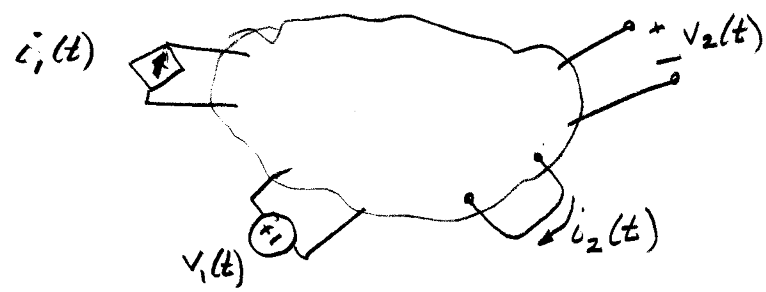
1 CIRCUITS IN THE TIME DOMAIN

- In this section, we consider circuits driven by one or more voltage or current sources, where we designate one or more voltages or currents as outputs.

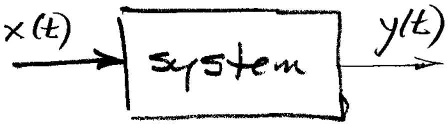


- All signals are functions of time and we work in time domain only (no transforms)
  - differential equation solution
  - convolution

- Several important concepts:
  - LTI, derivative & integral properties
  - state
  - ICs at $t = 0^-, 0^+$
  - unit impulse
  - convolution

1.1 Linearity, Time Invariance and Consequences

- Linearity



— A system is linear iff the response to

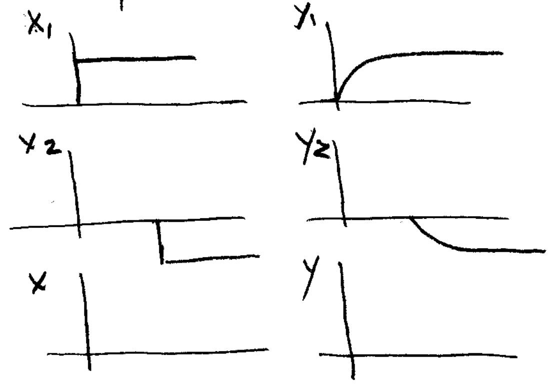
$$x(t) = x_1(t) + x_2(t)$$

is $$y(t) = y_1(t) + y_2(t)$$

where $x_1(t)$ produces $y_1(t)$

$x_2(t)$ produces $y_2(t)$

for <u>any</u> choice of $x_1(t)$, $x_2(t)$



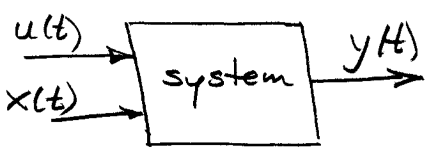
- Special case: if $x(t)$ produces $y(t)$

then $\alpha x(t)$ produces $\alpha y(t)$

- More generally,

$\alpha x_1(t) + \beta x_2(t)$ produces $\alpha y_1(t) + \beta y_2(t)$

— With multiple inputs

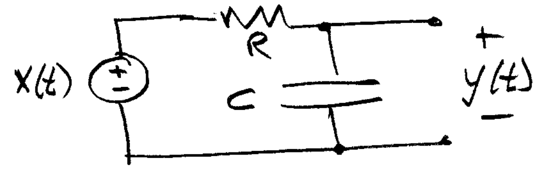


If $u(t)$ acting alone $(x(t)=0)$ produces $y_u(t)$

and $x(t)$ " " $(u(t)=0)$ " $y_x(t)$

then $u(t)$ and $v(t)$ together produce $y_u(t) + y_x(t)$

example



Generally

$$\dot{y}(t) + \frac{1}{\tau} y(t) = \frac{1}{\tau} x(t)$$

If $\dot{y}_1(t) + \frac{1}{\tau} y_1(t) = \frac{1}{\tau} x_1(t)$, $\dot{y}_2(t) + \frac{1}{\tau} y_2(t) = \frac{1}{\tau} x_2(t)$

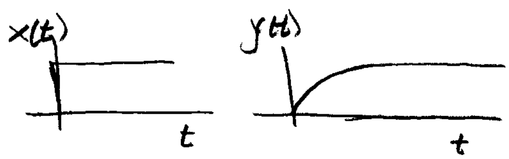
then

$$\frac{d}{dt}\left(y_1(t) + y_2(t)\right) + \frac{1}{\tau}\left(y_1(t) + y_2(t)\right) = \frac{1}{\tau}\left(x_1(t) + x_2(t)\right)$$

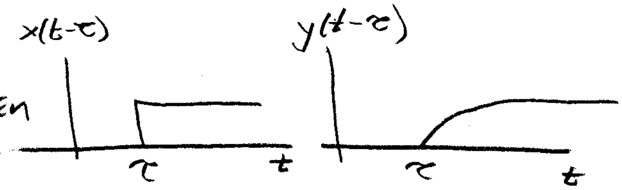
- Time invariance — the way the system responds does not change with time:
  - If $x(t)$ produces $y(t)$ then $x(t-\tau)$ produces $y(t-\tau)$ for any τ

example The circuit above

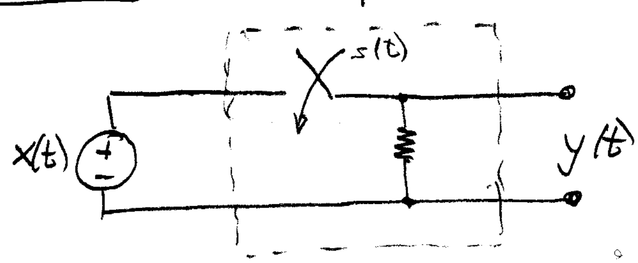
If

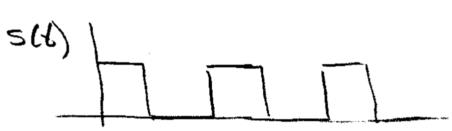


then



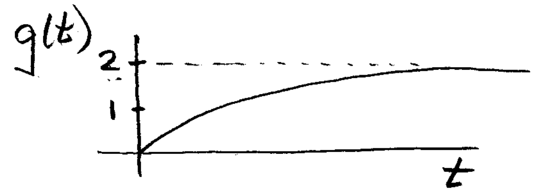
example This system is linear, but time varying



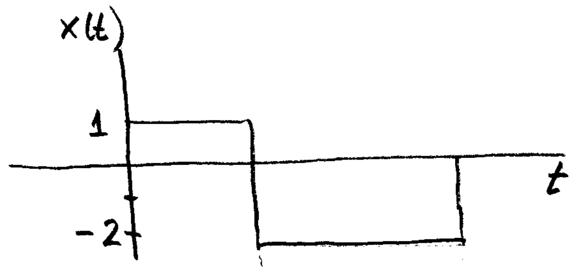


- You can exploit the LTI property for quick solutions. Doing so builds intuition.

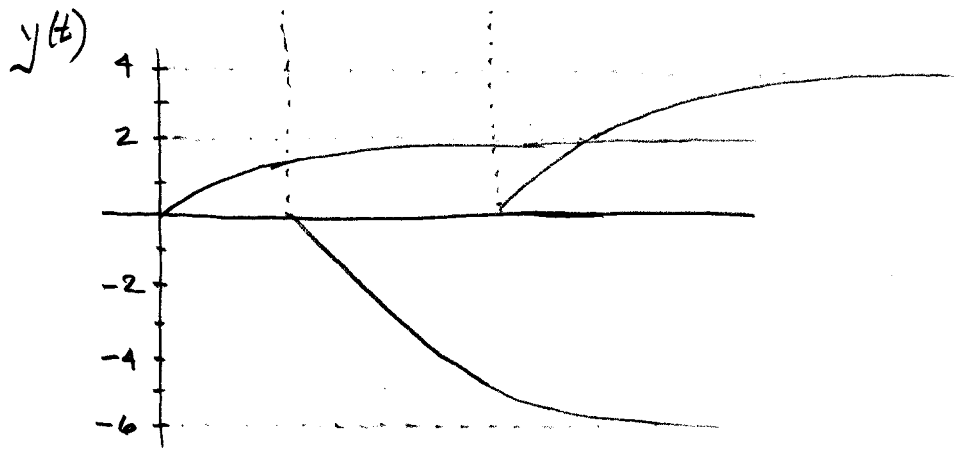
- Sketch solutions. If the step response is



then response to



is

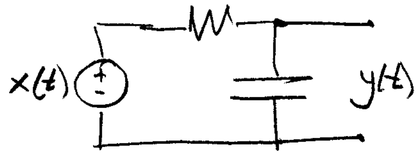


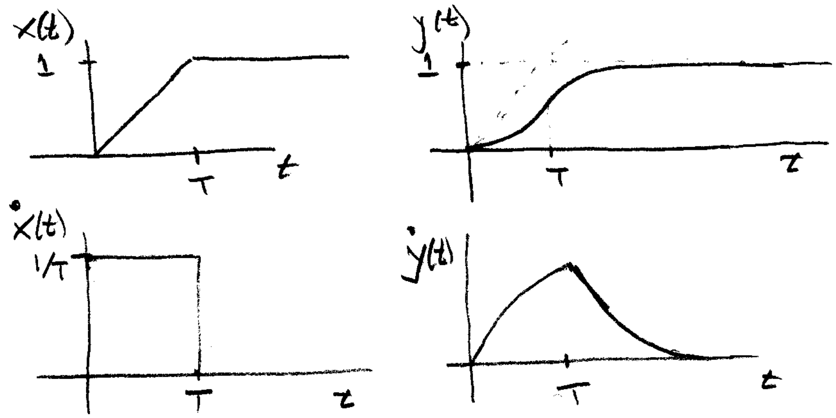
- Derivative property applies to all LTI systems.
  - If $x(t)$ produces $y(t)$

    then $\dfrac{x(t+\Delta t)-x(t)}{\Delta t}$ produces $\dfrac{y(t+\Delta t)-y(t)}{\Delta t}$

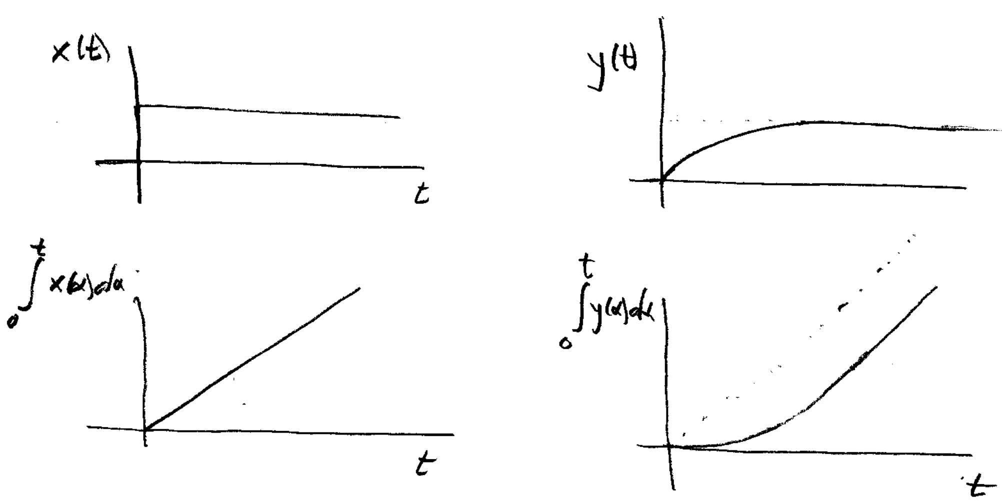
    Take $\lim_{\Delta t \to 0}$, and $\dot{x}(t)$ produces $\dot{y}(t)$

<u>example</u>

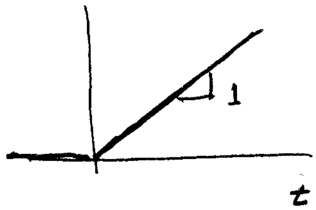




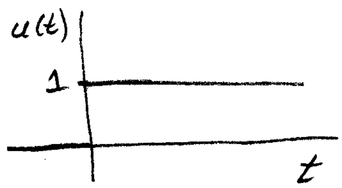
- Similarly, integral property $\int_0^t x(\alpha)\,d\alpha$ produces $\int_0^t y(\alpha)\,d\alpha$

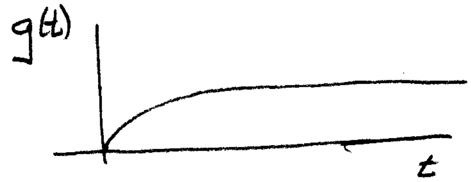


- So derivative of unit ramp response

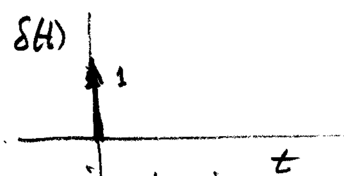


is the unit step response:





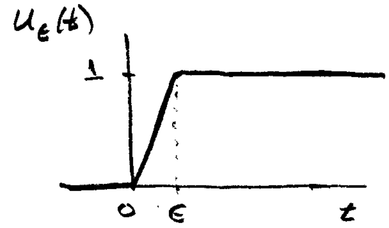
and the derivative of the unit step response is the unit impulse response:



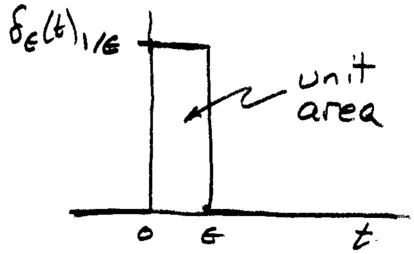
unit impulse is "derivative" of unit step.

The unit impulse represents a singularity — clearly a non-physical signal, with infinite height, zero width

- The impulse is best considered as shorthand notation for a limit, not as a physical signal

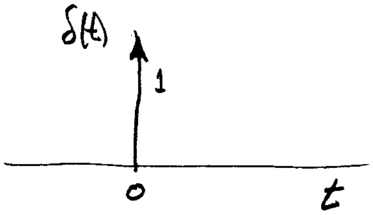


$u_\epsilon(t)$, a continuous function



approximate impulse

$$\delta_\epsilon(t) = \frac{d}{dt} u_\epsilon(t)$$



define unit impulse

$$\delta(t) = \lim_{\epsilon \to 0} \delta_\epsilon(t)$$

zero width, infinite height, unit area

- Properties of unit impulse

→ $\int_{-\infty}^{\infty} \delta(t)\,dt = 1$ since $\int_{-\infty}^{\infty} \delta_\epsilon(t)\,dt = 1$ for any ϵ

→ $\int_{-\infty}^{\infty} \delta(t) f(t)\,dt = f(0)$ since $\lim_{\epsilon \to 0} \int_{-\infty}^{\infty} \delta_\epsilon(t) f(t)\,dt = \lim_{\epsilon \to 0} \frac{1}{\epsilon} \int_0^{\epsilon} f(t)\,dt$

$$= \lim_{\epsilon \to 0} \frac{F(\epsilon) - F(0)}{\epsilon} = f(0)$$

→ similarly $\int_{-\infty}^{\infty} \delta(t-\tau) f(t)\,dt = f(\tau)$

→ $\int_{-\infty}^{t} \delta(\alpha)\,d\alpha = u(t)$ the unit step

→ $f(t)\,\delta(t-\tau) = f(\tau)\,\delta(t-\tau)$

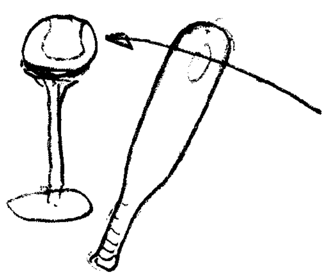
- The impulse response $h(t)$ may not be observed physically, but it is very useful in analysis.

- Origin of the term

  Newton's law $\quad f = \dot{p} \quad (p = mv,\ \text{momentum})$

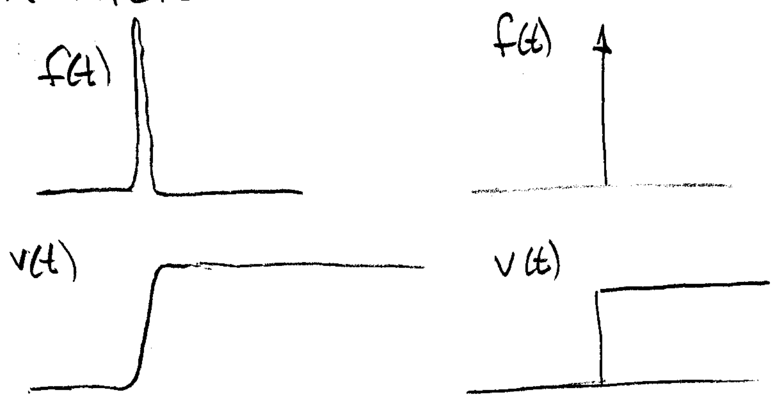
  "impulse" $\displaystyle\int_{-\infty}^{\infty} f(t)\,dt = \Delta p$

tee ball



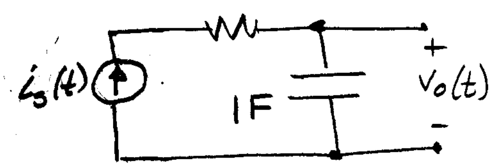
zoom microview

macroview

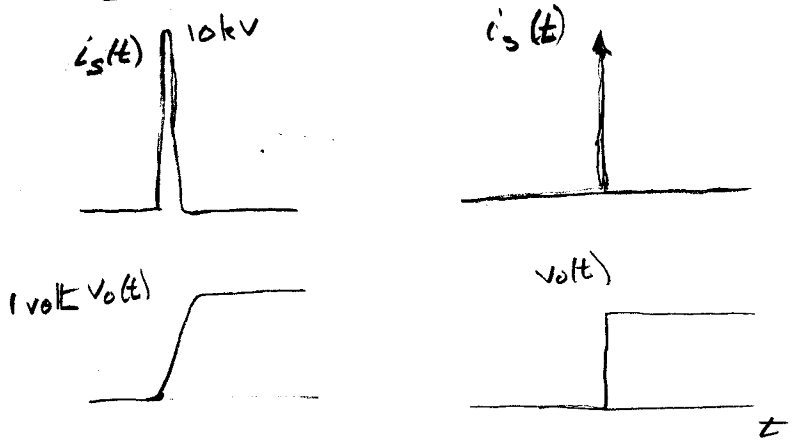


instantaneous change in velocity ??

RC lowpass



zoom



instantaneous change in capacitor voltage ??