4.5 Bode Plots of Frequency Response

DC&L, 15.8

- We have become used to sketching frequency response with linear scales for amplitude gain, phase shift and frequency.

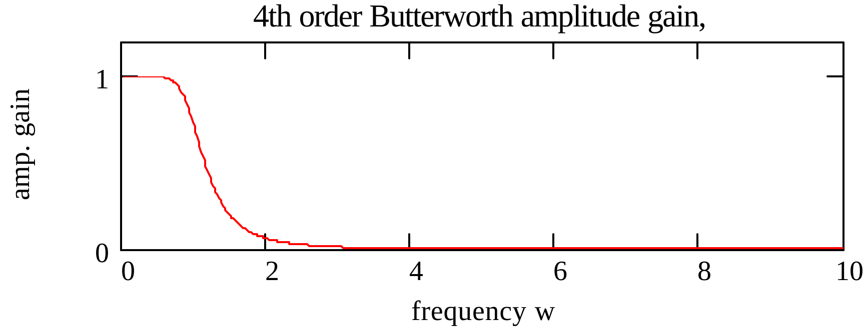
- It is common, though, to use a logarithmic measure (dB) for the gain:
  - A power ratio P_1/P_2 in dB is

    $$10 \log_{10}(P_1/P_2) \text{ dB}$$

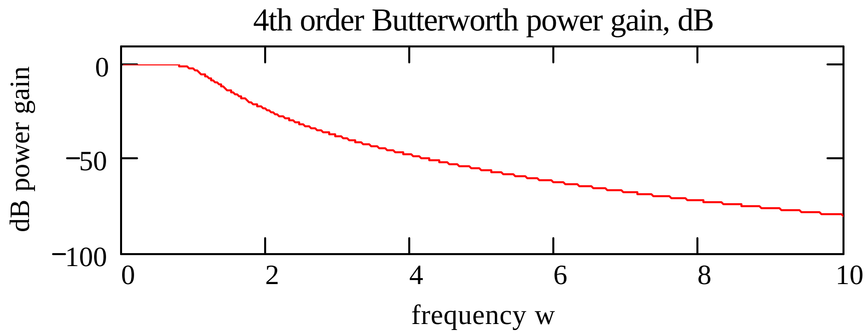
  - A filter's power gain is

    $$10 \log(|H(j\omega)|^2) \text{ dB}$$

- Why use dB?
  - Filters are often used to suppress unwanted interference or noise outside a designated passband, which admits the desired signal. We're interested in the ratio of junk power to desired power. A linear scale makes it hard to see just how much we suppress the junk.

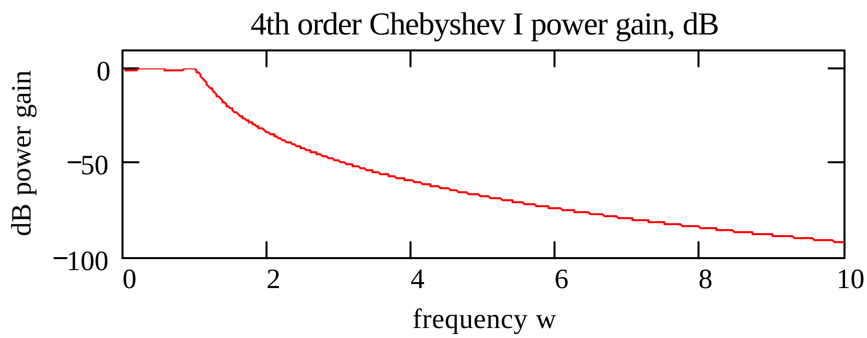


Where the gain is close to zero, it is hard to read just how close it is - and therefore what the suppression ratio is.

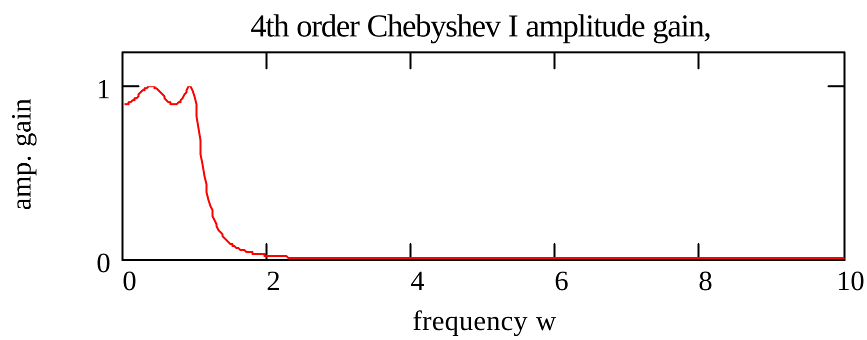


The logarithmic vertical scale expands small values of gain, so we can easily see the amount of suppression according to frequency.

On the other hand, the dB presentation over a wide frequency range tends to conceal damage to the desired signal in the passband. Have a look at a 4th order Chebyshev I design (we'll see it later).



It's hard to see the 1 dB passband ripple, since a log scale expands small amplitudes at the expense of compression of the larger ones.

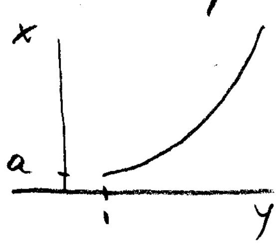


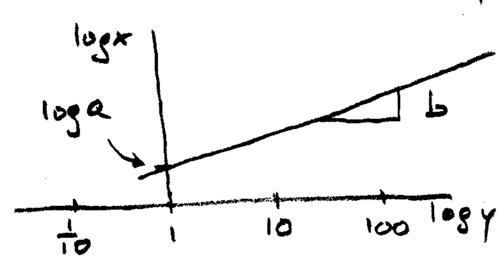
Here's a linear scale again, where we can see that there will be some distortion of the desired signal.

- In digital signal processing, it is common to present frequency response as dB vs. a <u>linear</u> frequency axis, since filters are often FIR, and the frequency range is finite $[-f_s/2, f_s/2]$, f_s = sampling frequency.

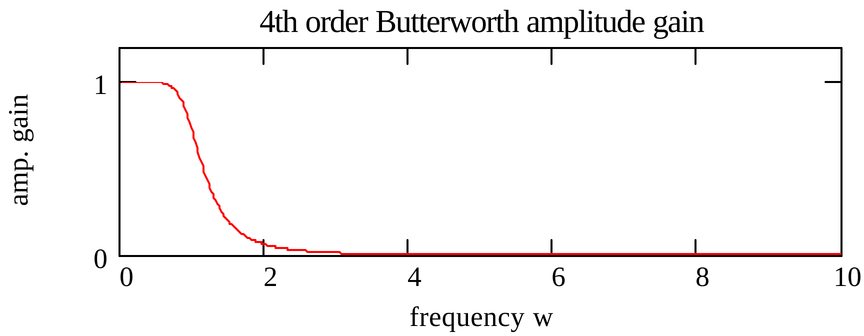
- In analog design, once we adopt dB presentation for the gain, a log frequency scale is almost irresistible. Why?
  - The frequency range is $[0, \infty)$
  - We have finite order filters with rational polynomial transfer functions. Frequency responses are asymptotic to: (e.g.) $\frac{1}{\omega}$, $\frac{1}{\omega^2}$, $\frac{1}{\omega^n}$ at high frequencies; const, ω, ω^n etc at low.
  - Algebraic variation looks linear on a log-log plot

$$x = a \cdot y^b \qquad \log x = \log a + b \log y$$



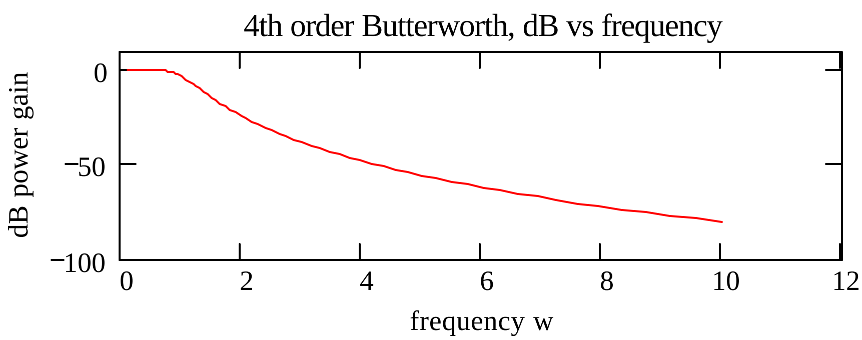


Trace it through. Here's the original linear-linear plot:



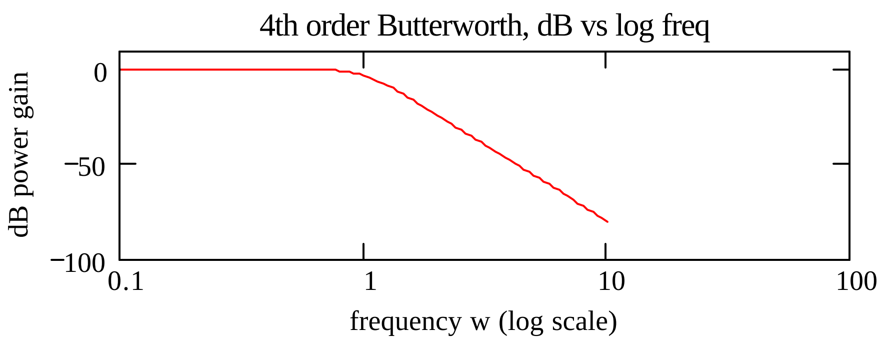
It's easy to see what's happening in the passband, but not the attenuation.

Now use dB for the vertical axis to expand the scale for the small gain values:



It's easier to see where there is a great deal of attenuation, but it's hard to summarize its behaviour at high frequencies.

Now use a log scale horizontally, too, for a log-log plot. The log scale compresses high frequencies, expands low frequencies, and doesn't show 0 rad/sec.



The asymptote is linear, so it's easy to identify high frequency bahaviour: the slope of -80 dB/decade and intercept of 0 dB means the amplitude response has asymptote $1/\omega^4$.

But **don't forget -** the log ω scale is warped, and it makes the gain drop-off look very fast. Don't be misled, and think you'll get that on a linear scale!

- These dB vs log frequency plots are called Bode plots.

- Bode plots are easy to sketch, because of the linear regimes. But why bother, if you have a computer? Because:
  - you can see how individual poles & zeros affect the response — more intuition;
  - in design, you can see which poles or zeros to move, remove or add.

In the rest of this section, you'll see how to sketch asymptotic (ie straight line) Bode plots.

- Step 1: Prepare the transfer function by factoring and normalizing.

ex: $$H(s) = \frac{120\,s}{s^2+32s+60} = \frac{120\,s}{(s+2)(s+30)}$$

$$= \frac{2\,s}{(1+s/2)(1+s/30)}$$

Note — this is the opposite of how you write it for residue inversion or pole-zero vector sketches.

- Since dB is logarithmic, the dB representation of the power gain $|H(j\omega)|^2$ is the sum (difference) of its factors. In our example,

$$|H(j\omega)|^2 = \frac{4 \cdot |j\omega|^2}{|1+j\omega/2|^2 \cdot |1+j\omega/30|^2}$$

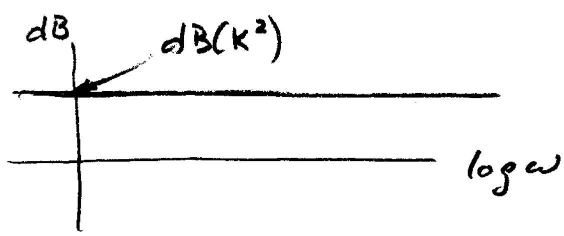
$$dB(|H(j\omega)|^2) = dB(4) + dB(|j\omega|^2) - dB(|1+j\omega/2|^2) - dB(|1+j\omega/30|^2)$$

and the phase is inherently a sum (difference):

$$\arg[H(j\omega)] = \arg[2] + \arg[j\omega] - \arg[1+j\omega/2] - \arg[1+j\omega/30]$$

- Each factor has a simple asymptotic plot.

<u>constant</u> A constant K has

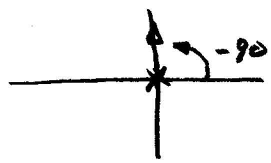


$10 \log K^2 = 20 \log K$

phase is zero

pole or zero at the origin

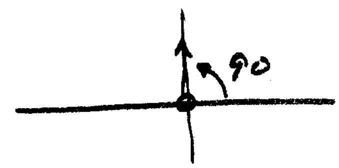
$$H(s) = \frac{\cdots}{s\,(\cdots)(\cdots)}$$



$$|H(j\omega)|^2 = \frac{1}{\omega^2}, \quad -20\ \text{dB/decade}$$

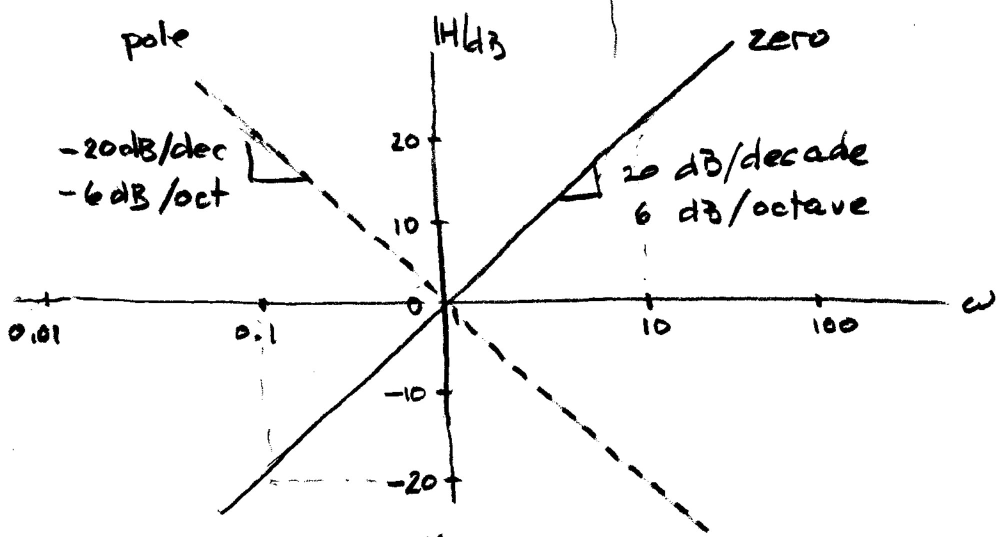
$$\measuredangle H(j\omega) = -90^\circ$$

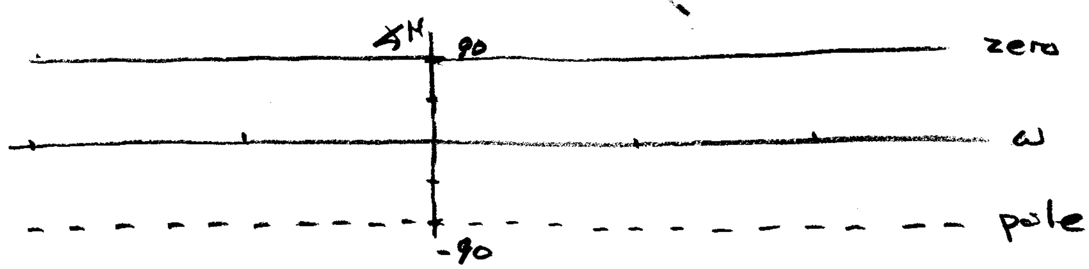
$$H(s) = \frac{\cdots s}{(\cdots)(\cdots)}$$



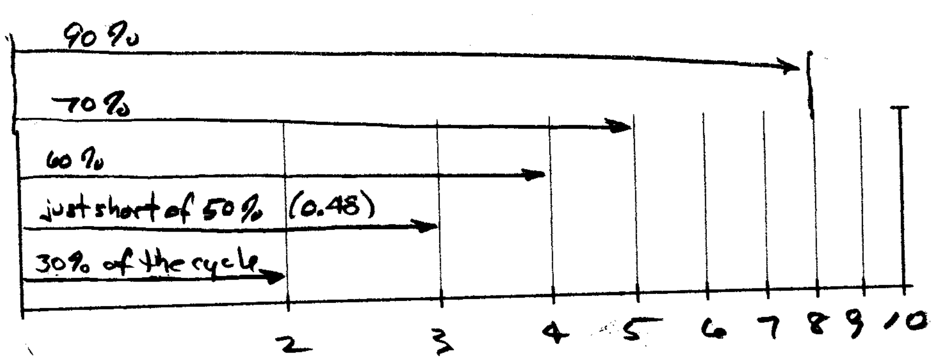
$$|H(j\omega)|^2 = \omega^2, \quad 20\ \text{dB/decade}$$

$$\measuredangle H(j\omega) = 90^\circ$$

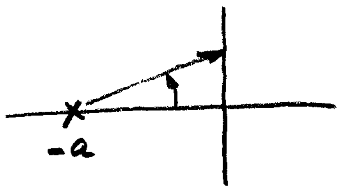




To sketch a logarithmic axis:



a real pole



$$H(s) = \frac{\cdots}{(1+s/a)(\cdots)}$$

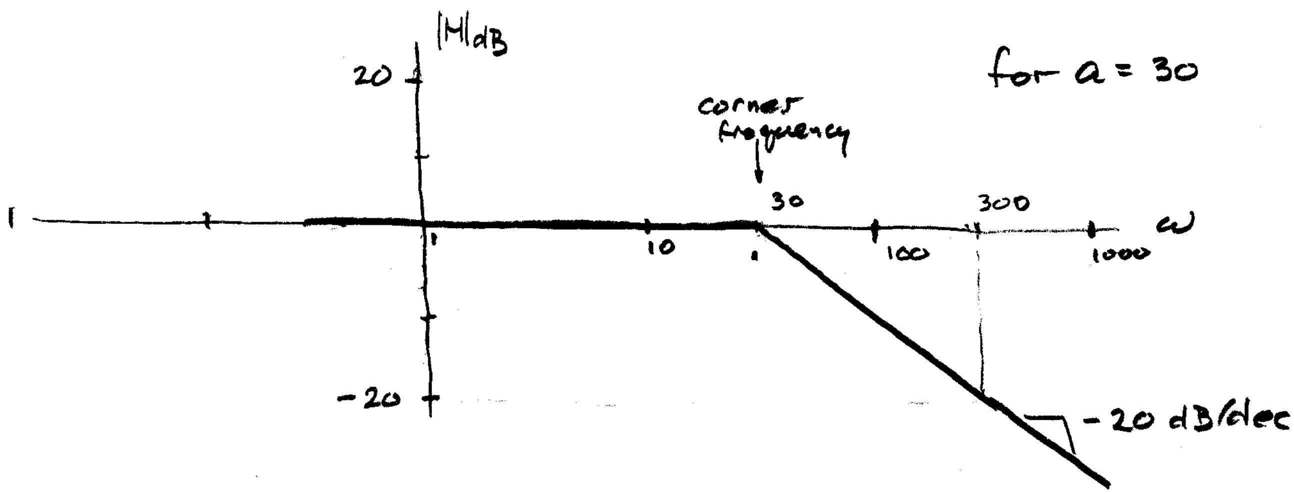
For the pole alone, $H(j\omega) = \dfrac{1}{1+j\omega/a}$

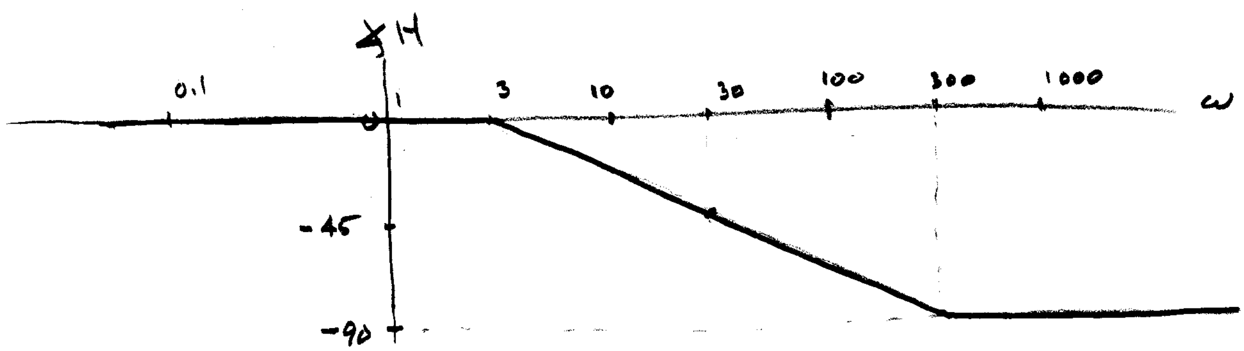
Three easy regimes: two asymptotes and a specific point

$\omega \ll a$: $H(j\omega) \approx 1$ $\quad |H|_{dB} \approx 0\,\text{dB}$ $\quad \measuredangle H \approx 0$

$\omega = a$: $H(j\omega) = \dfrac{1}{1+j}$ $\quad |H|_{dB} = -3\,\text{dB}$ $\quad \measuredangle H = -45°$

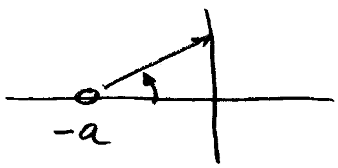
$\omega \gg a$: $H(j\omega) \approx \dfrac{1}{j\omega/a}$ $\quad |H_{dB}| \approx -20\log(\omega/a)$ $\quad \measuredangle H \approx -90°$





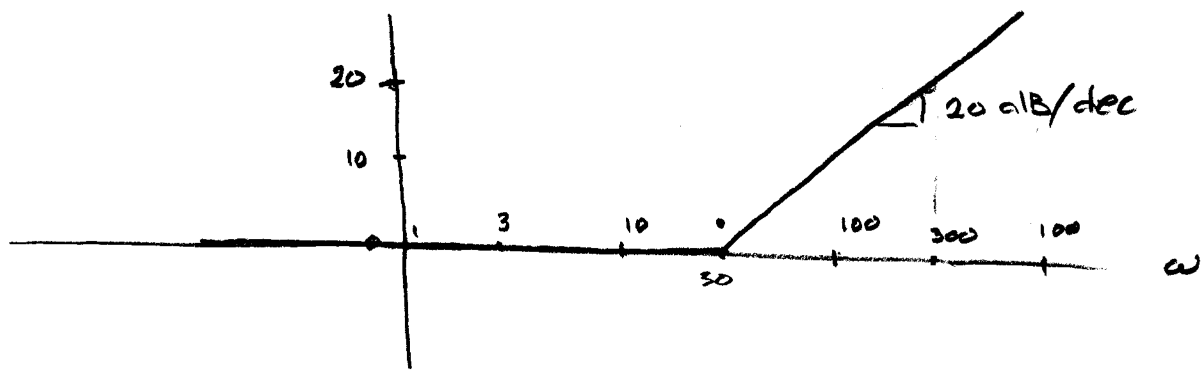
For phase approximation, go through $-45°$ at $\omega = a$ (exact) and hit $0, -90$ a decade down, a decade up, respectively (an approx).

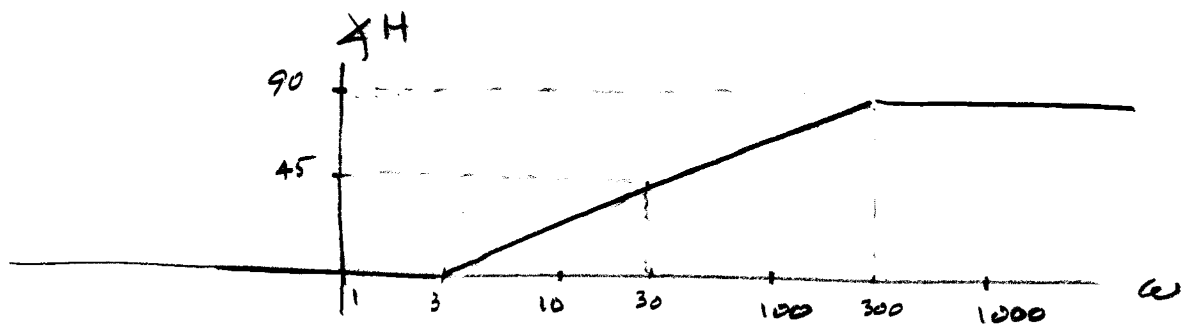
a real zero



$$H(s) = \frac{\cdots(1+s/a)}{(\cdots)(\cdots)(\cdots)}$$

Its behaviour, in dB and in phase, is the negative of that of the real pole.





Again, do the straight line approximation, and sketch the curve around it.

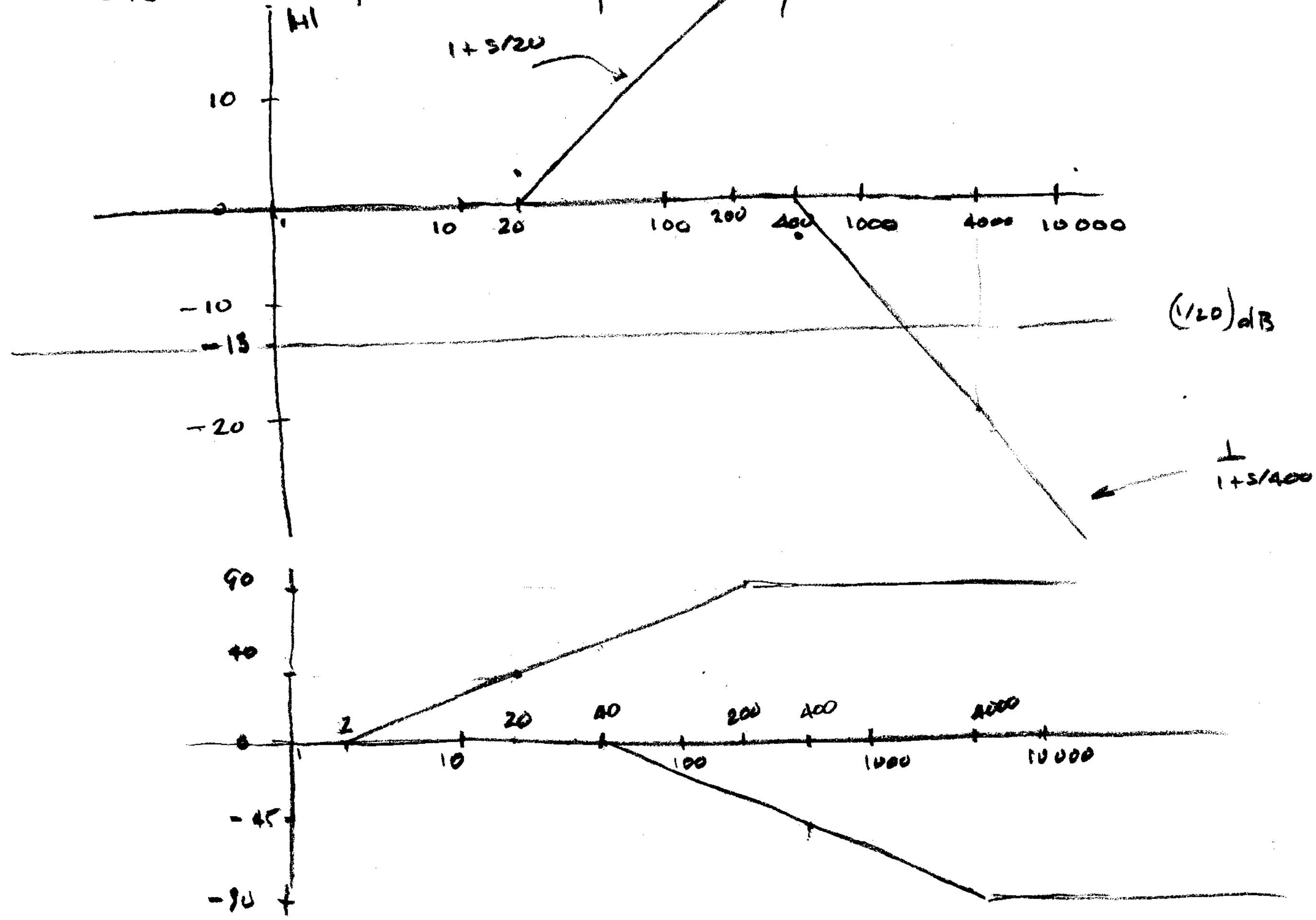
Read "Sketching Asymptotic Bode Plots" from Notes and Demos part of website

- example phase lead $H(s) = \dfrac{s+20}{s+400}$

Sketch an asymptotic (i.e straight line) Bode plot.

- Rewrite : $H(s) = \dfrac{1}{20} \dfrac{1+s/20}{1+s/400}$

- Sketch each factor separately and add them



- Bode Plots of Resonant Systems

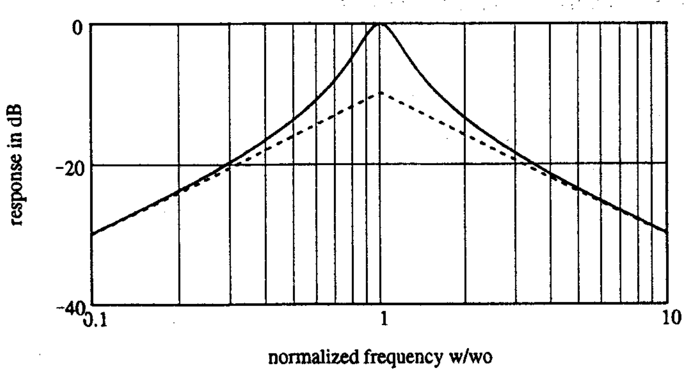
  - By manipulation, we can put it in the form $H(\omega) = \dfrac{1}{1 + jQ\left(u - \frac{1}{u}\right)}$

Check asymptotes:

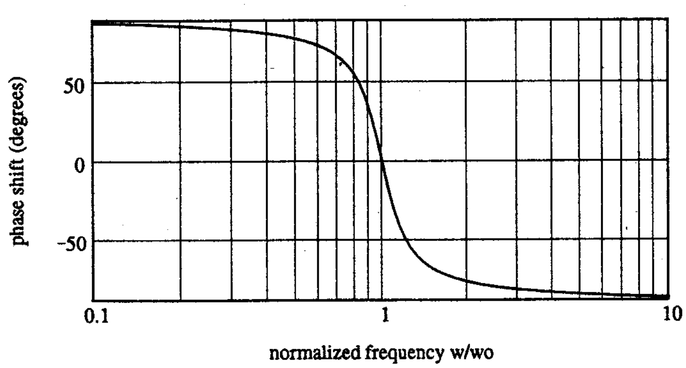
- for $\omega \to 0$ $H(\omega) \sim \dfrac{u}{jQ}$: 20 dB/dec, angle 90°

for $\omega \to \infty$ $H(\omega) \sim \dfrac{1}{jQu}$: −20 dB/dec, angle −90°

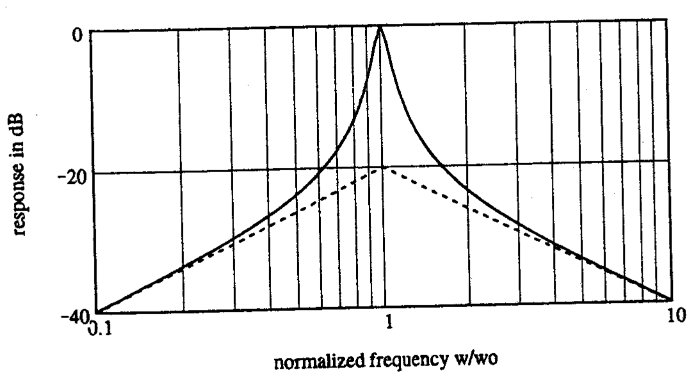
Intersection of asymptotes at $u = 1$ ($\omega = \omega_0$), where magnitude is $1/Q$. True value of $H(\omega_0) = 1$



Q = 3.16



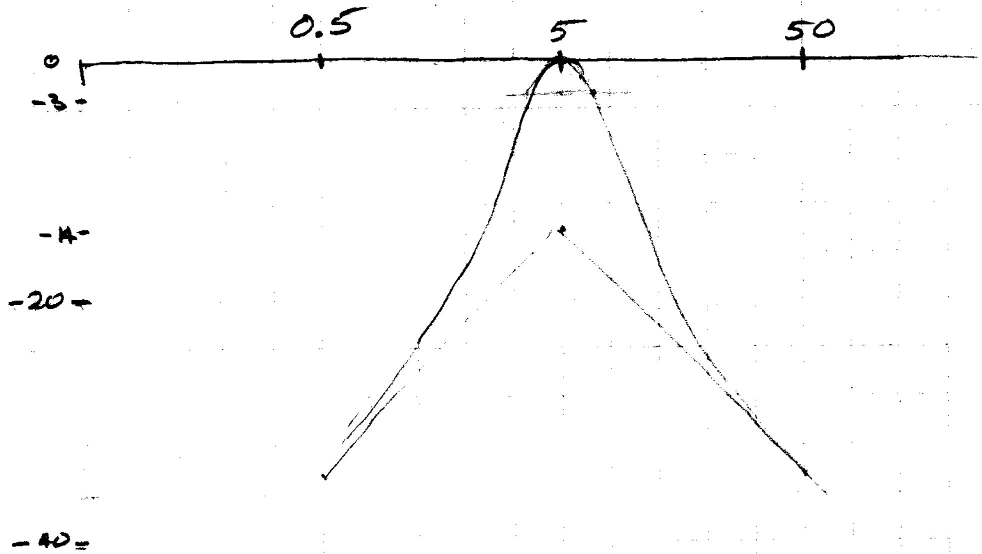
Q = 3.16



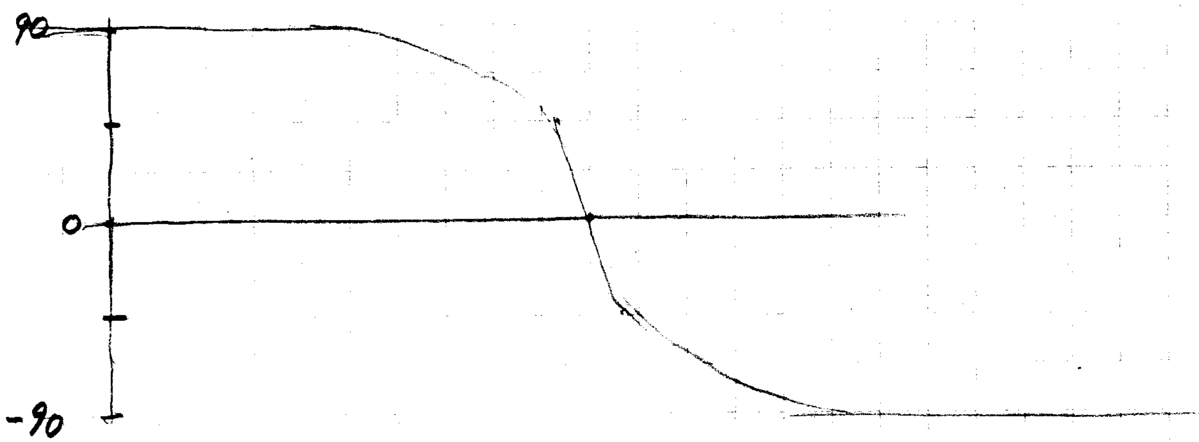
Q = 10

• <u>Example</u> $H(s) = \dfrac{s}{s^2 + s + 25}$ sketch Bode plot

$\omega_0 = 5$, $2\zeta\omega_0 = 1$, $\zeta = 0.1$, $Q = 5$, $\beta = \frac{\omega_0}{Q} = 2\zeta\omega_0 = 1$



$20 \log Q = 14\,dB$



<u>example</u> $H(s) = \dfrac{25}{s^2+s+25} = \dfrac{25}{s}\,\dfrac{s}{s^2+s+25} = 5\,\dfrac{5}{s}\,\dfrac{s}{s^2+s+25}$