Appendix 5A About Chebyshev polynomials.

- Warp the ω axis with $\theta = \cos^{-1}(\omega)$. The functions $C_n(\omega) = \cos(n\theta)$ are the Chebyshev polynomials. Why are they polynomials?
  - For $n=1$, $C_1(\omega) = \cos(\cos^{-1}(\omega)) = \omega$
  - For $n=2$, $C_2(\omega) = \cos(2\theta)$

$$= 2\cos^2(\theta) - 1$$

$$= 2C_1^2(\omega) - 1 \quad = 2\omega^2 - 1$$

Both are polynomials.

  - For higher n, trig identity

$$\cos(n\theta) = 2\cos(\theta)\cos((n-1)\theta) - \cos((n-2)\theta)$$

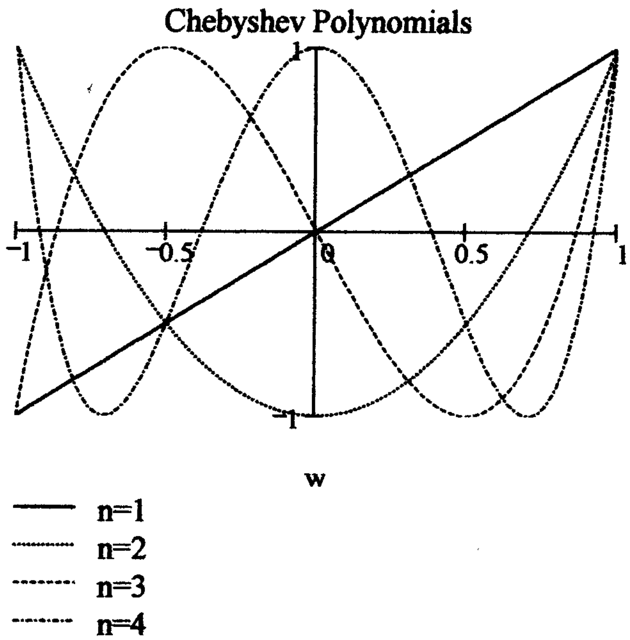
gives the recursion

$$C_n(\omega) = 2\omega C_{n-1}(\omega) - C_{n-2}(\omega)$$

which produces a polynomial of degree n.

$$C_1(\omega) = \omega$$
$$C_2(\omega) = 2\omega^2 - 1$$
$$C_3(\omega) = 4\omega^3 - 3\omega$$
$$C_4(\omega) = 8\omega^4 - 8\omega^2 + 1$$
$$\vdots$$

- Because Chebyshev polys can be expressed as a cosine, they have an equiripple property in $-1 \leq \omega \leq 1$



This property is very useful when we want to approximate a function over a given interval and minimize the maximum error — $C_n(\omega)$'s don't peak up in places; they are always bounded by $[-1, 1]$