ENSC380
Lecture 7

Objectives:

- Learn how to find the answer to the homogeneous difference equation
- Learn how to find the **impulse response** of the LTI DT system

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Homogeneous Response

- The difference equation:

$$a_n y[n] + a_{n-1} y[n-1] + \ldots + a_{n-D} y[n-D] = 0$$

  is called a homogeneous difference equation.

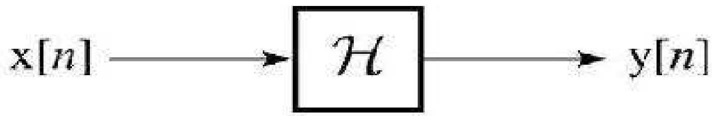
- The general form of the function that can satisfy the homogeneous difference equation is:

$$y_h[n] = A\alpha^n$$

- Replace $y[n]$ with $y_h[n]$ in the above equation to find α:

- $y_h[n] = A\alpha^n$ is called the **eigenfunction** of the difference equation.

Impulse Response of DT Systems



- Consider the following LTI-DT system:

$$a_n y[n] + a_{n-1} y[n-1] + \ldots + a_{n-D} y[n-D] = x[n]$$

- We want to find the **impulse response** of the system, i.e., the response to $x[n] = \delta[n]$. We call this response ($h[n]$)

$$a_n h[n] + a_{n-1} h[n-1] + \ldots + a_{n-D} h[n-D] = \delta[n]$$

Impulse Response (Cont.)

- Since the forcing function (input signal) to the system is zero for all $n < 0$, we can easily see that

$$y[n] = \qquad\qquad \text{for } \; n < 0$$

- Also for $n > 0$, the difference equation is:

$$a_n y[n] + a_{n-1} y[n-1] + \ldots + a_{n-D} y[n-D] =$$

  which means

$$y[n] = \qquad\qquad\qquad \text{for } \; n > 0$$

- How about for $n = 0$? For $n = 0$ it is easy to find $y[0]$ directly from the equation:

  This can be used as the initial condition for $y[n]$. We will see this through examples.

Example 1

Recall the example in Lecture 6, where we were given the impulse response:

$$8y[n] + 6y[n-1] = x[n] \quad , \quad h[n] = \frac{1}{8}(-\frac{3}{4})^n u[n]$$

Now, let's find this impulse response for ourselves!

Example 2

Find the impulse response of this system:

$$y[n] = x[n] - x[n-1]$$

Example 3

Find the impulse response of this system: