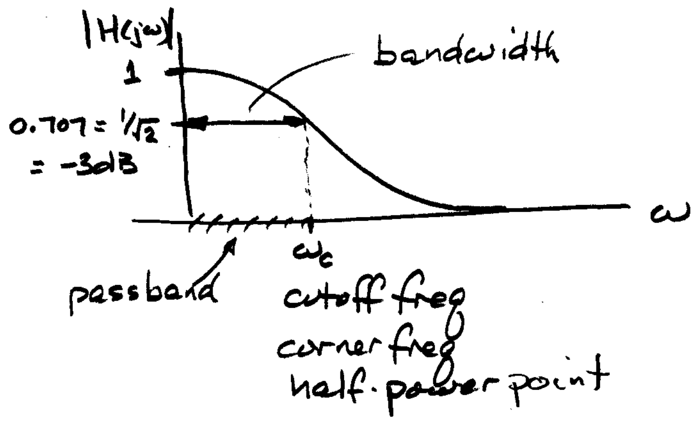
4.3 Parameters Derived From Frequency Response

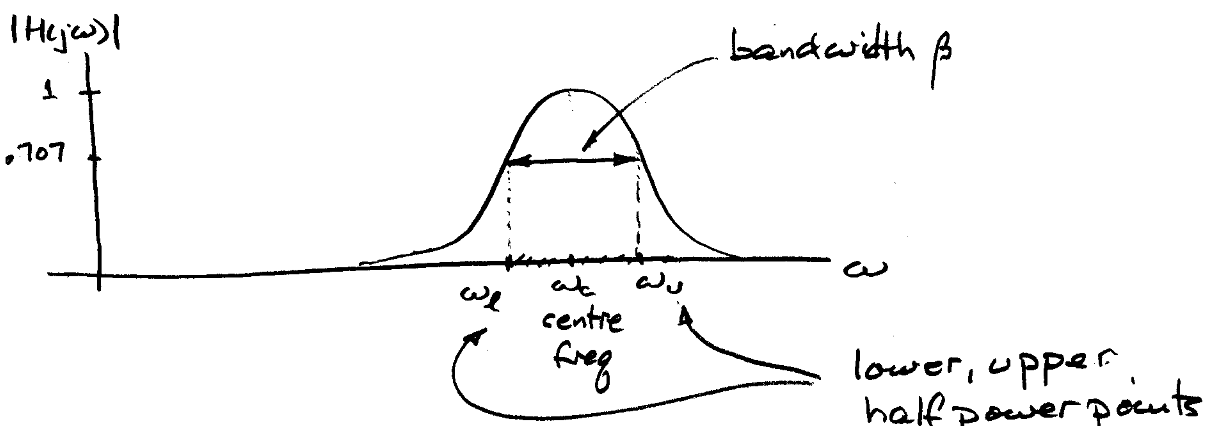
- We often summarize the features of the frequency response that are most important for many applications by a few key parameters.

- Bandwidth of a lowpass filter



many other definitions are possible

- Bandwidth of a bandpass filter



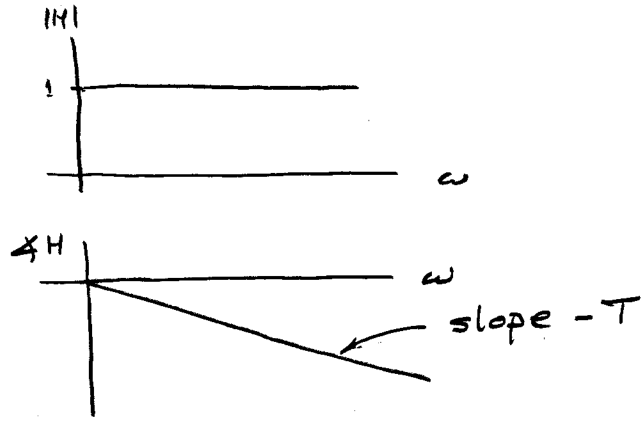
For simple bandpass, quality factor defined:

$$Q = \frac{\omega_c}{\beta}$$ reciprocal of fractional bandwidth

- Group delay
  - Recall that ideal delay has $h(t) = \delta(t-T)$

    and

    $$H(j\omega) = e^{-j\omega T}$$

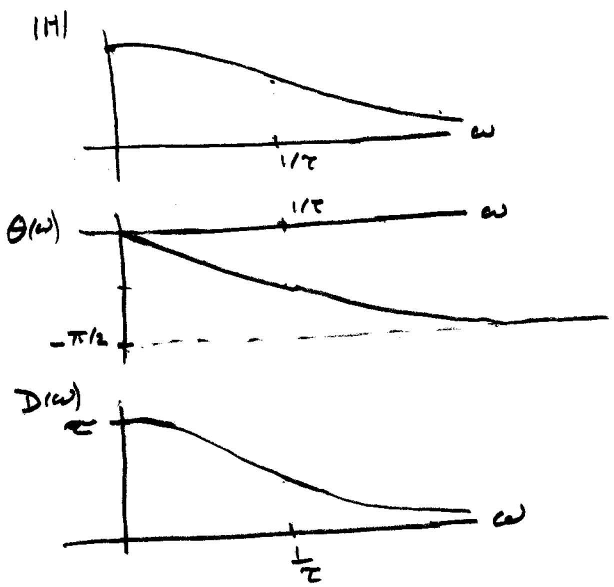


It's an allpass filter, and the delay is the negative of the slope of the phase curve.

  - By analogy, if a filter phase $\theta(\omega)$ is not linear in ω, speak of delay as the negative of the derivative of the phase curve:

$$\text{delay } D(\omega) = -\frac{d\theta(\omega)}{d\omega} \qquad \text{frequency-dependent delay.}$$

For example, $H(s) = \dfrac{1}{s + 1/\tau}$



$$\theta(\omega) = -\tan^{-1}(\omega\tau)$$

$$D(\omega) = \frac{\tau}{1 + (\omega\tau)^2}$$

$$\approx \tau, \quad \omega\tau \ll 1$$

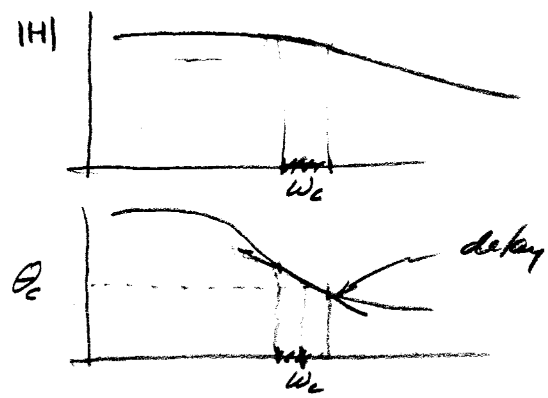
see p. 4.2.8

two consequences of $D(\omega)$:

- if not constant across interesting range of signal frequencies, then signal suffers "delay distortion" or "phase distortion".

- if a narrow band modulated signal passed thru filter, then

eg $m(t)\cos(\omega_c t)$



linear approx
$$\theta(\omega) = \theta(\omega_c) - D(\omega_c)(\omega - \omega_c)$$

group delay
envelope delay

- modulation is delayed by $D(\omega_c)$
- carrier is shifted by $\theta(\omega)$

output is $\approx m(t - D(\omega_c))\cos(\omega_c t + \theta(\omega_c))$

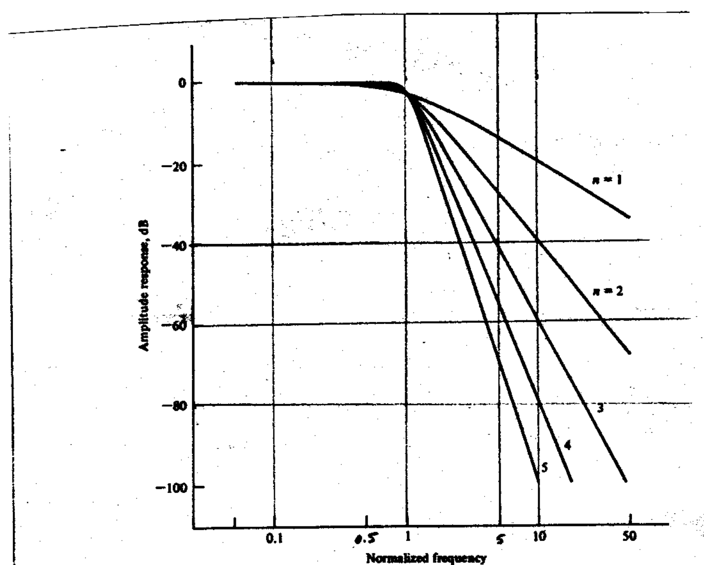


FIGURE B-4. Amplitude response of Butterworth filters.

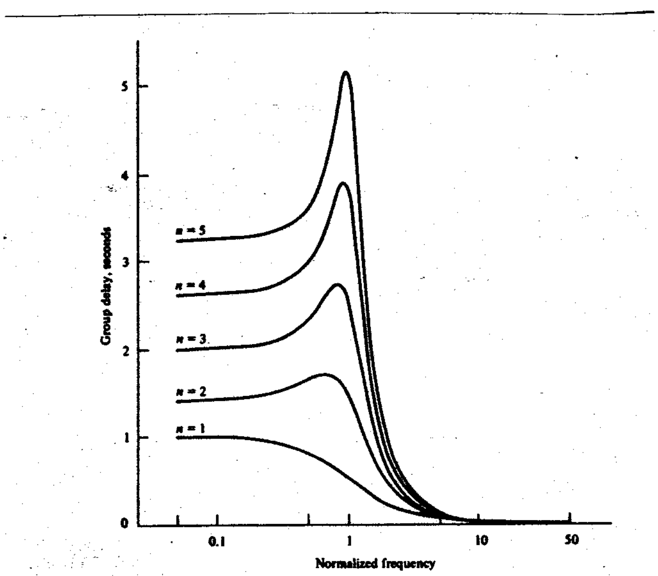
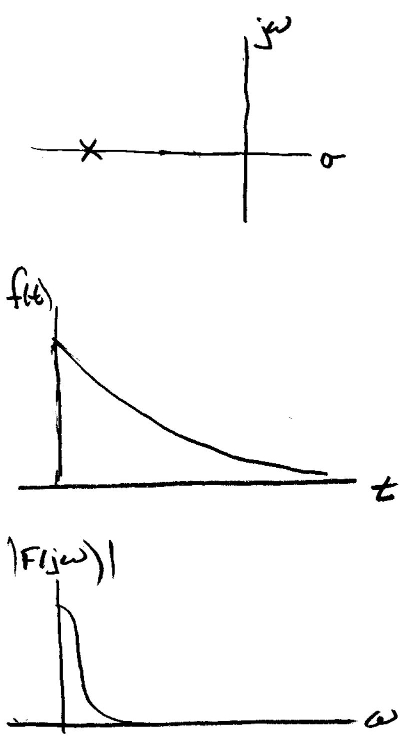
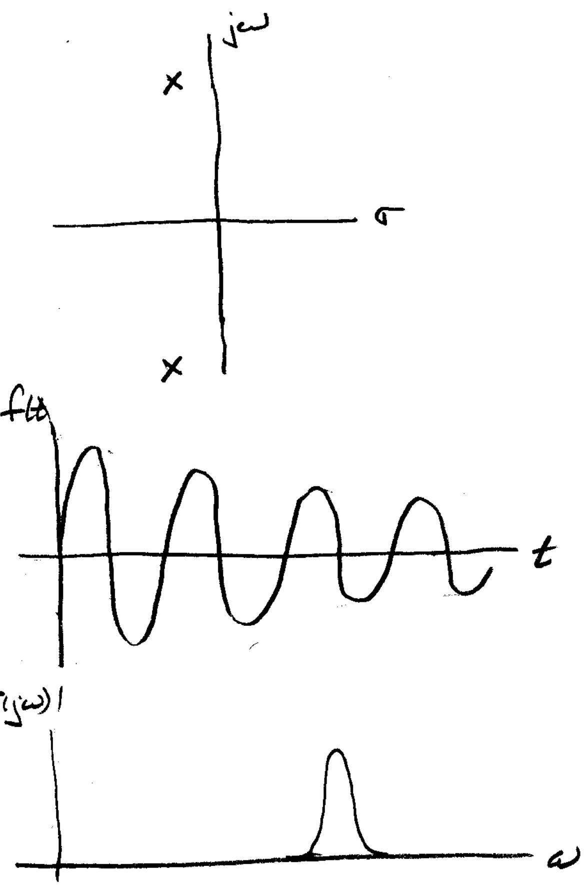


FIGURE B-5. Group delay of Butterworth filters.

- Time-frequency scaling
  - We have seen that as poles get closer to the imaginary axis, the duration of the time function increases and the bandwidth decreases.
  - Also, as poles move away from the real axis, the oscillation period of the time function decreases, but the range of frequencies increases





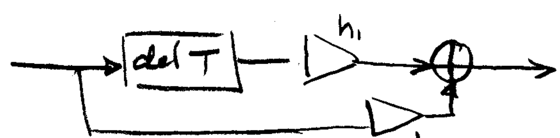
- This illustrates a general time-frequency scaling property, one that isn't tied to pole-zero diagrams and rational polynomial transforms. Define (roughly)
  - the extent
  - the fine structure size

  in each domain (see figures).

  → the extent in $\left\{\begin{matrix}\text{time}\\ \text{frequency}\end{matrix}\right\}$ is inversely proportional to the fine structure size in $\left\{\begin{matrix}\text{frequency}\\ \text{time}\end{matrix}\right\}$.

- For compact functions, the extent and fine structure size are about the same. Not true for others.

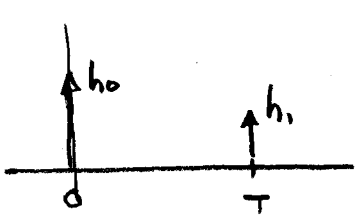
- example

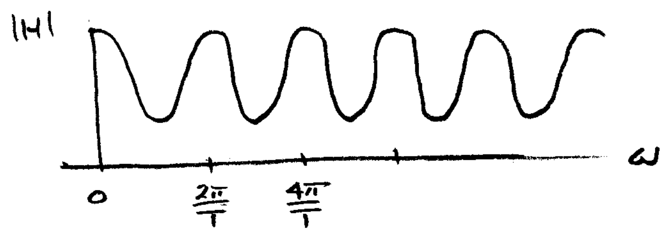


$h(t) = h_0\,\delta(t) + h_1\,\delta(t-T)$

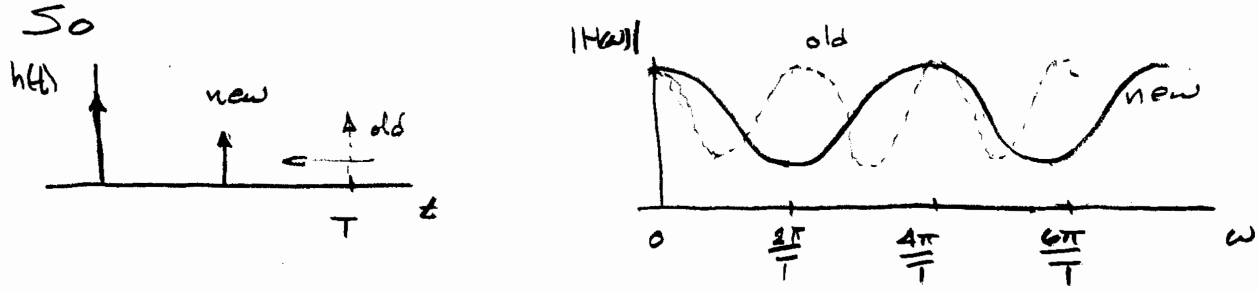
$H(s) = h_0 + h_1 e^{-sT}$ $\quad H(j\omega) = h_0 + h_1 e^{-j\omega T}$

$|H(j\omega)| = \left(h_0^2 + h_1^2 + 2h_0h_1\cos(\omega T)\right)^{1/2}$





So



and

- A corollary:

A signal with $\left\{\begin{matrix}\text{large bandwidth}\\ \text{long duration}\end{matrix}\right\}$ can resolve close spacing in $\left\{\begin{matrix}\text{time}\\ \text{frequency}\end{matrix}\right\}$.