# 3.2 Circuits With Non-Zero Initial Conditions

DC & L 14.5

- Often, we deal with circuits that are <u>not</u> initially at rest — particularly when there are switches in the circuit (e.g. switched cap filters)

- ICs are straightforward to handle using LT.

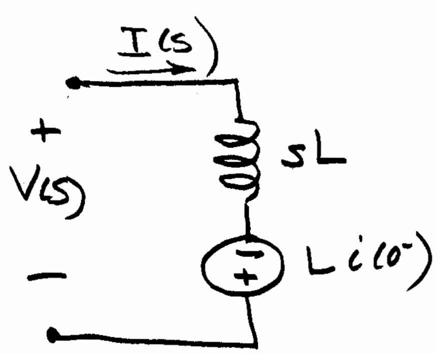
  – inductor $\quad v = L\frac{di}{dt}$

  or

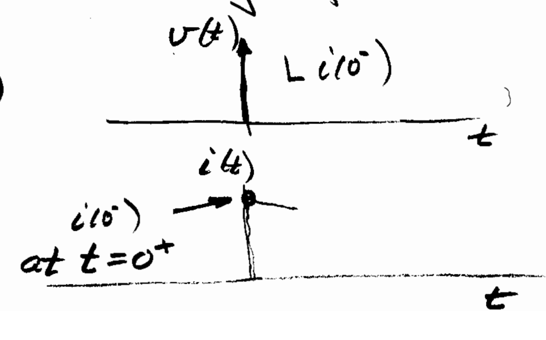
$$V(s) = L\left(sI(s) - i(0^-)\right)$$

$$= sL\,I(s) - L\,i(0^-)$$

second term is a constant, the LT of an impulse, and it adds an extra component of $V(s)$ — so add a fictitious <u>voltage</u> source (for a <u>current</u> IC)



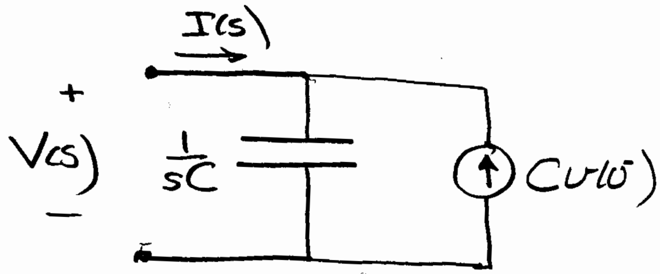

infinite voltage for zero time causes current to jump to IC value



– capacitor $i = C \frac{dv}{dt}$

$$I(s) = C\left(sV(s) - v(0^-)\right) = sC\,V(s) - \underbrace{C\,v(0^-)}_{\text{transform of impulse}}$$

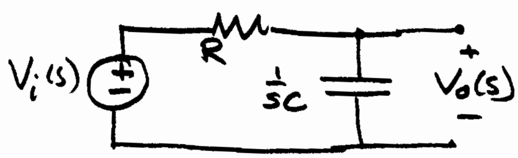
A fictitious current source to represent voltage IC




The impulse in current dumps an instant $C\,v(0^-)$ coulomb onto the cap, causing a voltage jump to $v(0^-)$

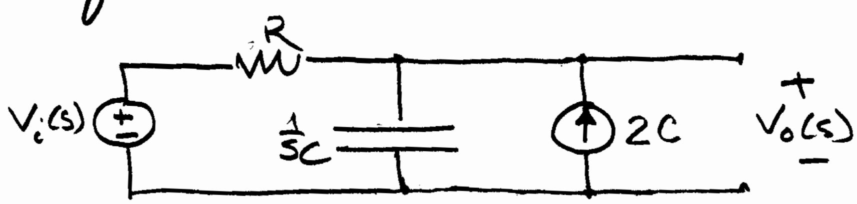
– and solve circuits using superposition, as if they were at rest

- example

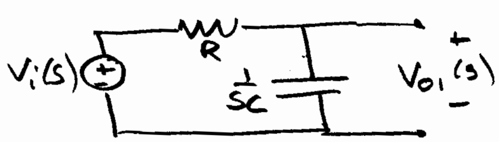


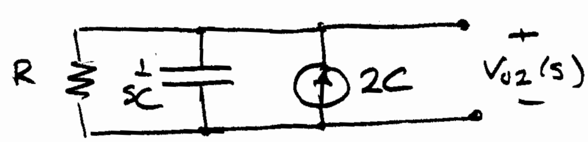

with $v_o(0^-) = 2$ V

equivalent circuit:




superposition:



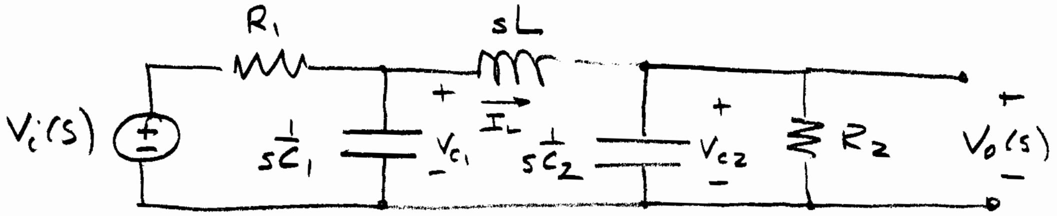




$$V_{o1}(s) = \frac{1/sC}{R + \frac{1}{sC}} V_i(s) \qquad V_{o2}(s) = \frac{R \cdot \frac{1}{sC}}{R + \frac{1}{sC}} \cdot 2C$$

$$= \frac{1}{1 + sRC} V_i(s) \quad + \quad = \frac{2RC}{1 + sRC}$$

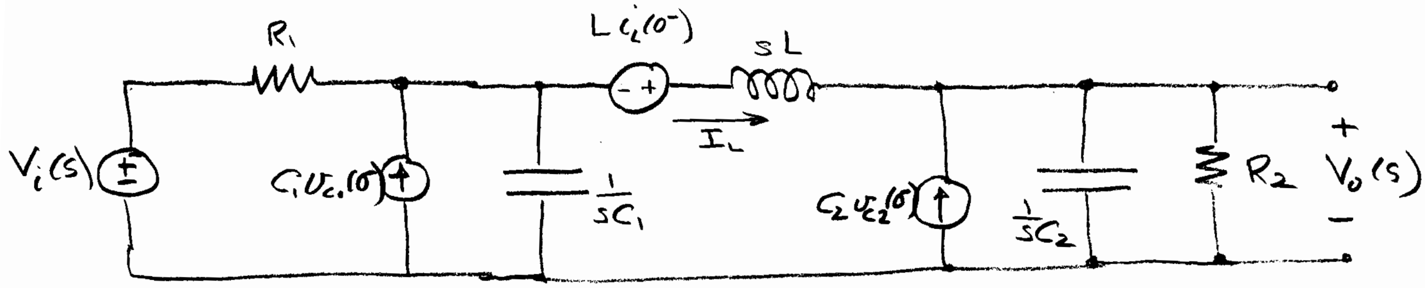
what is H(s) in this example?

- A complicated example – a $3^{rd}$ order lowpass

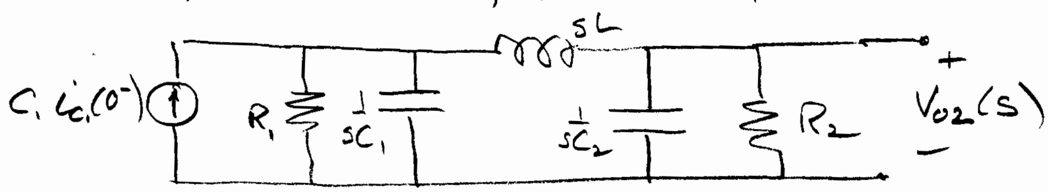


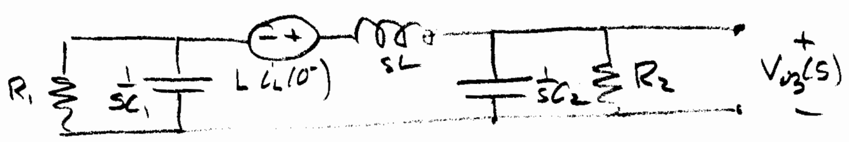

- If zero ICs, then solve by circuit reduction, or loop and/or node equations, all with impedances $R$, $sL$, $1/sC$.

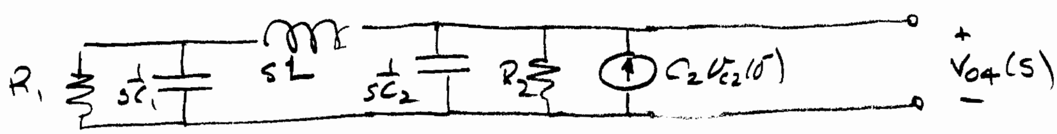
- If non-zero IC's, redraw circuit



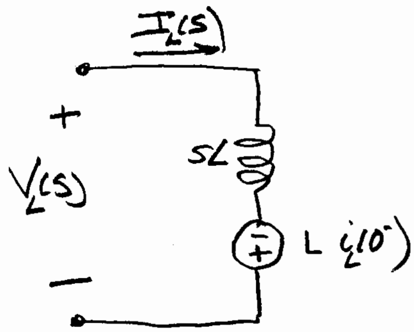

Total response is the superposition of responses to four sources; for example

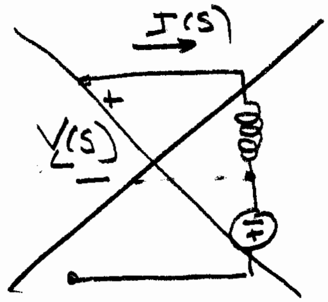


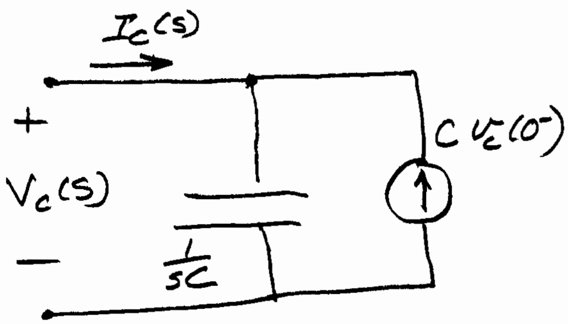



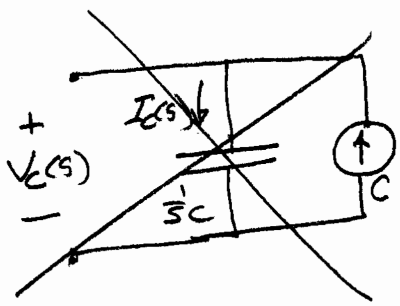
- Be careful if the "output" is a circuit element with a non-zero IC



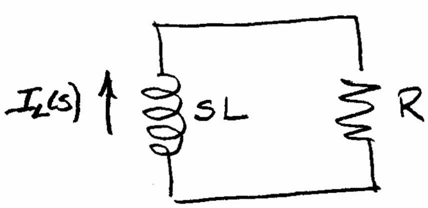

The inductor voltage $V_L(s)$ <u>includes</u> the IC source



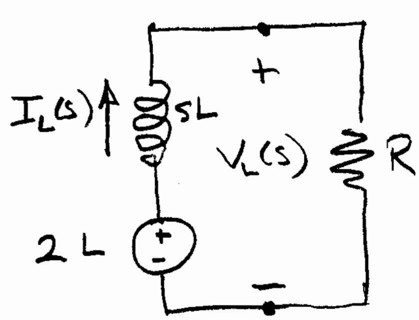

The cap current must <u>include</u> the IC source

<u>example</u>




Initial inductor current is $i_L(0^-) = 2$ A. Solve for inductor voltage $V_L(s)$.

equiv cct




① By voltage divider:

$$V_L(s) = \frac{R}{R+sL} \cdot 2L$$

② By current then voltage

$$I_L(s) = \frac{2L}{sL+R}$$

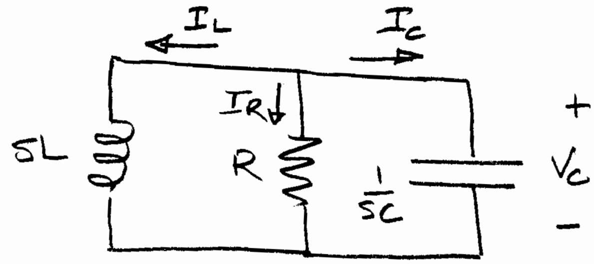
so

$$V_L(s) = 2L - sL\,I_L(s)$$

$$= 2L - \frac{2sL^2}{sL+R}$$

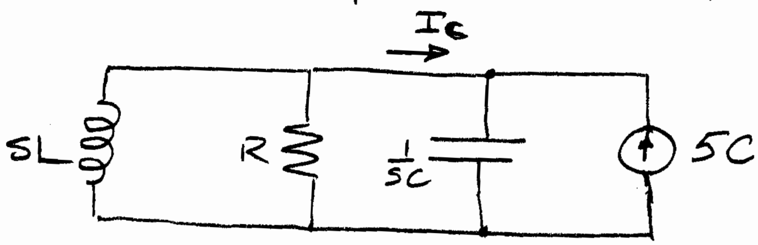
$$= \frac{2RL}{sL+R}$$

another example




The capacitor is initially charged to 5 V and the initial inductor current is 0.

Solve for capacitor current.




The cap current is <u>not</u> simply a current divider applied to 5C; it must also include 5C.

current divider alone:

$$5C \frac{sC}{sC + \frac{1}{R} + \frac{1}{sL}} = \frac{5C\, s^2}{s^2 + \frac{1}{RC}s + \frac{1}{LC}}$$

plus IC source:

$$I_c(s) = \frac{5C\, s^2}{s^2 + \frac{1}{RC}s + \frac{1}{LC}} - 5C \qquad \text{(why "$-$"?)}$$

$$= \frac{-\frac{5}{R}s - \frac{5}{L}}{s^2 + \frac{1}{RC}s + \frac{1}{LC}}$$

or $I_c(s) = -$ current to $sL \| R = \frac{sLR}{sL+R}$, obtain by current div

$$= -5C \cdot \frac{\frac{1}{sC}}{\frac{sLR}{sL+R} + \frac{1}{sC}} = \frac{-5\left(\frac{s}{R} + \frac{1}{L}\right)}{s^2 + \frac{1}{RC}s + \frac{1}{LC}}$$