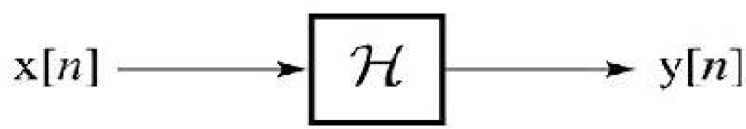
ENSC380
Lecture 5

Objectives:

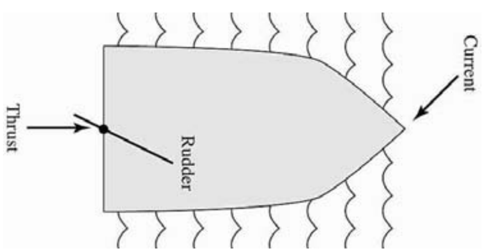
- Systems:
  - What is a system?
  - Briefly study two simple examples of CT and DT lowpass filters
  - Learn about different system characteristics: **Linearity, Time Invariance**, Causality, Memory, Stability, Invertibility

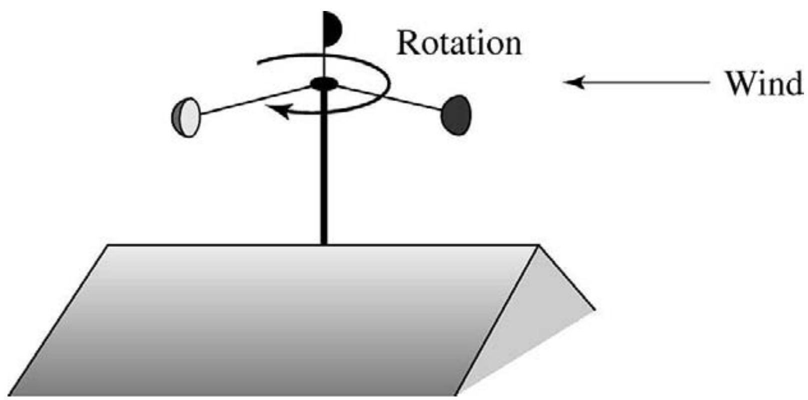
Systems

- Broad definition: System is any thing that operates on something and produces something else.
- Systems can be electrical, mechanical, chemical, economical, organic, . . .
- Systems can be natural or artificial. For example: Emma's system is natural, a low pass filter designed by an engineer using a resistor and a capacitor is artificial.
- Systems can be CT or DT, e.g., have CT or DT input/output signals.
- Block diagrams:



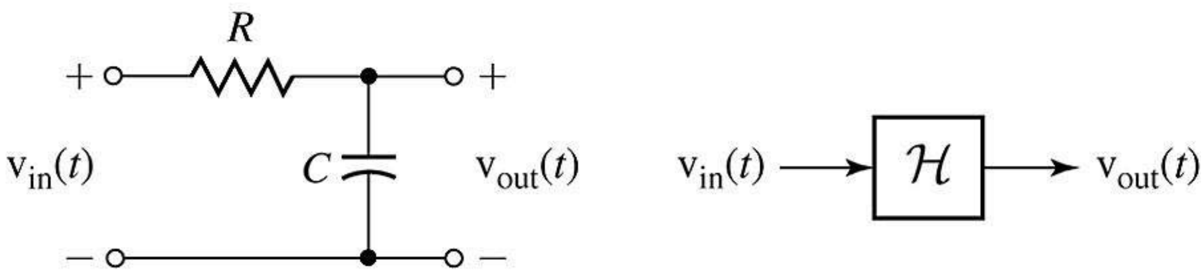
- Examples:





Circuit Example

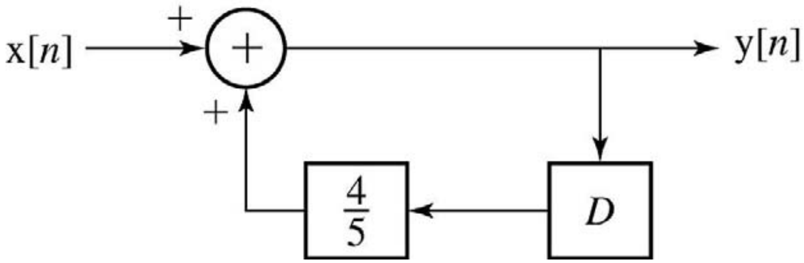
- This circuit (or any electrical circuit) can be considered as a system. CT or DT?



- What kind of filter is this system?
- What is the differential equation that relates the output and input voltages?
- What is the output voltage (system's response) to a step function (excitation signal)? $V_{out}(0) = 0$.

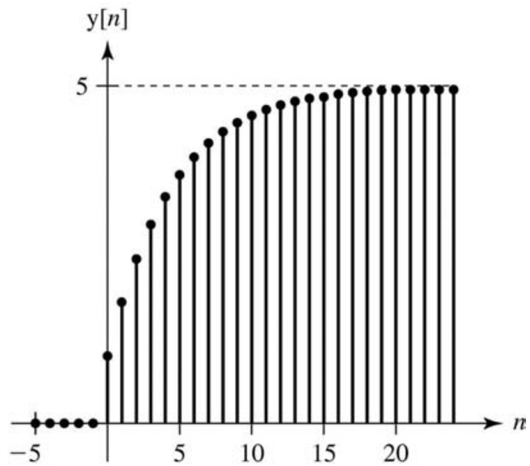
DT Example

- Consider the following system, where the “D block” represents a delay component whose output signal is its input signal delayed by 1 unit of time.



- Write and equation which relates $y[n]$ to $x[n]$. This is called a difference equation.
- If the system is initially at rest ($y[n] = 0$ for $n < 0$) and if the input signal is $u[n]$, the output signal is:

$$y[n] = \begin{cases} 5 - 4(\frac{4}{5})^n & n \geq 0 \\ 0 & n < 0 \end{cases}$$



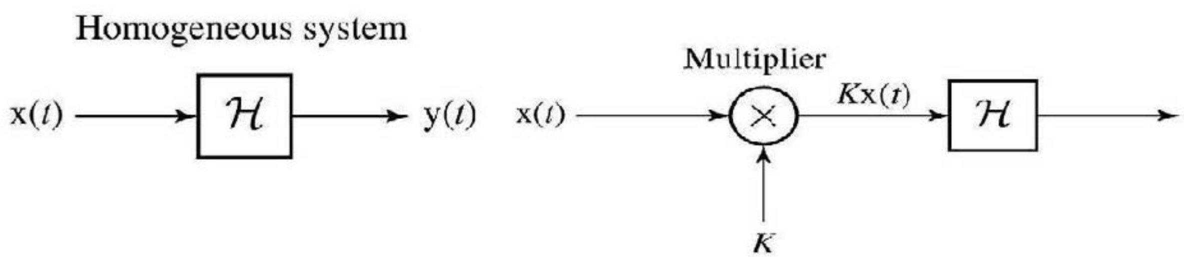
- We will come back to this example soon to see how the above solution can be found using the system’s characteristics.

System Characteristics

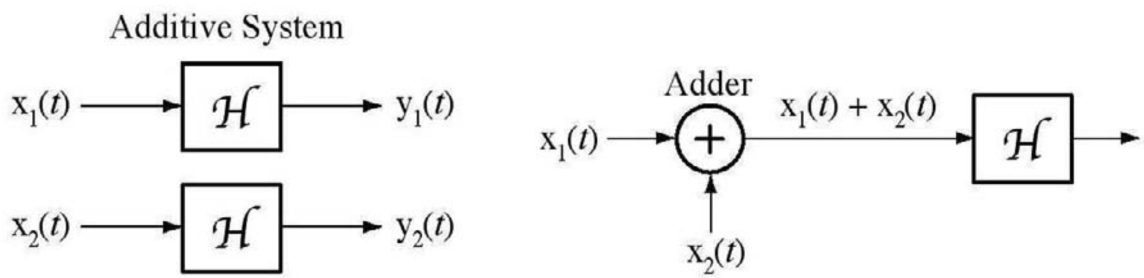
- Systems can be categorized based on their characteristics.
- We will look at the following characteristics: Homogeneity, Additivity, Linearity, Time Invariance, Stability, and Causality
- Our focus however will be on systems with **Linearity** (Homogeneity $+$ Additivity) and **Time Invariance**, i.e., **LTI systems**.
- The reason for studying only LTI systems in this course, is that there is a simple way for their analysis, which does not apply to non-LTI systems!
- All system characteristics studied here apply to both CT and DT systems.

Linearity

- Linearity is the combination of two characteristics:
- **Homogeneity**: If the input is multiplied by a constant, the output multiplies by the same constant as well:



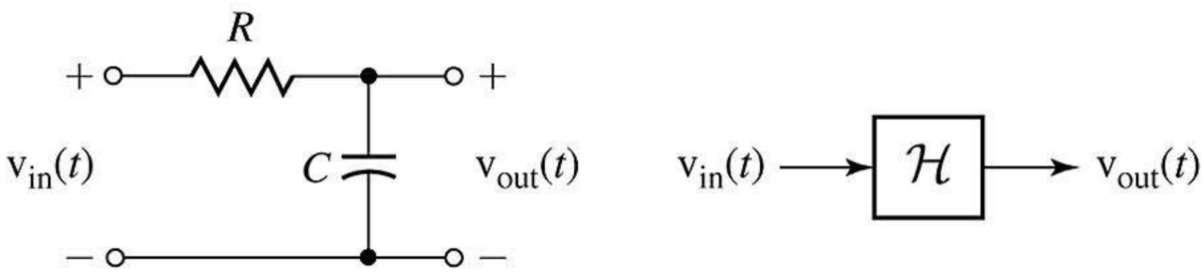
- **Additivity**: If the response of the system to $x_1(t)$ is $y_1(t)$ and to $x_2(t)$ is $y_2(t)$, then its response to $x_1(t) + x_2(t)$ is



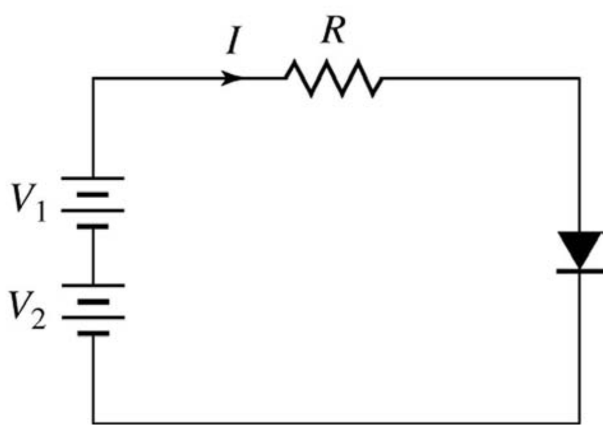
- **Linearity**: If a system is both homogeneous and additive, it is called linear. i.e., if the response of the system to $x_1(t)$ is $y_1(t)$ and to $x_2(t)$ is $y_2(t)$, then its response to $ax_1(t) + bx_2(t)$ (a and b are constants) is

Linearity Examples

- Linear example: The RC lowpass filter is a linear system:

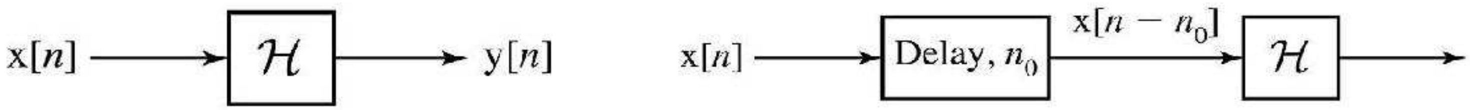


- Non-linear example : The electric diode

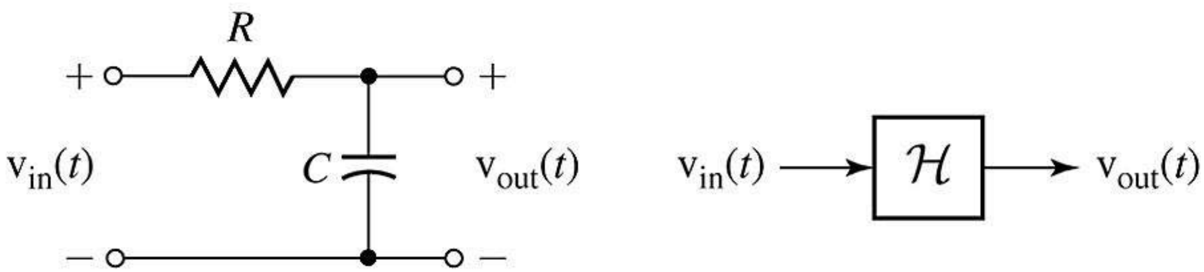


Time Invariance

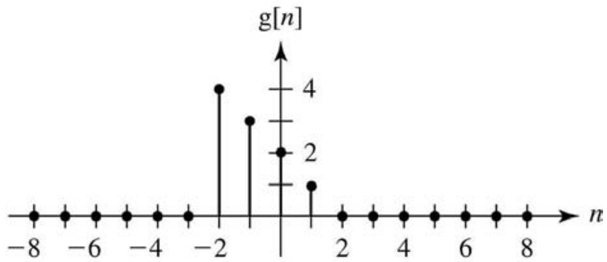
- **Time Invariance** (TI): If the response of the system to $x(t)$ is $y(t)$, then the impulse of the system to $x(t - t_0)$ is



- Example:



Example of a **non-TI** system: Consider the DT system defined by: $y[n] = x[2n]$. Assume the input signal $g[n]$ is given. What is the response of the system to $g[n]$? What is the response to $g[n-1]$. Is the system TI?

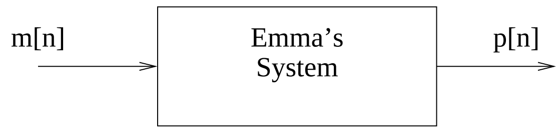


Other Characteristics

- **Causality**: A system whose output depends only on the current and past (not future) values of the input is called causal. All physical systems are causal.
- **Memory**: A system whose output at time t_0 depend on its input only at time t_0 is called **memory less**. If the systems output depend on the input value at times other than the present, the system has memory and is also called **dynamic**. Any system that has delay elements in it is dynamic.
- **Stability**: If a system's response to a bounded input ($|x(t)| < \infty$), is bounded, the system is **stable**, also called bounded-input-bounded-output (BIBO). For example, for the RC lowpass filter, as long as the input voltage is finite the output is finite too.
- **Invertibility**: A system which has a unique response to every unique excitement is invertible. For such systems, we can find the input from a given output signal. Most practical systems are invertible but some aren't! an example of a non-invertible system is an electrical rectifier. A rectifier is a circuit using diodes, whose output voltage is the absolute value of its input voltage: $v_{out}(t) = |v_{in}(t)|$.

Example

Let's consider Emma again and study all the above characteristics about it:



$$p[n] = 0.8\, m[n-1]$$