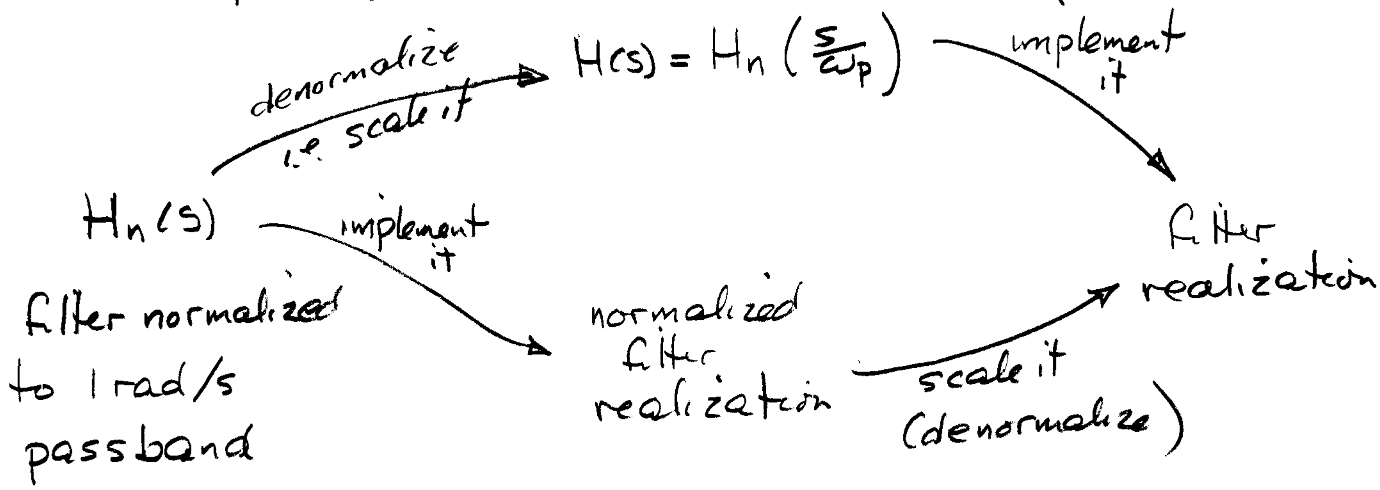
5.4 Filter Implementation With Passive Components

DC+L, 21.5

- In the previous two sections, we saw how to find Butterworth and Chebyshev I transfer functions to meet specs. Here we learn how realize them with a working filter constructed from R, L, C but no op amps.

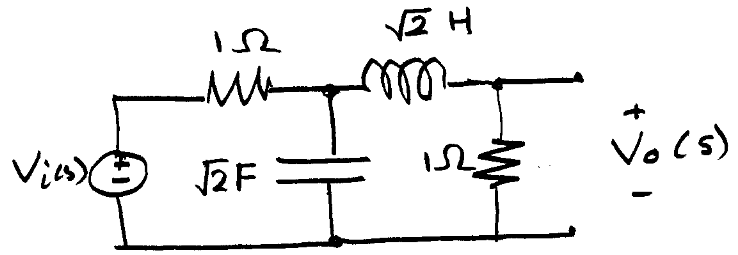
- We need two things:
  - a filter structure
  - a way to pick component values

- A couple of paths to the implementation



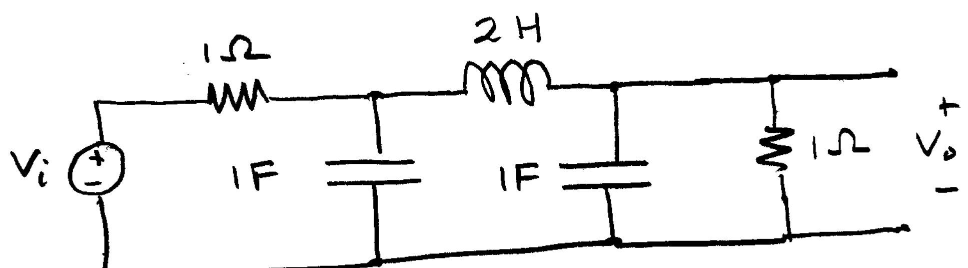
- The lower path is the usual way:
  - Just as there are tables of filter polynomials for $\omega_p = 1$ rad/s, there are compendia of filter structures for $\omega_p = 1$ rad/s.
- So we have to get a repertoire of a couple of designs and learn how to scale them.

- Two Butterworth lowpass circuits with $\omega_c = 1$ rad/s:



$$H(s) = \frac{1}{2} \; \frac{1}{s^2 + \sqrt{2}s + 1}$$

2^{nd} order



3^{rd} order $\quad H(s) = \frac{1}{2} \; \frac{1}{s^3 + 2s^2 + 2s + 1}$

(see Table, p. 5.2.2)

Check them yourself.

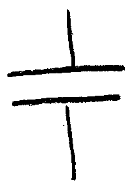
But so what? We want something other than $\omega_c = 1$ rad/s. The solution lies in scaling:

- frequency scaling
- impedance scaling

$\boxed{\text{DC + L 17.8}}$

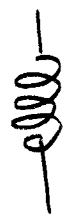
- **Frequency scaling**

  - The dynamic elements are caps and inductors:



$Z_c = \frac{1}{sC}$

$Y_c = sC$



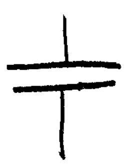
$Z_L = sL$

$Y_L = \frac{1}{sL}$

  - To scale up the frequency by a factor K_f, do what we have done before:

$$s \leftarrow \frac{s}{K_f}$$

so



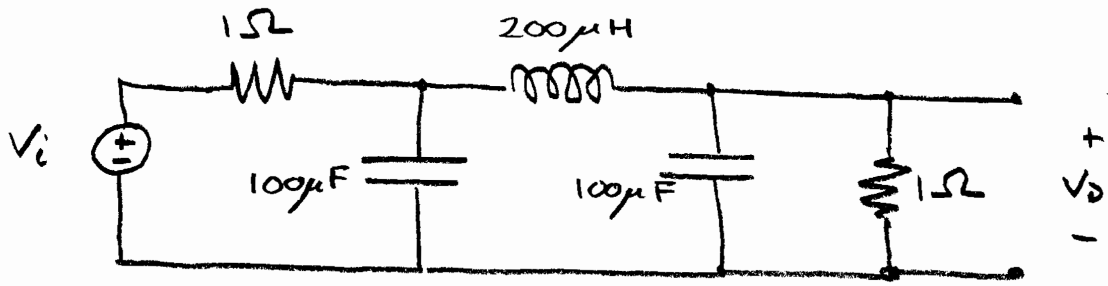
$$\frac{1}{\left(\frac{s}{K_f}\right)C} = \frac{1}{s(C/K_f)}$$

$$\left(\frac{s}{K_f}\right)L = s\left(\frac{L}{K_f}\right)$$

Divide both caps and inductors by K_f.

- example: Devise a 3rd order Butterworth lowpass with $\omega_c = 10$ krad/s $(= 1.6\text{ kHz})$.

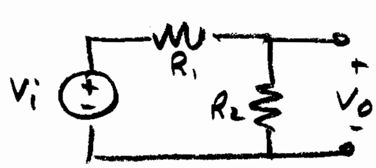
  We have $K_f = 10^4$, so modify the unit cutoff frequency prototype.



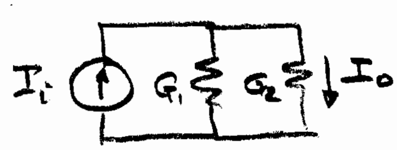
- Frequency scaling in the example gave a filter that is still unrealistic. 1Ω ??

- Impedance scaling

  – Scale up all impedances by K_i and transfer functions that relate voltage to voltage (or current to current) are unaffected.



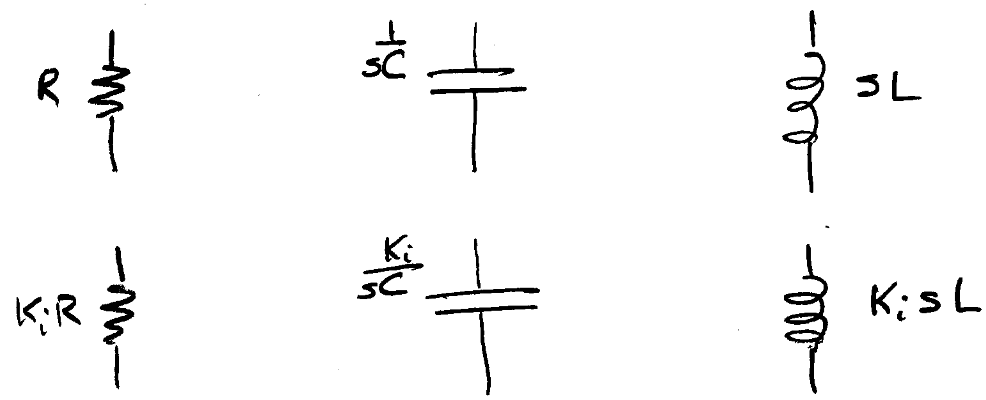
$$V_o = H V_i = \frac{R_2}{R_1+R_2} V_i$$



$$I_o = H I_i = \frac{G_2}{G_1+G_2} I_i$$

examples: The transfer functions are dimensionless

– Impedance scaling of circuit elements:

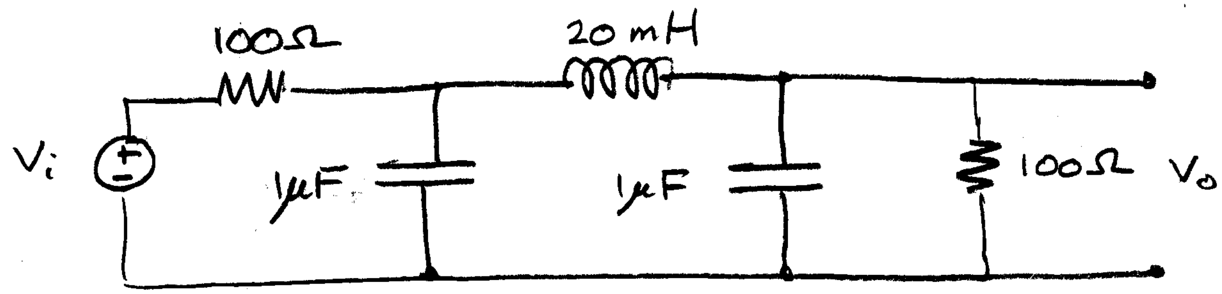


so

$R \leftarrow K_iR \qquad C \leftarrow C/K_i \qquad L \leftarrow K_iL$

- example (continued).

  We are more comfortable with 100 Ω resistors than 1Ω. Use $K_i = 100$. Note $V_o/V_i = H$ is dimensionless.



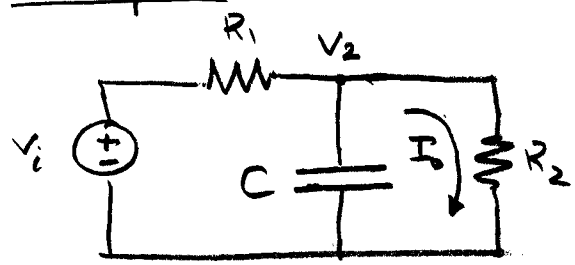
That's better! This is a usable 3rd order Butterworth.

– If the transfer function is not dimensionless, it scales with K_i:

$$V_o(s) = H(s)\, I_i(s) \quad \text{becomes} \quad V_o(s) = K_i H(s)\, I_i(s)$$

$$I_o(s) = H(s)\, V_i(s) \quad \text{becomes} \quad I_o(s) = \frac{1}{K_i} H(s)\, V_i(s)$$

<u>example</u>



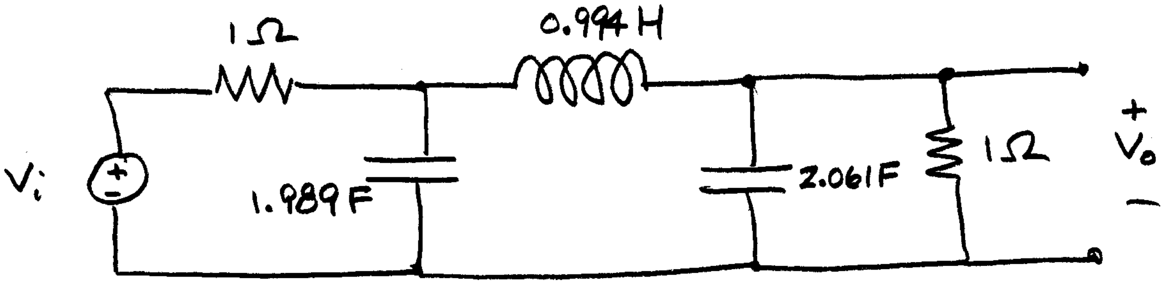
$$H(s) = \frac{I_o(s)}{V_i(s)} = \frac{V_2(s)}{R_2 V_i(s)} = \frac{1}{s R_1 R_2 C + R_2 + R_1}$$ after tidying

now impedance scale:

$$H_{new}(s) = \frac{1}{s (K_i R_1)(K_i R_2) \frac{C}{K_i} + K_i R_2 + K_i R_1}$$

$$= \frac{1}{K_i} H(s)$$ since $H(s)$ is a transfer admittance.

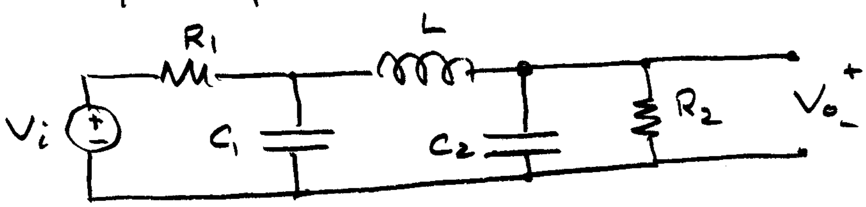
• The third order prototype circuit on p. 5.4.2 can be modified for 3rd order Chebyshev I with 1 dB ripple:



$$H(s) = \frac{1}{2}\;\frac{0.4913}{(s^2+0.4942s+0.9942)(s+0.4942)}$$

see Table p. 5.3.7

– These value were obtained from the general analysis of this circuit [DC+L, Example 21.4]



$$H(s) = \frac{\frac{1}{LC_1C_2}}{s^3 + \frac{C_1+C_2}{C_1C_2}s^2 + \frac{C_1+C_2+L}{LC_1C_2}s + \frac{2}{LC_1C_2}}$$

then equating the denominator coefficients to the expanded denominator from the Table

$$s^3 + 0.988\,s^2 + 1.238\,s + 0.491$$

and solving numerically for R_1, R_2, C_1, C_2, L.

- Final note:
  - Don't forget to include the source and load resistances in your final circuit values.
  - For example, the fully scaled circuit on p. 5.4.5