4.2 Frequency Response By Pole Zero Diagrams

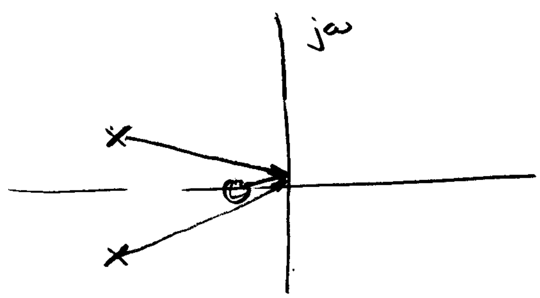
DCEL 15.5

- In this section, we'll develop a simple method to determine the frequency response of a circuit. In design, it lets you determine roughly where you want to put the poles and zeros.

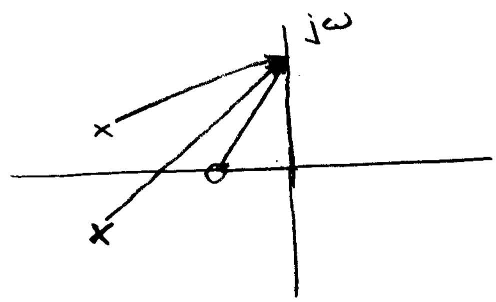
- Use the fact that $H(j\omega)$ is $H(s)$ on the $j\omega$ axis.

$$H(s) = K \frac{(s-z_1)(s-z_2)\cdots(s-z_m)}{(s-p_1)(s-p_2)\cdots(s-p_n)}$$

$$H(j\omega) = K \frac{(j\omega-z_1)\cdots(j\omega-z_m)}{(j\omega-p_1)\cdots(j\omega-p_m)} = K \frac{\prod_{i=1}^{m}(j\omega-z_i)}{\prod_{k=1}^{n}(j\omega-p_k)}$$



each factor is a "vector" on the s-plane



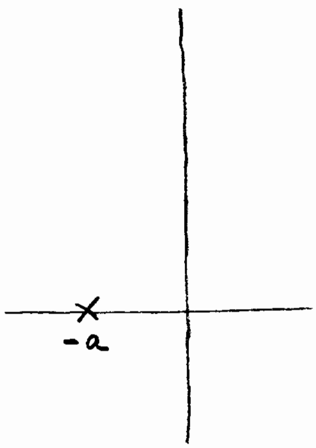
- So the magnitude $|H(j\omega)| = K \dfrac{\prod_{i=1}^{m} |j\omega - z_i|}{\prod_{k=1}^{n} |j\omega - p_k|}$

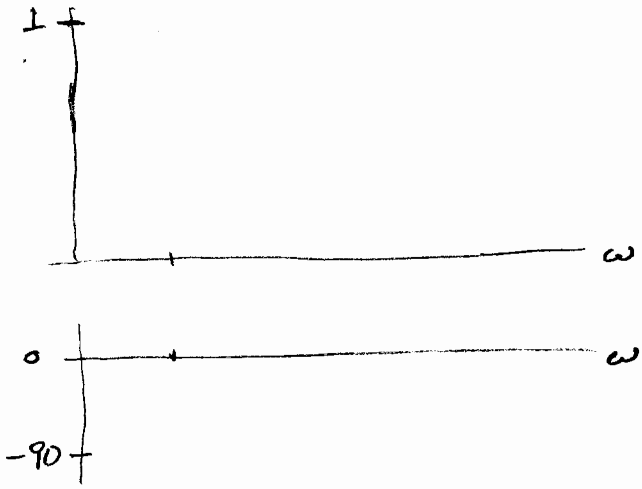
  is the product of zero-vector lengths divided by product of pole-vector lengths.

  and the phase $\arg[H(j\omega)] = \sum_{i=1}^{m} \arg(j\omega - z_i) - \sum_{k=1}^{n} \arg(j\omega - p_k)$

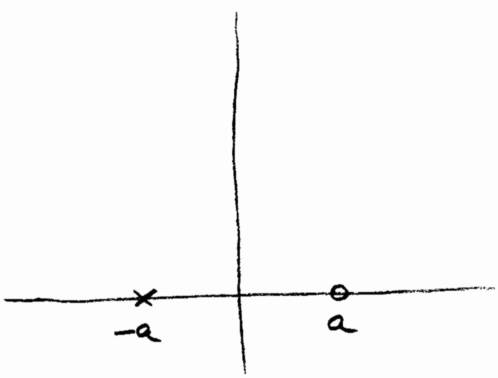
  is the sum of the zero-vector phases less the sum of the pole-vector phases.

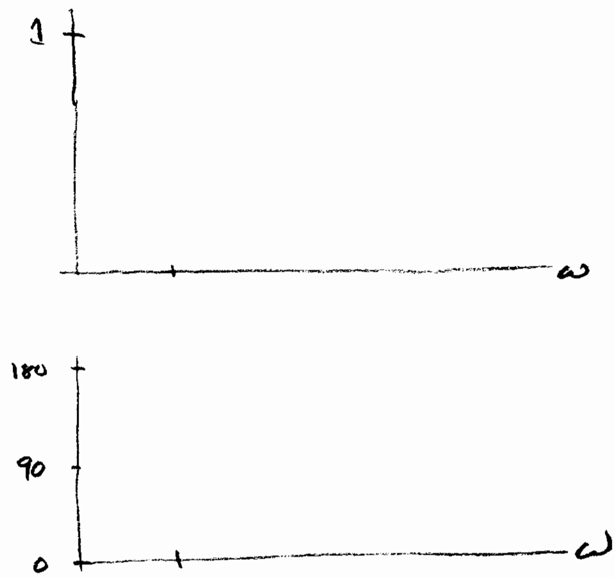
Single pole lo pass



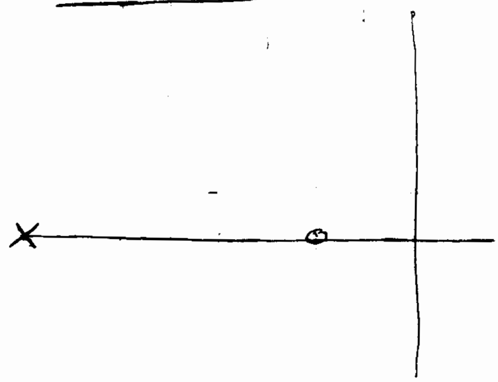


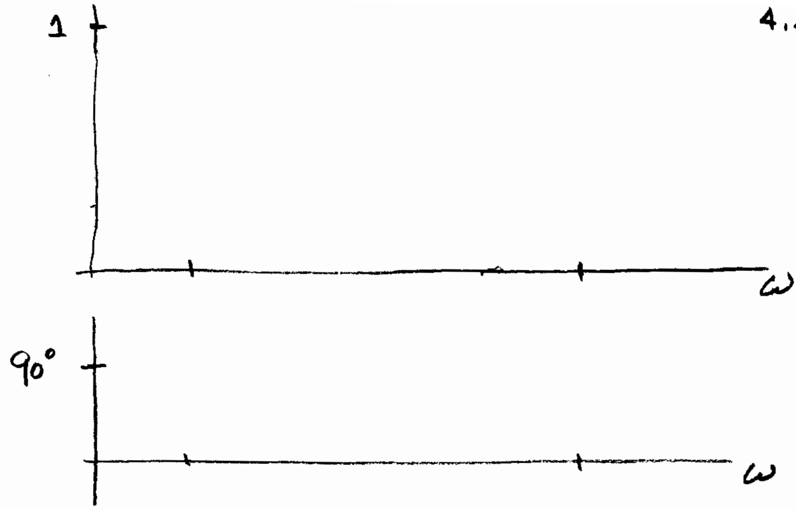
First order all pass



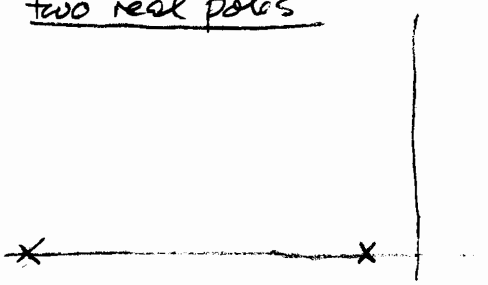


Lead filter

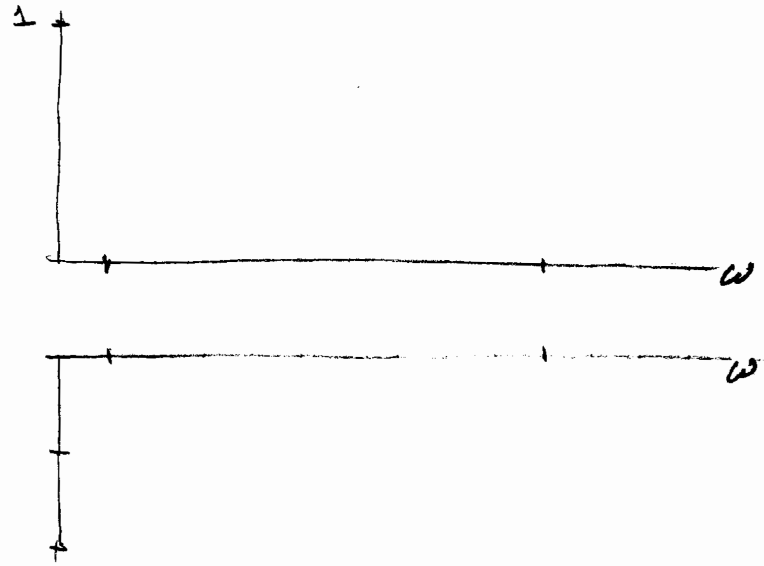




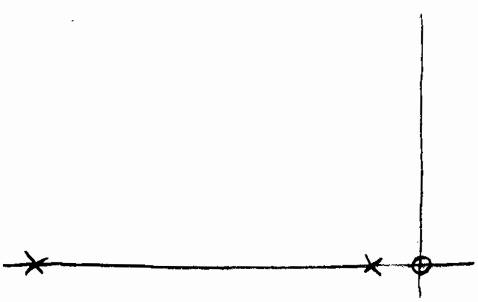
two real poles

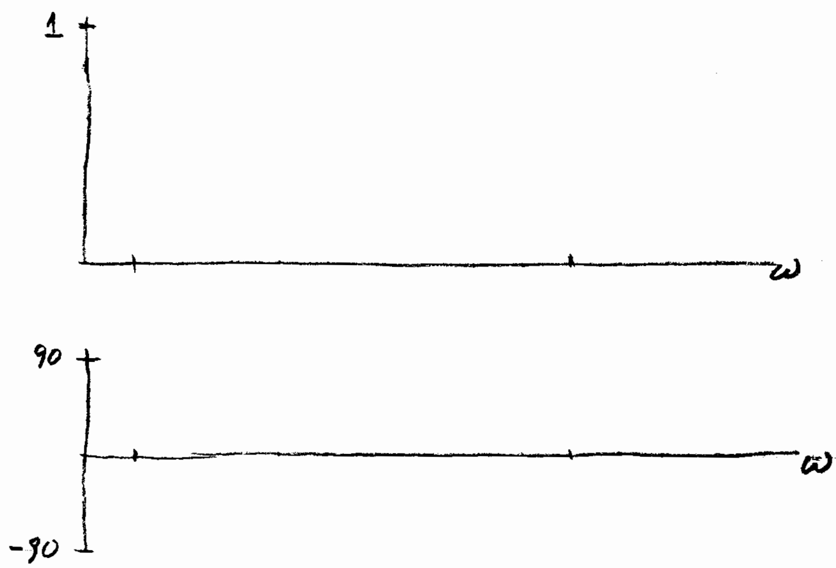


use
K=8
for
convenience

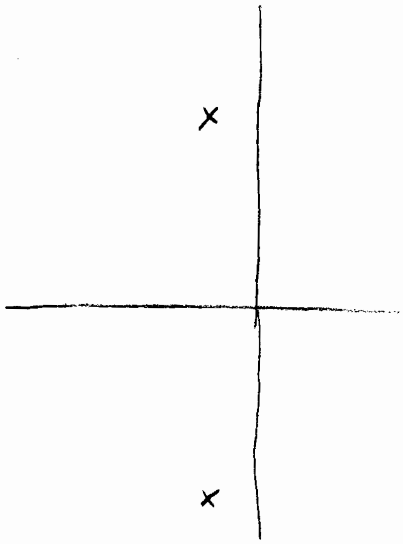


two real poles & a zero

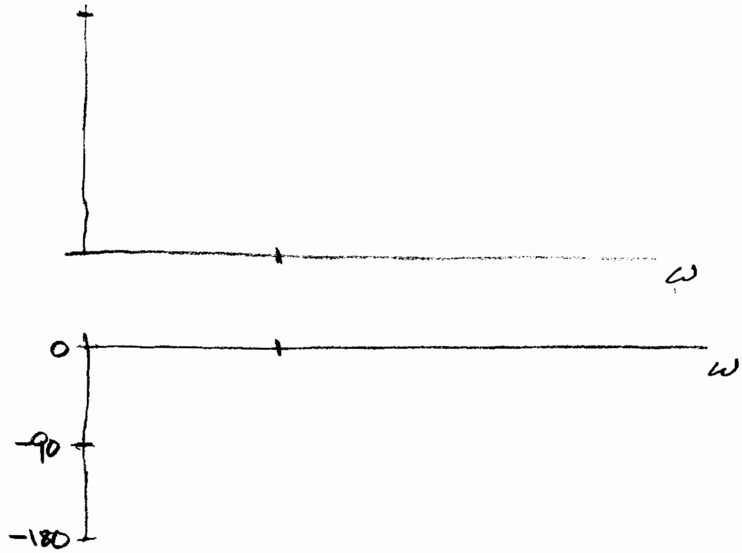




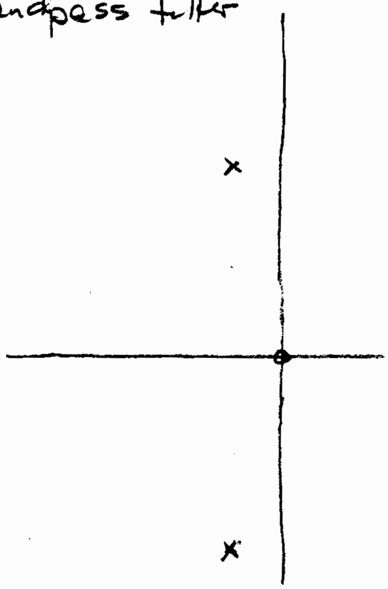
two lightly damped poles

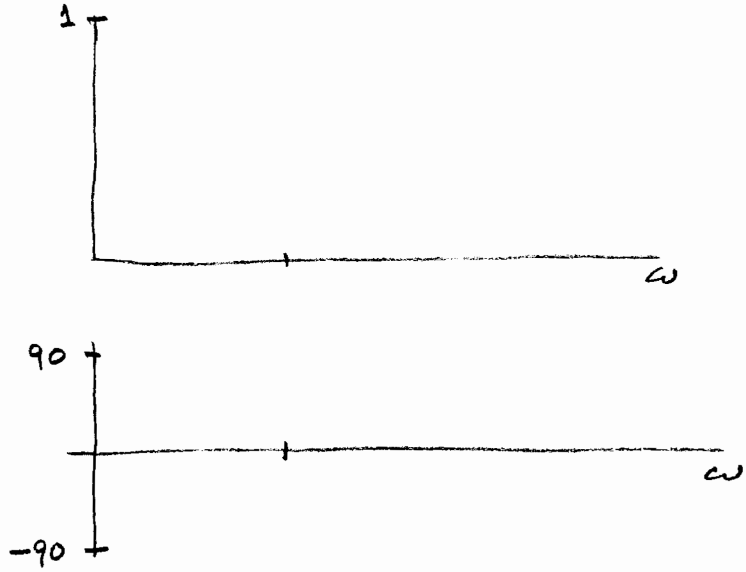


use K=8

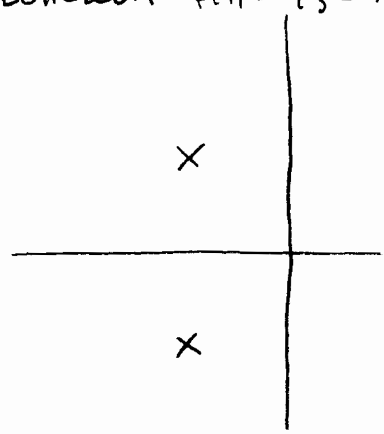


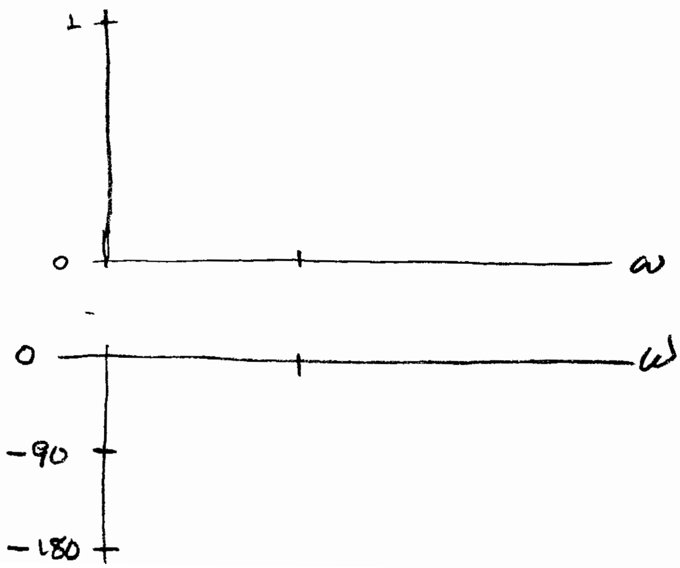
bandpass filter





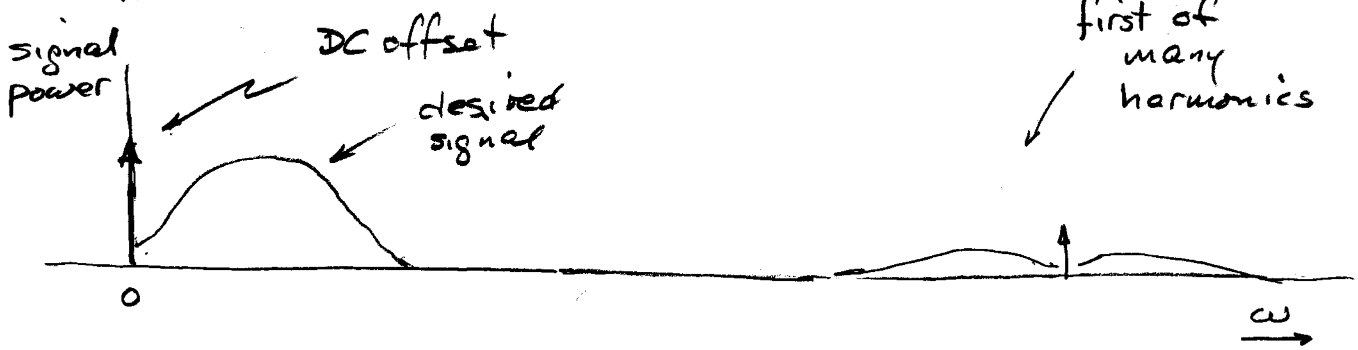
Butterworth filter ($\zeta = 1/\sqrt{2}$)

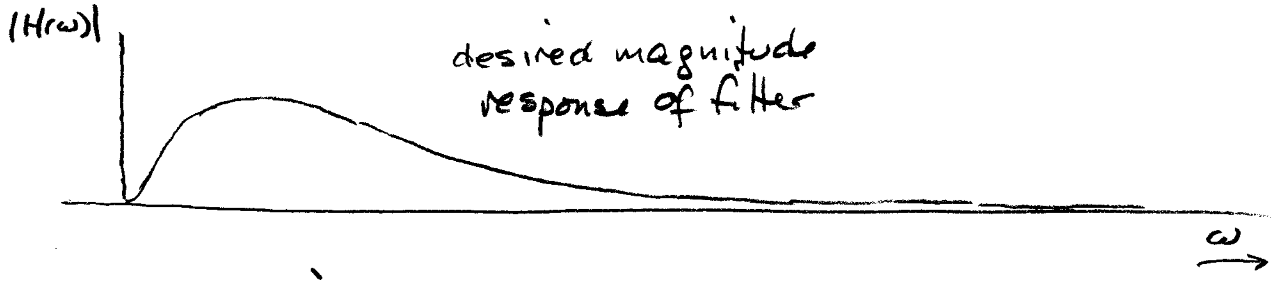




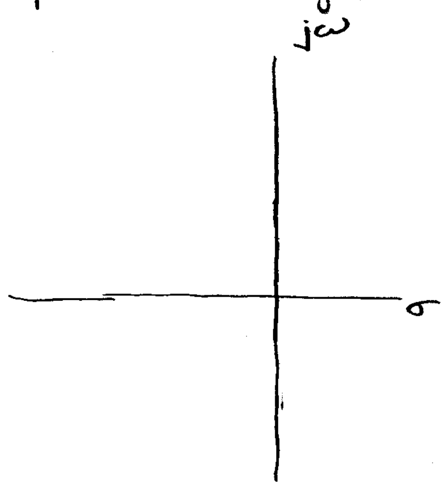
- Now use this idea for design.

We want a lowpass filter to suppress harmonics in a signal, which also contains an unwanted DC offset





pole-zero diagram

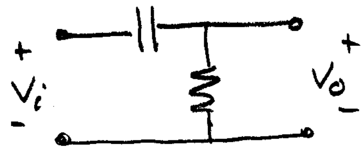


Note: there will be some distortion of the desired signal

- Approximations of the elementary circuits on page 4.1.4. The rough equivalences below are useful in themselves and as a way to understand the action of other circuits.

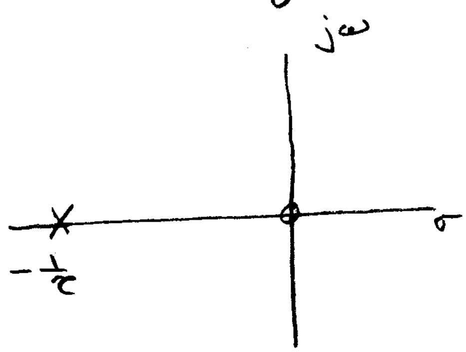
  - differentiator.

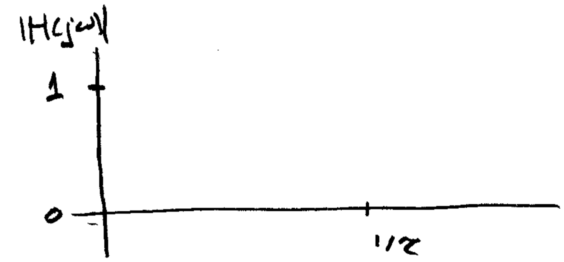
    This circuit

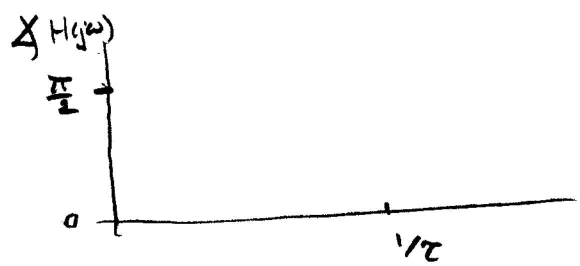


$$H(s) = \frac{s}{s + 1/\tau}$$

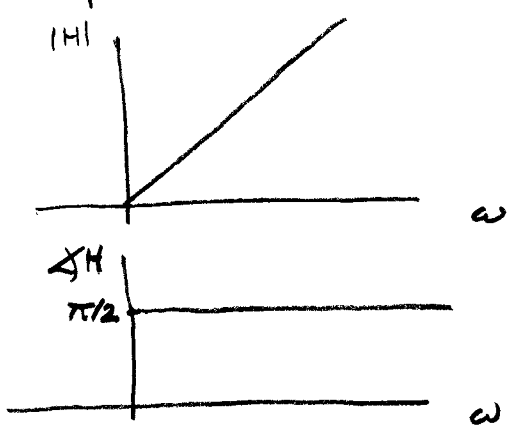
is not a true differentiator, but look at its frequency response







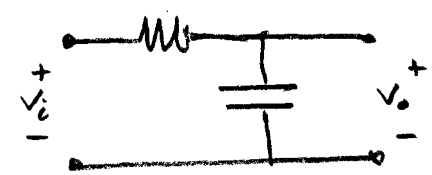
Compare with true differentiator



slope 1

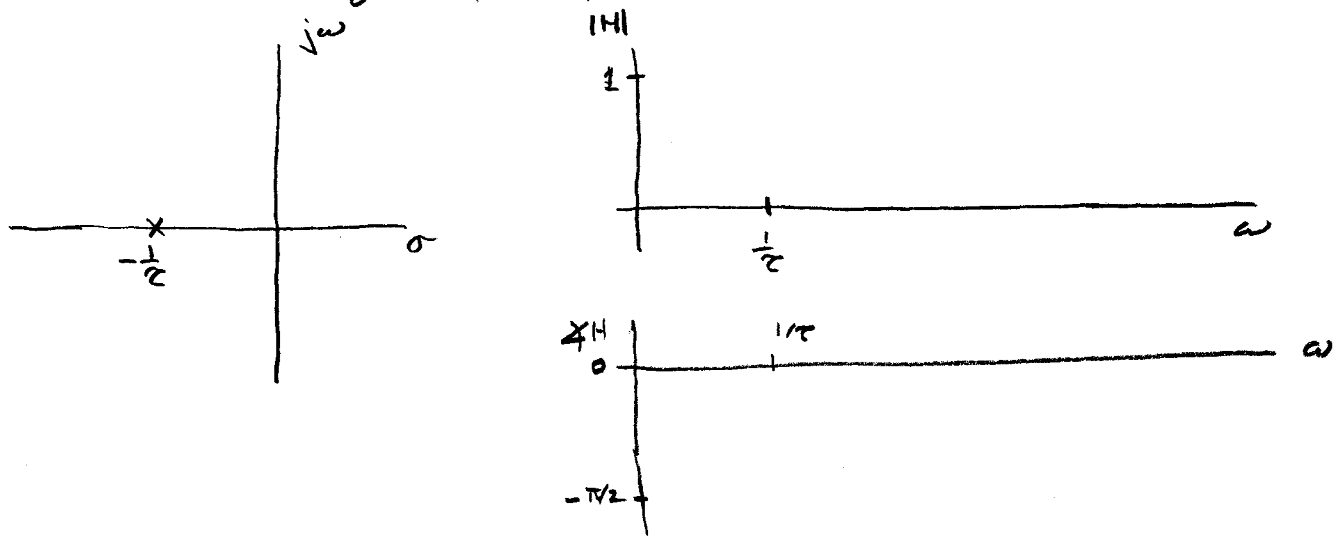
– integrator

Again, a circuit that is not a true integrator

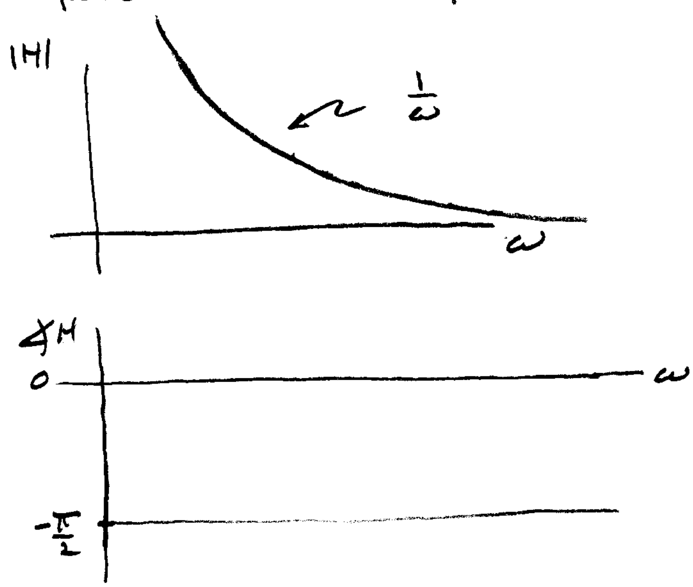


$$H(s) = \frac{1/\tau}{s + 1/\tau}$$

but its frequency response is

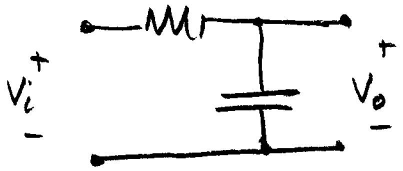


Compare with true integrator



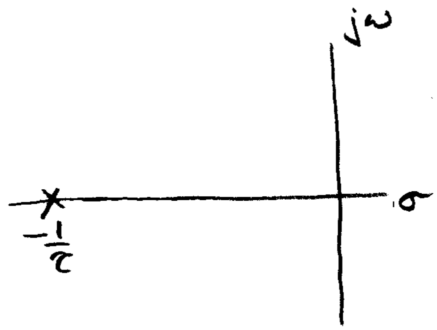
– delay

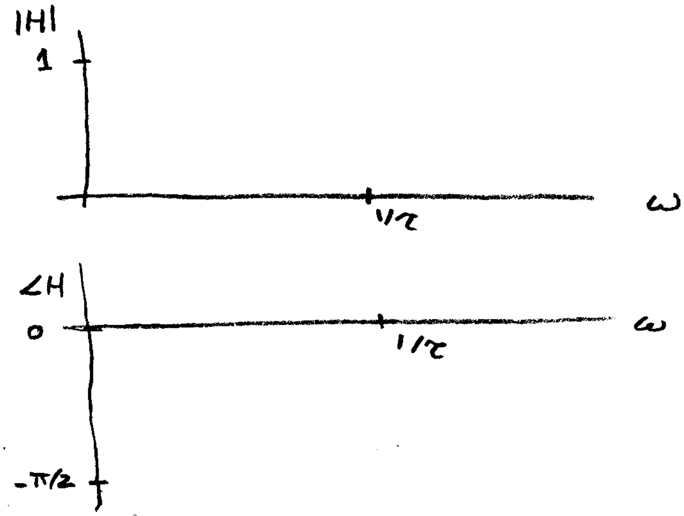
The same low pass circuit as in the last example can look rather like a delayer



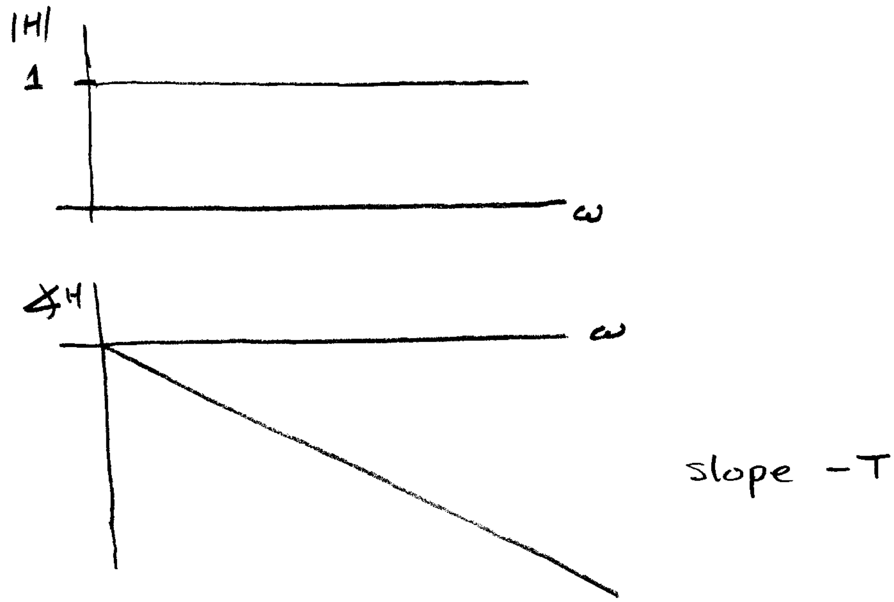
$$H(s) = \frac{1/\tau}{s + 1/\tau}$$

Its frequency response is

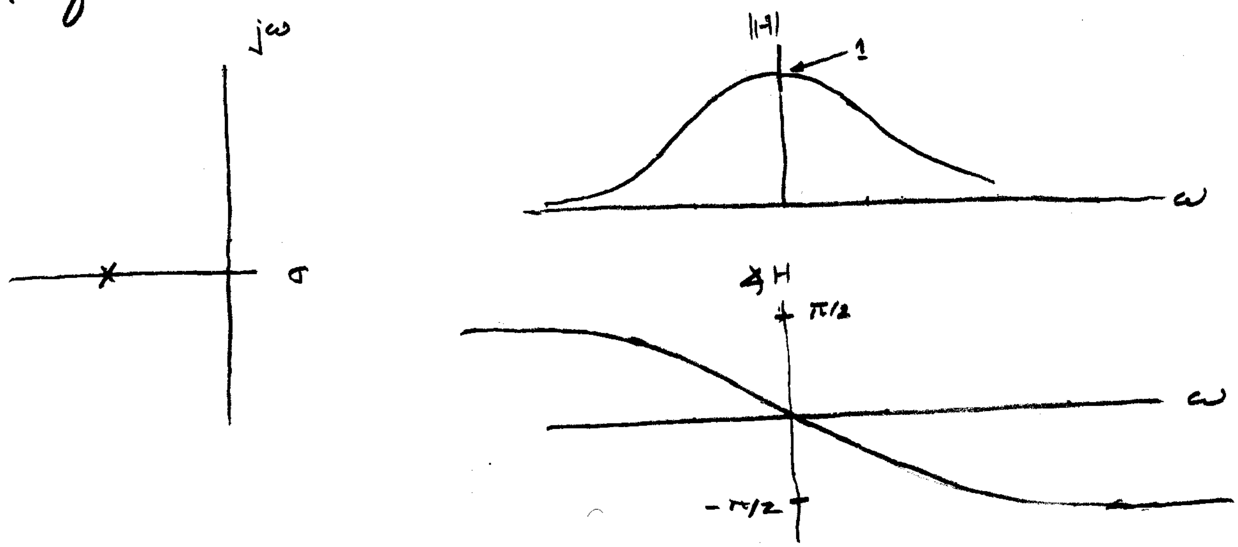




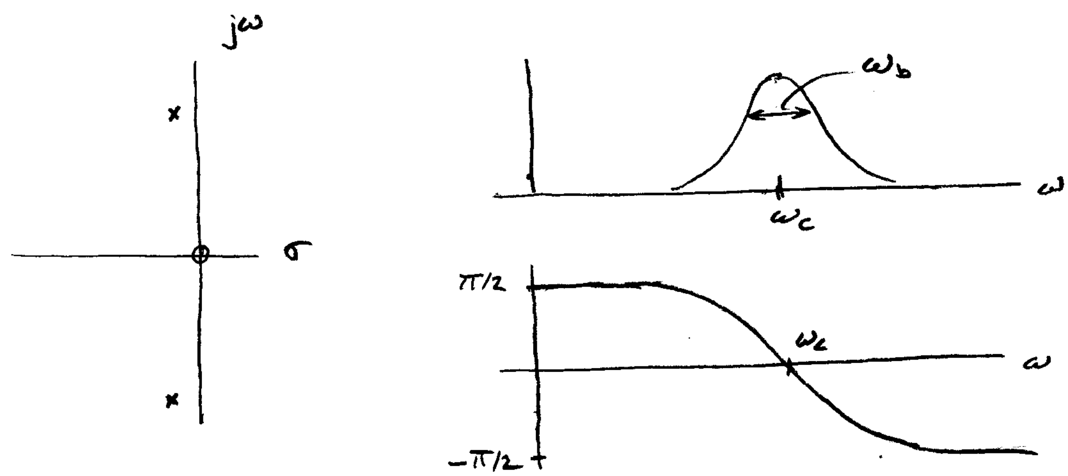
Compare with true delayer



- Lowpass - Bandpass transformation
  - This is a nice trick to generate a good bandpass filter design from a good low pass design
  - Consider single pole lowpass. Sketch its frequency response for positive and negative frequencies

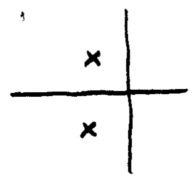


  - This is the same behaviour as in resonant circuits, but centred at 0 rad/s.



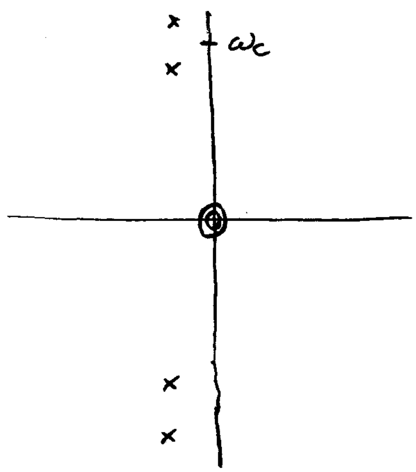
It's an approximate equivalence, but not bad for narrowband ($\omega_b / \omega_c \ll 1$) filters.

- This suggests the following design approach
  - → select a lowpass design with the right bandwidth — for example



$\zeta = \frac{1}{\sqrt{2}}$ 2nd order Butterworth

  - → translate the pattern up to $j\omega_c$ and down to $-j\omega_c$



  - → and put in a zero at the origin for every lowpass pole, to compensate near ω_c for the image at $-\omega_c$.

– Later we'll see formally how to do it – but here's a preview.

→ given a lowpass design of bandwidth 1 rad/sec $H_{lp}(s)$

→ replace s with $\frac{s^2+\omega_c^2}{\omega_b s}$

$$H_{bp}(s) = H_{lp}\left(\frac{s^2+\omega_c^2}{\omega_b s}\right)$$

where ω_c is the centre frequency

ω_b is the bandwidth

– An example: our 1st order lowpass with unit bandwidth

$$H_{lp}(s) = \frac{1}{s+1}$$

From this,

$$H_{bp}(s) = \frac{1}{\frac{s^2+\omega_c^2}{\omega_b s}+1} = \frac{\omega_b s}{s^2+\omega_b s+\omega_c^2}$$

$$= \underbrace{\frac{2\zeta\omega_0 s}{s^2+2\zeta\omega_0 s+\omega_0^2}}_{\text{the standard 2nd order bandpass}}$$