ENSC380
Lecture 9

Objectives:

- Learn about the impulse response of a CT and LTI system ($h(t)$)
- Learn how to find the response of an LTI system to $x(t)$, having $h(t)$
- Learn how to find the convolution integral of two signals
- Learn the properties of convolution integral
- Learn about the response of LTI systems to unit-step, complex exponential, and sinusoidal functions

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CT Impulse Response

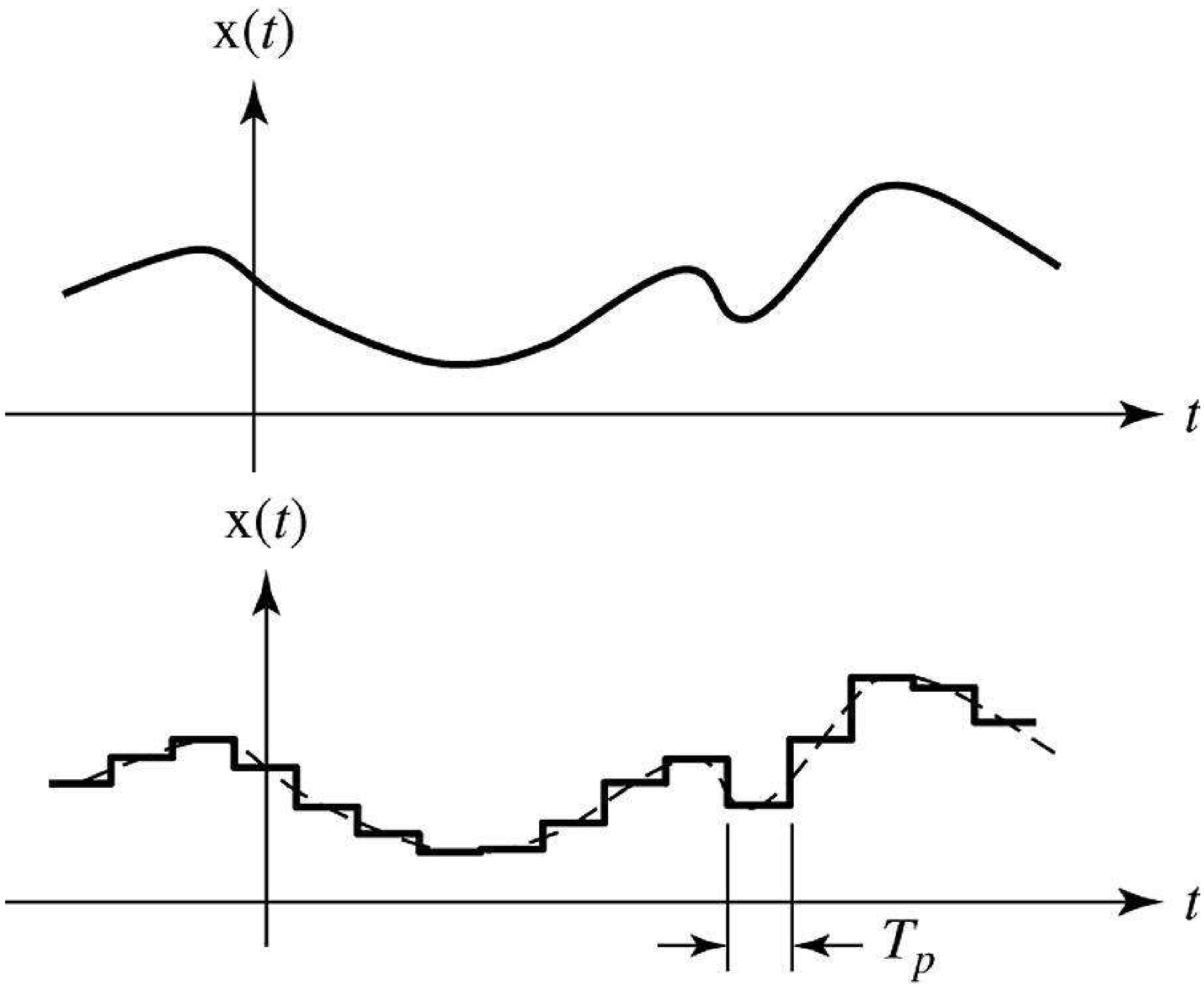
- Any CT and LTI system can be shown with a linear constant coefficient differential equation of:

$$a_n y^{(n)}(t) + a_{n-1} y^{(n-1)}(t) + \ldots + a_1 y'(t) + a_0 y(t) =$$
$$b_m x^{(m)}(t) + b_{m-1} x^{(m-1)}(t) + \ldots + b_1 x'(t) + b_0 x(t)$$

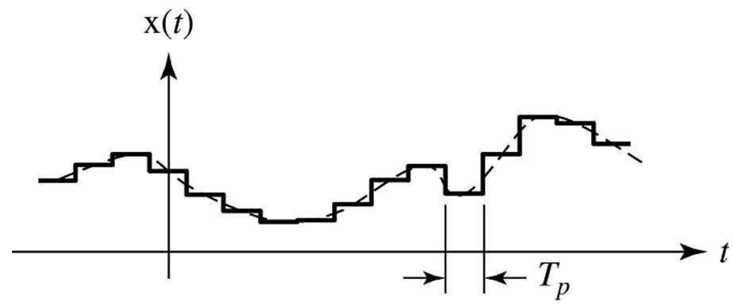
- the impulse response of the system is the answer to the above equation when $x(t) = \delta(t)$.
- We will learn how to find the impulse response of the CT-LTI system later, but for now we assume that we know the impulse response of the system and call it $h(t)$.
- Now we want to find the response of the system to an arbitrary excitation $x(t)$.

Approximating $x(t)$

- Let's approximate $x(t)$ with a train of weighted rectangular pulses of width T_p:



Approximating $x(t)$ (Cont.)



- $x(t)$ can be approximately written as:

$x(t) \simeq$

- The area under each pulse is , let's write the approximation in terms of unit-area pulses of width T_p:

Response to $x_p(t)$

As T_p approaches zero:

- Each unit-area pulse approaches:

- The approximation for $x(t)$ approaches its exact expression:

$$x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)\, d\tau$$

- Assuming the response of the system to $\delta(t)$ is $h(t)$, and using the LTI characteristics of the system, the response to $x(t)$:

Convolution Integral

$$x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(n-\tau)d\tau$$

- If the impulse response of an LTI system is $h(t)$, then the response of the system to an arbitrary input $x(t)$ is:

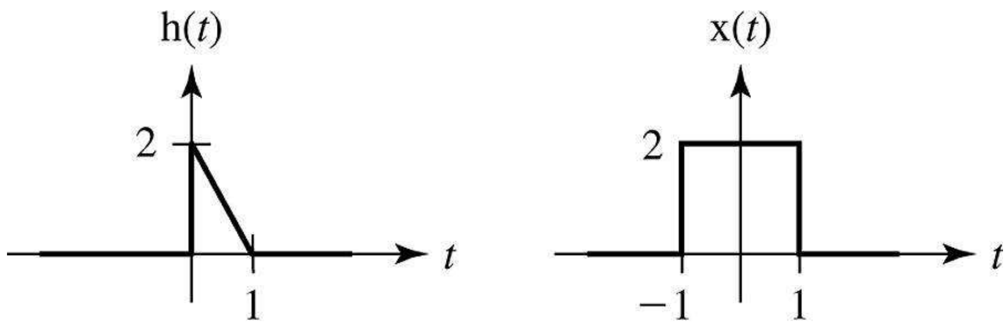
$$y(t) = x(t) * h(t)$$

- Compare with the DT case:

$$y[n] = x[n] * h[n] =$$

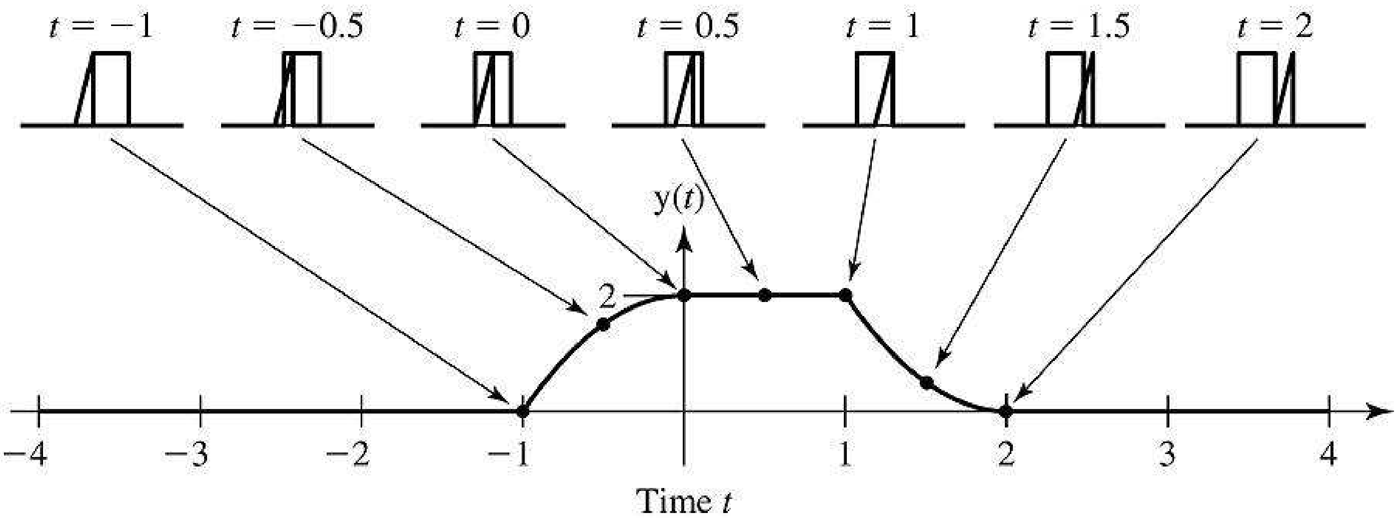
Example

Find $x(t) * h(t) =$



- First find $h(t-\tau)$:

- Then for each t, we should find the area under the function $x(t)h(t-\tau)$:



Convolution Properties

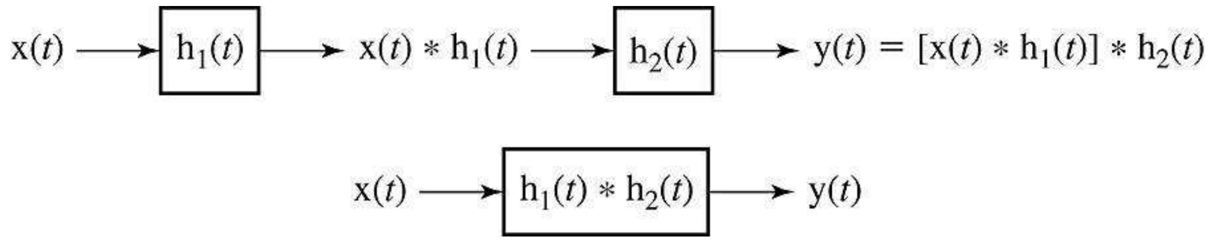
- $x(t) * \delta(t - t_0) =$. Proof?

- If $y(t) = x(t) * h(t)$ then $y'(t) = x'(t) * h(t) = x(t) * h'(t)$. Proof?

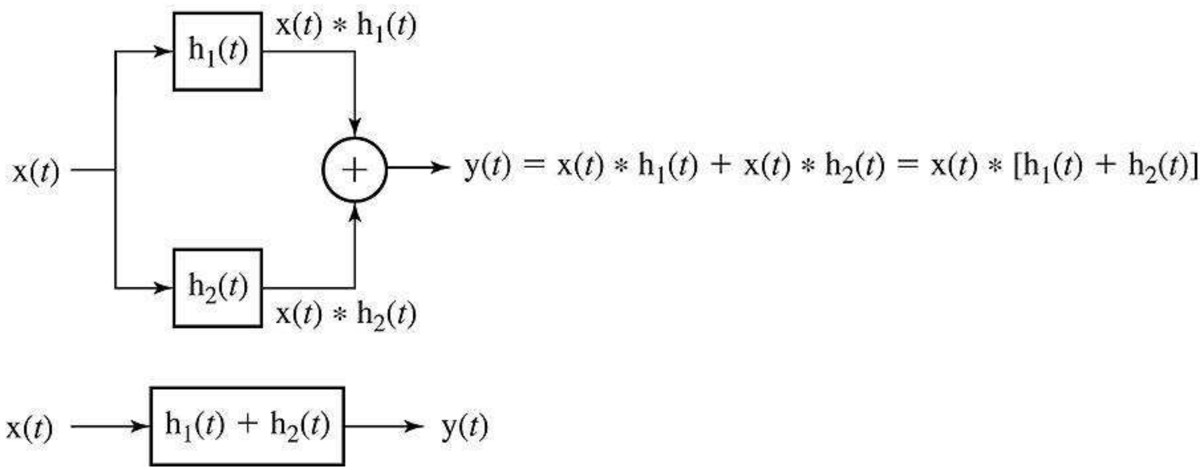
- If $y(t) = x(t) * h(t)$ then $y(at) = |a|x(at) * h(at)$. Proof?

Properties (Cont.)

- Commutative, Associative, and Distributive (same as for DT convolution sum).
- Systems is series :



- Systems in parallel:



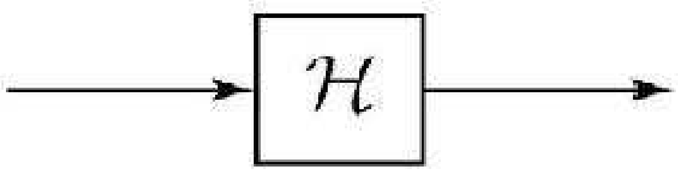
- Stability (BIBO) condition:

$$\int_{-\infty}^{\infty} |h(t)|dt < \infty$$

Unit Step Response

- Recall from properties of convolution integral:
  If $x(t) * h(t) = y(t)$, then, $x'(t) * h(t) =$
- Also recall the relationship between $u(t)$ and $\delta(t)$:
- Thus, if the impulse response of a system is $h(t)$, the step response is:
- Conversely, if the step response of a system is $h_u(t)$, the impulse response is:

Complex Exponential Response



$$y(t) = h(t) * e^{st} =$$

Sinusoidal Response

- $\cos(\omega t)$ can be written in terms of complex exponentials:

- Find the response of the system to each exponential and combine:

- It can be shown that the result is:

$$y(t) = \cos(\omega t) * h(t) = \cos(\omega t) \int_{-\infty}^{\infty} h(\tau) \cos(\omega \tau) d\tau - \sin(\omega t) \int_{-\infty}^{\infty} h(\tau) \sin(\omega \tau) d\tau$$