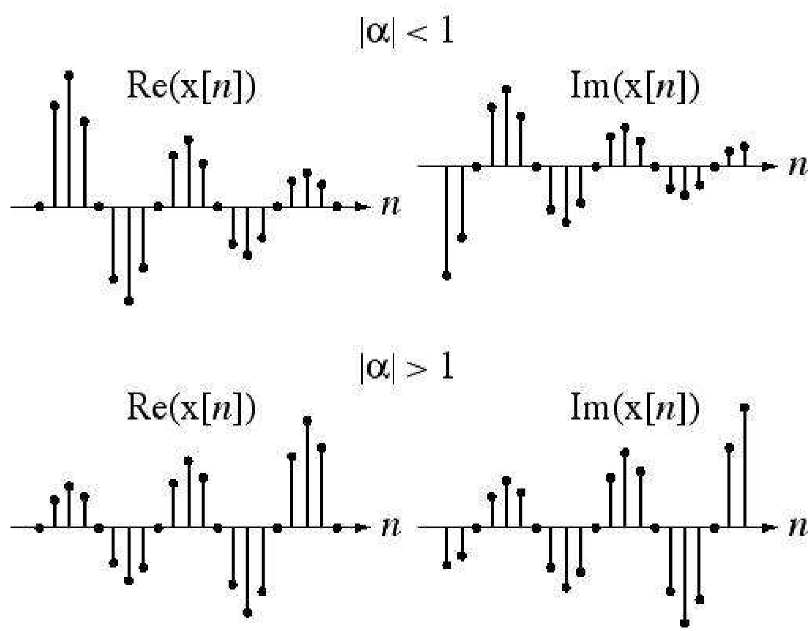
$$g[n] = Ae^{\beta n} \text{ or } g[n] = A\alpha^n, \alpha = e^{\beta}$$

where A is a real constant and β is a complex constant.

Real α



Complex α



DT Sinosoids

- General form:

$$g[n] = A\cos(2\pi K n + \theta)$$

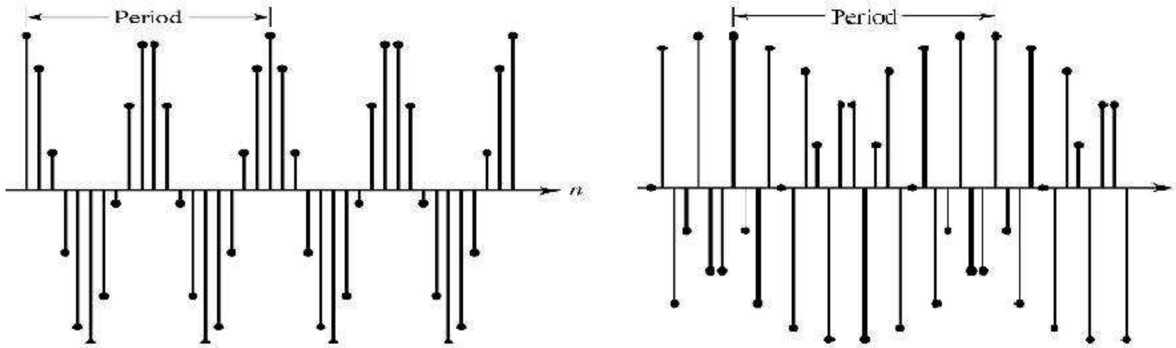
- What is the condition for $g[n]$ to be periodic?

- The fundamental period of a DT periodic sinusoid, is the smallest integer N which solves $KN = m$ for an integer m.
- Example: what is the fundamental period of:

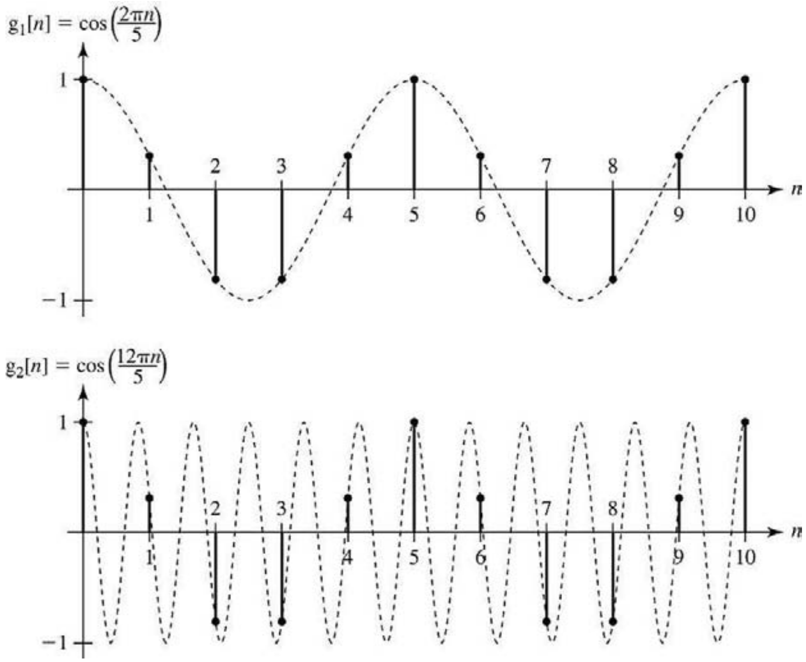
$$g[n] = 3\cos(2\pi \frac{15}{6} n + \theta)$$

DT Sinosoids (Cont.)

- Two DT sinusoids:



- An interesting point to note is that sometimes two DT sinusoids can be identical even if they have different Ks. For example: $g_1[n] = \cos(\frac{2\pi}{5}n)$ and $g_2[n] = \cos(\frac{12\pi}{5}n)$



DT Singularity Funcs.

- Unit sequence (very similar to CT unit step):
  $$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

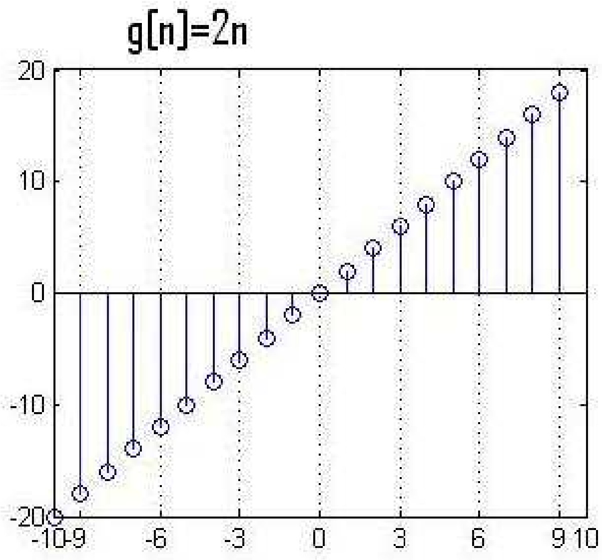
- Unit impulse (not too similar to CT impulse!): $\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$

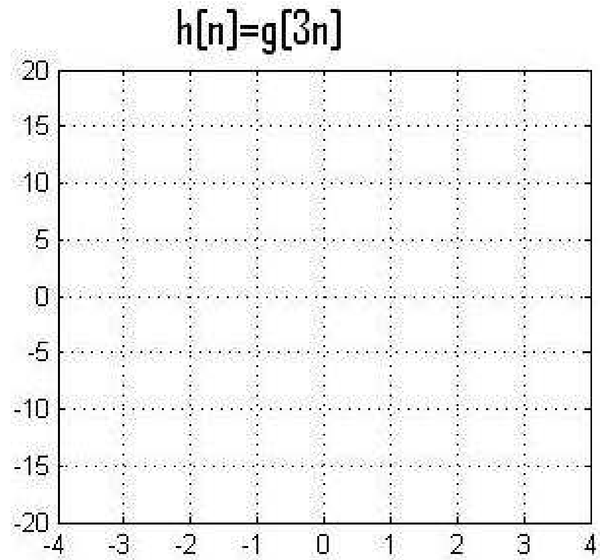
- Recall: $\delta(5t) =$?
  Now: $\delta[5n] =$?

- See your Text for other DT singularity functions;
  ramp$[n]$, rect$_{N_w}[n]$,comb$_{N_0}[n]$

DT Transformations

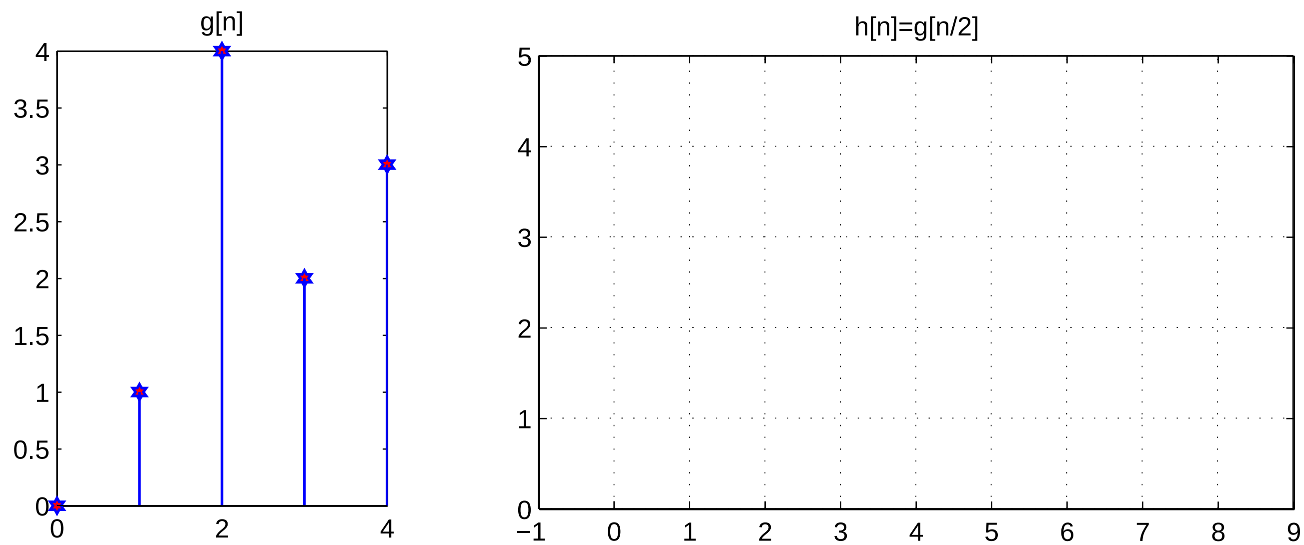
- Amplitude scaling and time shifting for DT signals are the same as for CT signals.
- Time scaling for DT signals:
  - **Time compression**: $h[n] = g[Kn]$ where $K > 1$ (integer)
    Some of the samples from the original function get lost (decimation):





DT Transformations (Cont.)

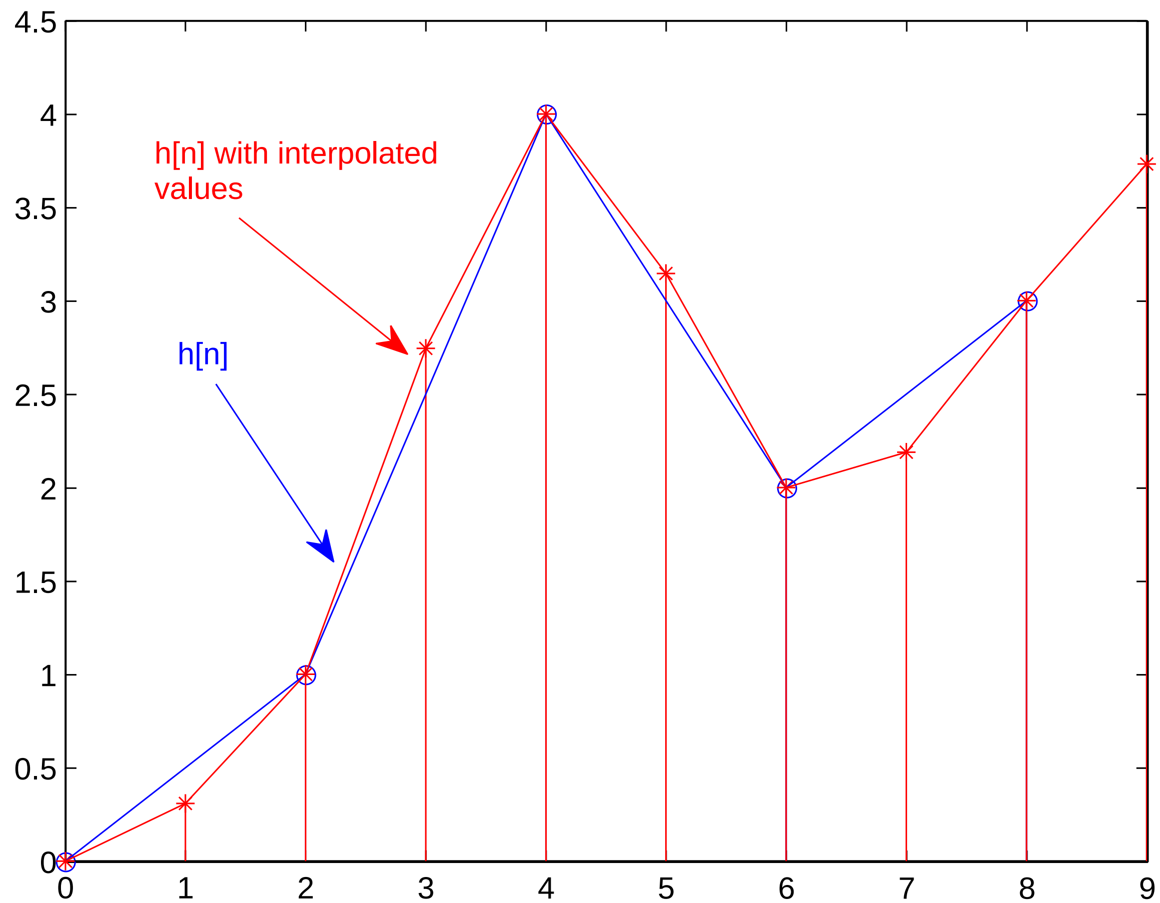
- Time scaling:
  - **Time expansion**: $h[n] = g[\frac{n}{K}]$ where $K > 1$ (integer) Some samples in the transformed signal are **undefined**:



  - The unknown samples can be left as “undefined” or they can be interpolated, based on the the known values before and after them. (Matlab function: “interp”)

Example

We have interpolated the previous example, by adding one sample between every two values:



DT Transformations (Cont.)

- Two more transformations for DT signals are **differencing** and **accumulation**.
- These are similar to CT differentiation and integration, but much simpler.
- For example, the first forward difference of a DT signal is defined as:

$$\Delta g[n] = g[n+1] - g[n]$$

- And the first backward difference of a DT signal is defined as:

$$\Delta g[n-1] = g[n] - g[n-1]$$

- If $h[n] = \Delta g[n-1] = g[n] - g[n-1]$, then the accumulation of $h[n]$ results in $g[n]$:

$$g[n] = \sum_{-\infty}^{n} h[m]$$

- Read pages 81 - 85 of your text, to learn more.
- DT signals also can be divided into **even** and **odd** functions. Again, very similar to CT signals. (Text: 85-89)

Signal's Energy

- The **energy** of a signal ($x(t)$ or $x[n]$) is defined as:

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt \qquad\qquad E_x = \sum_{-\infty}^{\infty} |x[n]|^2$$

- Example: Find the energy contained in $x[n] = (\frac{1}{2})^n u[n]$

- A signal which has a **finite energy** is called an "**energy signal**".
- But many signals (CT or DT) don't have a finite energy. In this case we consider their average signal power (averaged over time).

Signal's Power

- The **average power**(or power for short) of a signal ($x(t)$ or $x[n]$) is defined as:

$$P_x = \lim_{T\to\infty} \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} |x(t)|^2 dt \qquad\qquad P_x = \lim_{N\to\infty} \frac{1}{2N}\sum_{-N}^{N-1} |x[n]|^2$$

- Are periodic signals "energy signals"? Why?
- For periodic signals, their average power is equal to:

$$P_x = \frac{1}{T}\int_{t_0}^{t_0+T} |x(t)|^2 dt = \frac{1}{T}\int_T |x(t)|^2 dt$$

$$P_x = \frac{1}{N}\sum_{n=k}^{k+N-1} |x[n]|^2 = \frac{1}{N}\sum_{n=<N>} |x[n]|^2$$

- Signals which do not have finite energy, but have finite power are called "**power signals**".