ENSC380
Lecture 6

Objectives:

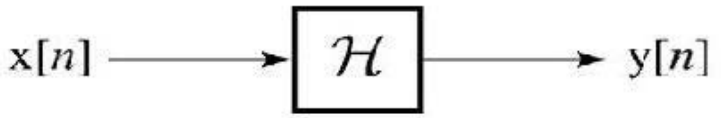
- Focus on DT systems
- Learn the general form of a DT **difference equation** describing an **LTI** system
- Learn how to write an arbitrary signal in terms of $\delta[n]$
- Learn the meaning of the **impulse response** of a system
- Learn that the response of a DT system to a general input, is the **convolution sum** of the input and the impulse response of the system
- Solve examples using what you learned

LTI System

- Any DT linear and time invariant (LTI) system can be described with a difference equation of the general form:

$$a_n y[n] + a_{n-1} y[n-1] + \ldots + a_{n-D} y[n-D] = x[n]$$

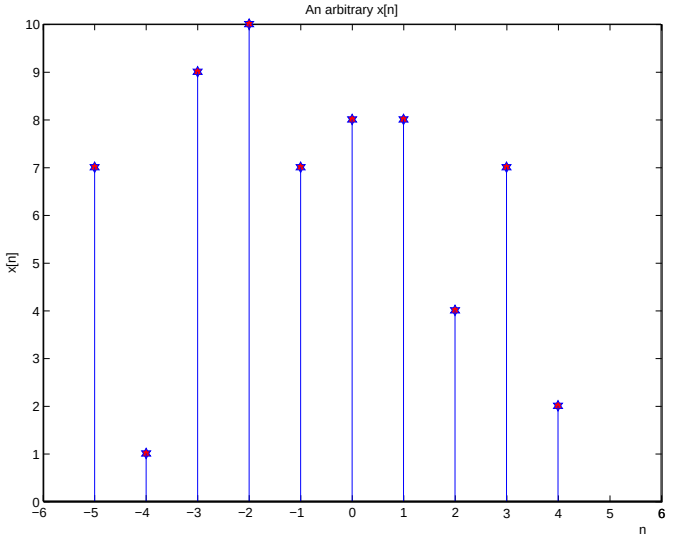
Where $x[n]$ is the input (excitation) and $y[n]$ is the output (response) of the system.



- If $x[n] = \delta[n]$ then the answer to the differential equation is called the **impulse response** of the system, and is usually shown with $h[n]$
- The **impulse response** of an LTI system is very important because it simplifies finding the response of the system to an arbitrary $x[n]$. How? Let's see!

$x[n]$ and $\delta[n]$

- Every DT signal $x[n]$ can be written as a linear combination of the unit impulse and its delayed versions:



$x[n] =$

Convolution Sum

- If the response of the system to $\delta[n]$ is $h[n]$, what is the response of the system to
  - $x[0]\delta[n]$?
  - $x[1]\delta[n-1]$?
  - $x[2]\delta[n-2]$?
  - ...
  - $x[-1]\delta[n+1]$?
  - $x[-2]\delta[n+2]$?
  - $x[-3]\delta[n+3]$?
  - ...
- What is the response of the system to $x[n] = \sum_{m=-\infty}^{m=\infty} x[m]\delta[n-m]$?

- The above summation is called the **convolution sum** and is shown with

$$y[n] = x[n] * h[n]$$

- Thus, the response of a DT and LTI system to a general input $x[n]$, is the convolution sum of $x[n]$ and the impulse response of the system, $h[n]$.

Example

A DT system is defined with the following difference equation:

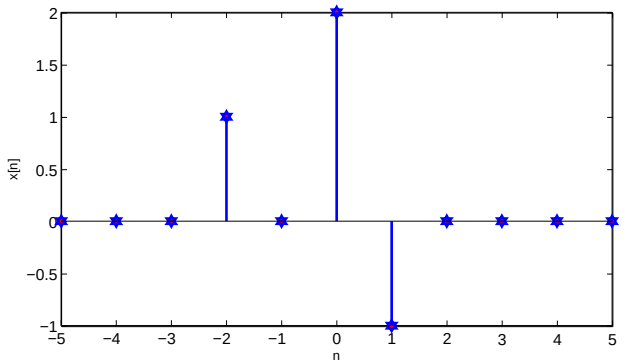
$$8y[n] + 6y[n-1] = x[n]$$

- What is the impulse response of the system?

$$h[n] = \frac{1}{8}(-\frac{3}{4})^n u[n]$$

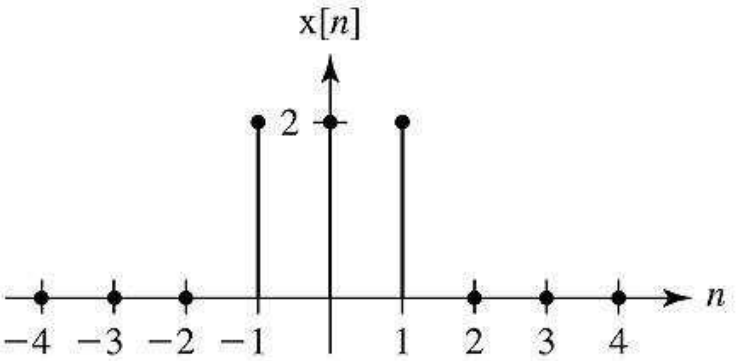
(We will soon learn how to find this impulse response!)

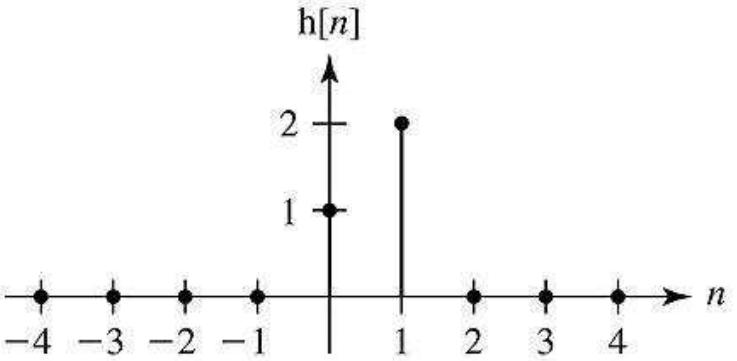
- What is the response of the system to $x[n]$ given below:



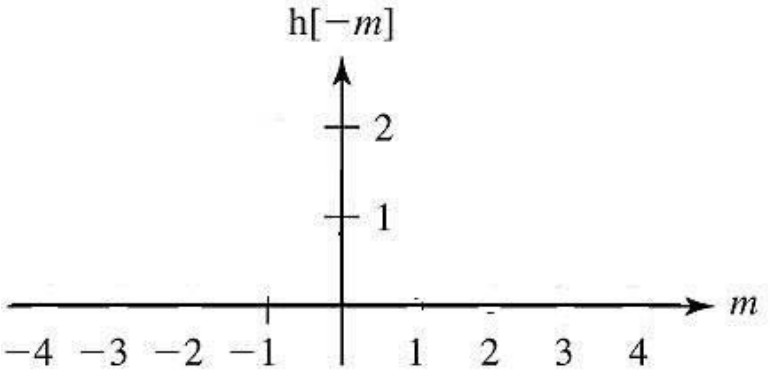
Example

Find $x[n] * h[n]$



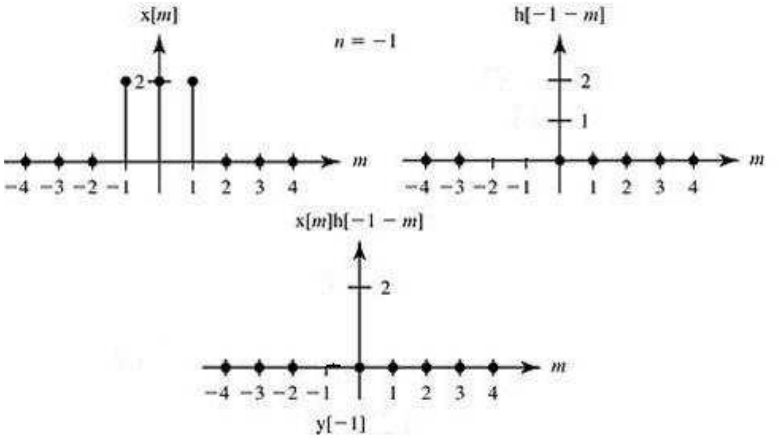


- Now find $h[-m]$

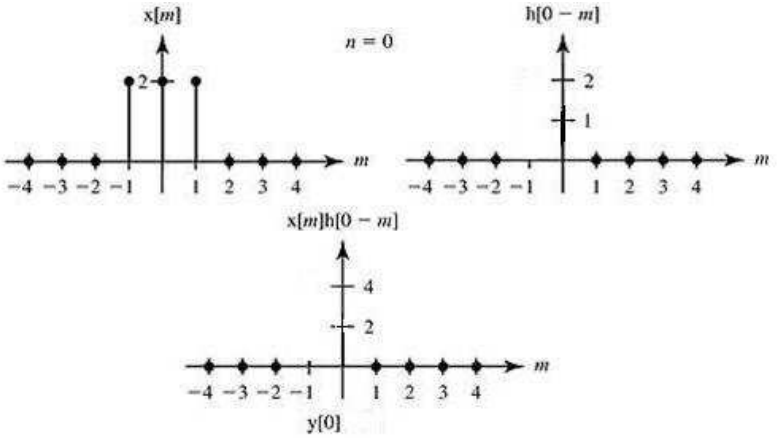


Example (Cont.)

- Now find $y[-1] = \sum_m x[m]h[-1-m]$

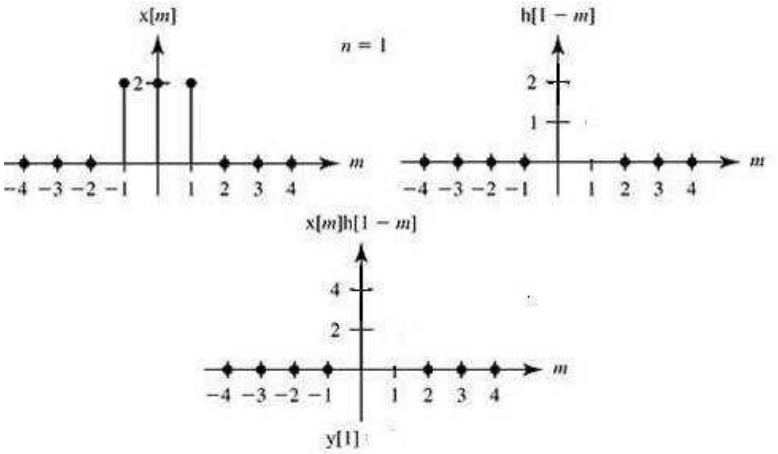


- Now find $y[0] =$

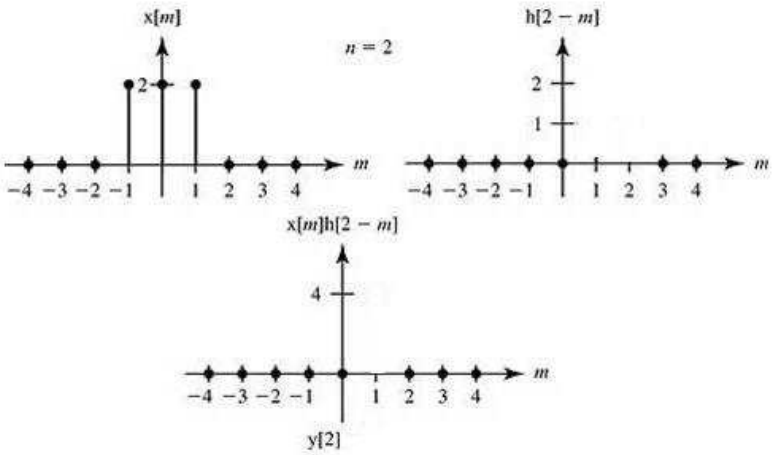


Example (Cont.)

- Now find $y[1] =$



- Now find $y[2] =$



Example (Cont.)

- What is $y[n]$ for $n \leq -2$ and $n \geq 3$?

- Plot $y[n]$: