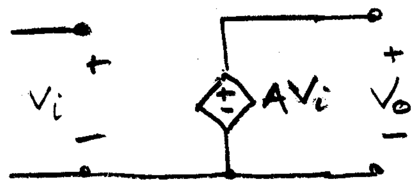
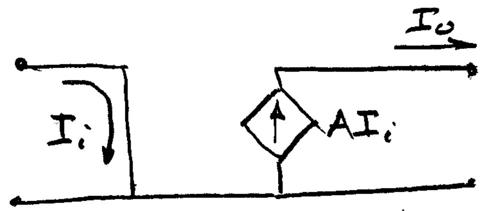
5.5 Sallen-Key and Related Active Filters

- Although passive filters are the best (or only) choice for some applications, their inductors are a nuisance for most purposes:
  - not completely linear
  - relatively expensive to make
  - bulky — very hard to put on an IC
- Active filters incorporate dependent sources — usually op amps, sometimes transistors — that allow complex poles with just R, C elements.
- The approach is similar to the way we do passive filters: select a normalized prototype of some structure, then scale it to your specs.

- Revisit impedance scaling with dependent sources.

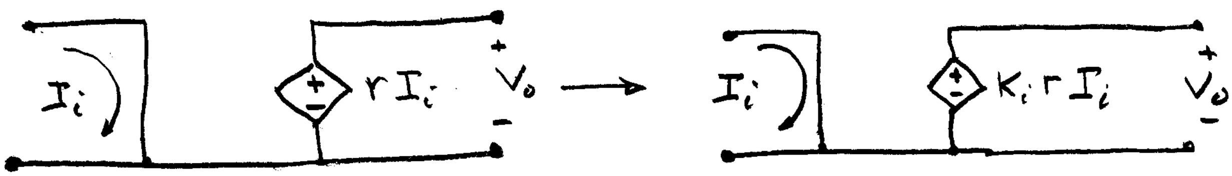


voltage gain

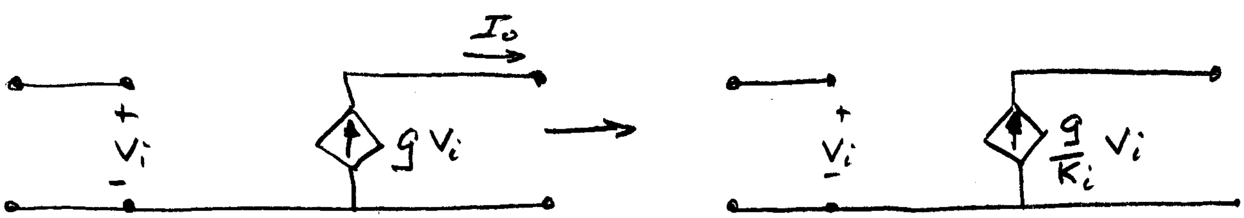


current gain

dimensionless gains are unaffected



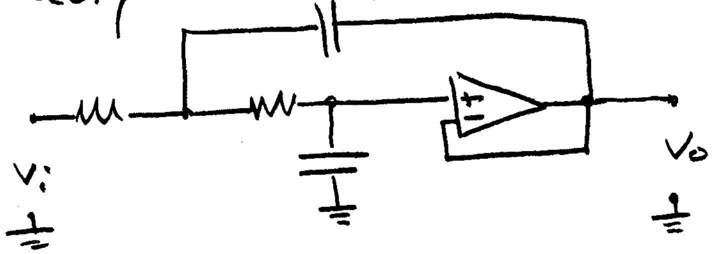
transresistance is scaled proportional to K_i



transconductance is scaled inversely proportional to K_i

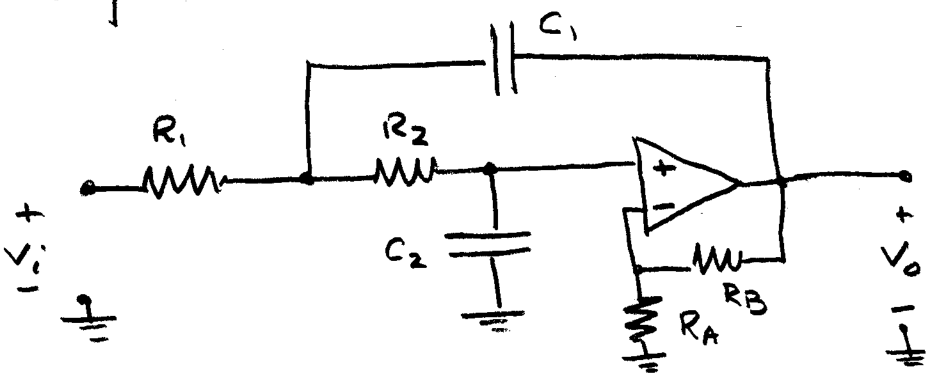
• Sallen and Key designs $\boxed{\text{DC+L 21.6}}$

– Sallen and Key designs incorporate an op amp configured as a voltage controlled voltage source (VCVS). You have seen one example already. From p. 3.1.10, a Sallen & Key lowpass



unit gain VCVS, non-inverting

– The general S+K lowpass is



→ R_A, R_B serve to set the gain K of the embedded VCVS.

→ R_1, R_2, C_1, C_2 take various values depending on K

– Its transfer function:

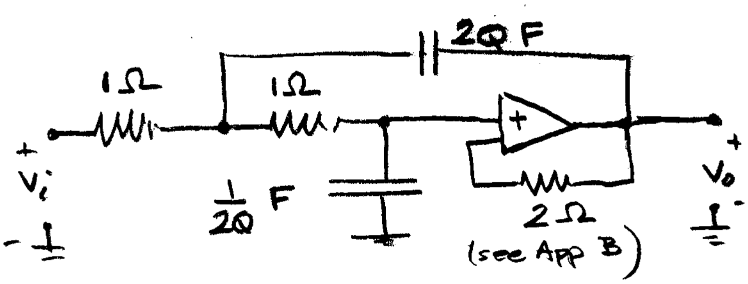
$$H(s) = \frac{\dfrac{K}{R_1 R_2 C_1 C_2}}{s^2 + \left(\dfrac{1}{R_1C_1} + \dfrac{1}{R_2C_1} + \dfrac{1-K}{R_2C_2}\right)s + \dfrac{1}{R_1R_2C_1C_2}}$$

and in normalized form

$$H(s) = \frac{K}{s^2 + \frac{1}{Q}s + 1}$$

for $\omega_0 = 1$,
$2\zeta\omega_0 = 2\zeta = \frac{1}{Q}$
Note DC gain K

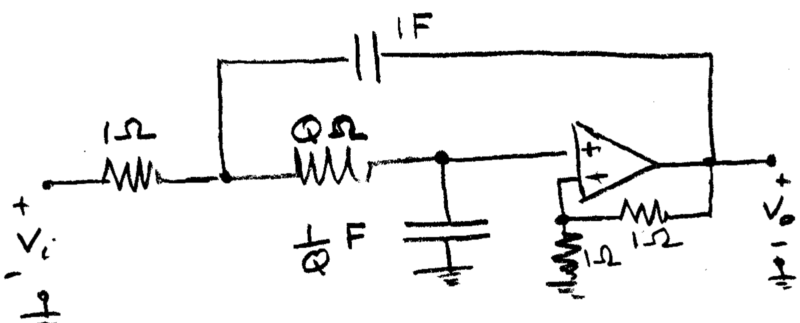
– Three common forms: $K = 1, 2, 4/3$ and match coeffs.



$K = 1$

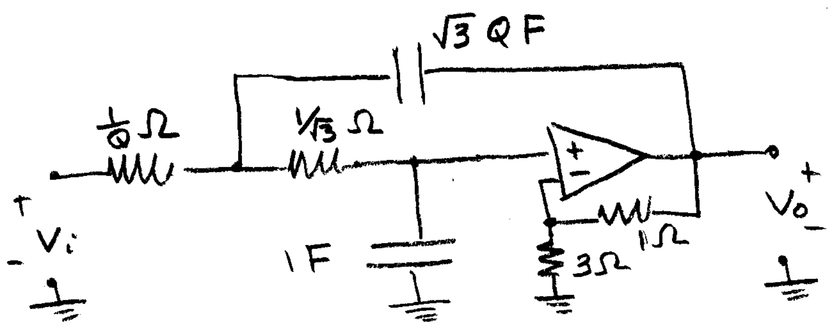
Not bad, but relative cap sizes can get large

$\frac{2Q}{1/2Q} = 4Q^2$ bias currents?



$K = 2$

component ratio is now $\frac{R_2}{R_1} = \frac{C_1}{C_2} = Q$, quite tolerable

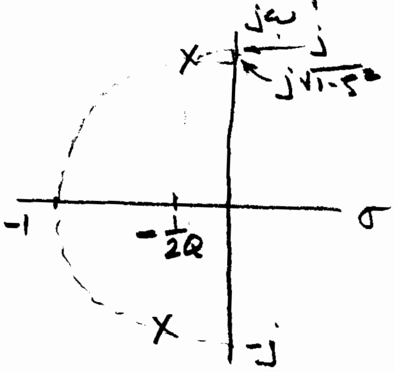


$K = 4/3$

less sensitive to component error. component ratio tolerable.

– Note $K=2$, $K=4/3$ designs should ideally choose $R_a \| R_b \approx R_1 + R_2$ respectively, to minimize bias current effects.

- These three Sallen-Key designs implement the lowpass form of a (possibly) resonant filter. Two poles, no zero



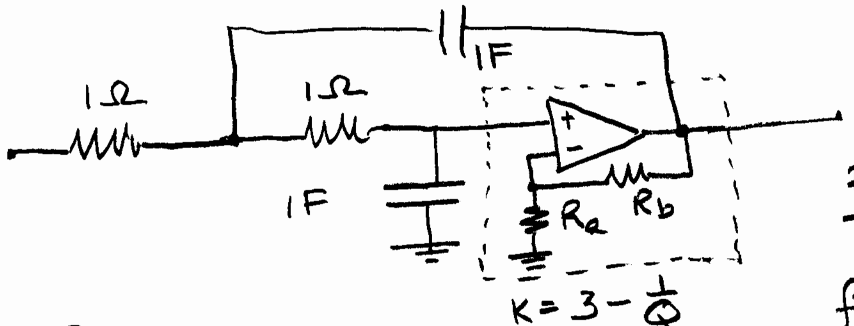
on unit circle, since normalized to $\omega_0 = 1$

$$\tau\omega_0 = \zeta = \frac{1}{2Q}$$

- These are not the only Sallen-Key designs. We have five parameters: R_1, R_2, C_1, C_2, K, and three criteria: ω_c, Q, reasonable impedances.
  - Suppose you have implemented a quad section using one of the three, but component value variation causes ω_c, Q to be off a little. Then

$$H(s) = \frac{\frac{K}{R_1R_2C_1C_2}}{s^2 + \underbrace{\left(\frac{1}{R_1C_1} + \frac{1}{R_2C_1} + \frac{1-K}{R_2C_2}\right)}_{\text{② The result of step ① gives wrong } Q\text{. Adjust } K \text{ to fix it.}}s + \underbrace{\frac{1}{R_1R_2C_1C_2}}_{\text{① adjust these to get } \omega_0}}$$

- Another canned design (but more sensitive to component variation:



$\frac{R_b}{R_a} = 2 - \frac{1}{Q}$

for $K = 3 - \frac{1}{Q}$

$R_1 = R_2 = R$

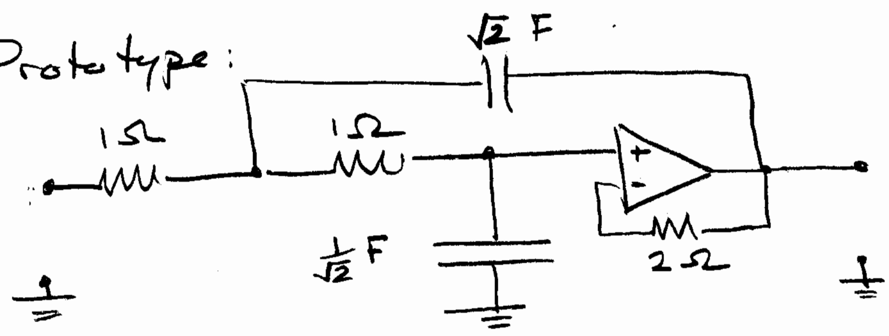
$C_1 = C_2 = C$

$K = 3 - \frac{1}{Q}$

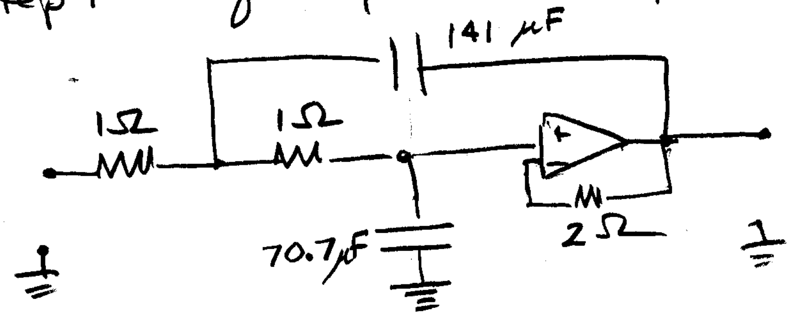
$\omega_0 = \frac{1}{RC} = 1$ for proto section.

- example.
  - Design a 2^{nd} order Butterworth active filter with cutoff frequency $\omega_c = 10$ krad/s.
  - Choose Sallen and Key structure because it's the only one we know so far. The Q factor $Q = \frac{1}{2\zeta} = \frac{1}{\sqrt{2}}$, so the $K=1$ design should work.

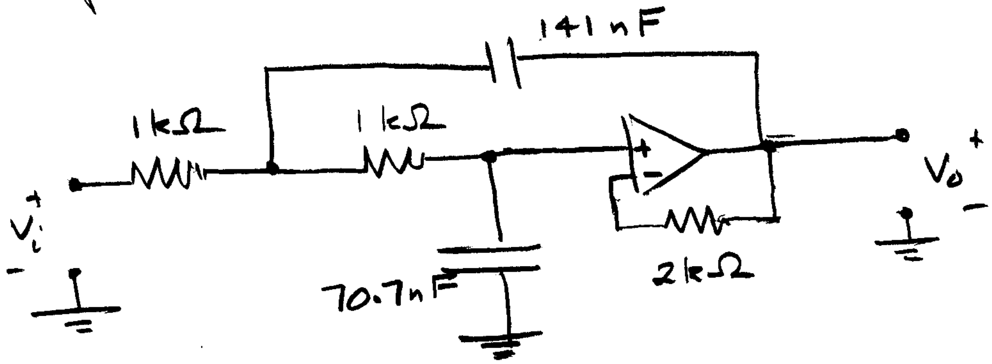
- Prototype:



– Step 1: Frequency scale $K_f = 10^4$.



– Step 2: Impedance scale (and compensate input bias current) $K_i = 1000$



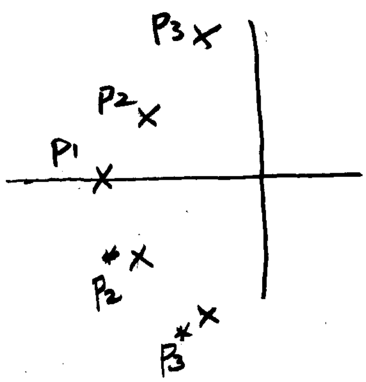
– The dc gain is $K = 1$ here. Your text shows how to change this without changing other aspects of the transfer function $\boxed{\text{DC+L 21.7}}$

• So far this has been easy. However, all we have tackled is a second order filter

- ## Higher order filters

  - How to structure a 5^{th} order filter? Trying complex arrangements of 5 capacitors, several resistors and some op amps is not attractive.

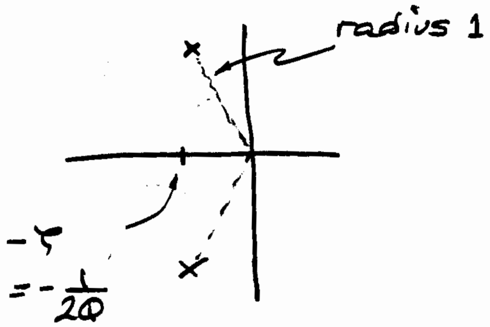
  - Solution: Structure it as a cascade of quadratic sections (plus a first order section if the order is odd).



$$\longrightarrow \boxed{\frac{K_1}{s-p_1}} \longrightarrow \boxed{\frac{K_2}{s^2-2\mathrm{Re}[p_2]s+|p_2|^2}} \longrightarrow \boxed{\frac{K_3}{s^2-2\mathrm{Re}[p_3]s+|p_3|^2}} \longrightarrow$$

This is why tables of filter coefficients are often presented as products of quadratics.

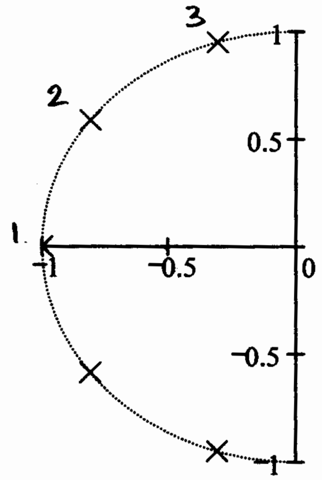
– We already know how to implement



$$H(s) = \frac{K}{s^2 + \frac{1}{Q}s + 1} = \frac{K}{s^2 + 2\zeta s + 1}$$

for unit natural frequency.

- example Start with Butterworth, since all poles have the same radius. Try for a 5^{th} order Butterworth, 3 dB bandwidth $\omega_c = 10$ krad/s.



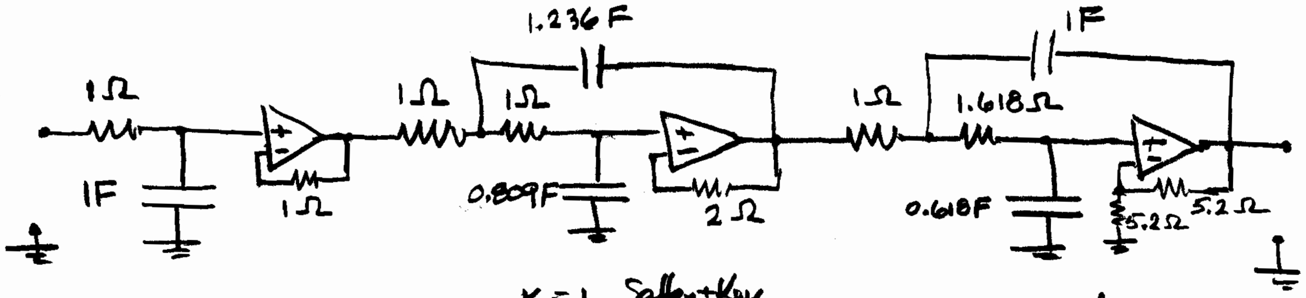
$$\begin{matrix} p_3 \\ p_2 \\ p_1 = \\ p_2^* \\ p_3^* \end{matrix} \begin{pmatrix} -0.309 + 0.951j \\ -0.809 + 0.588j \\ -1 \\ -0.809 - 0.588j \\ -0.309 - 0.951j \end{pmatrix} \longrightarrow Q = \begin{pmatrix} 1.618 \\ 0.618 \\ \\ \\ \\ \end{pmatrix} \begin{matrix} Q_3 \\ Q_2 \\ \\ \\ \\ \end{matrix}$$

Hmm. Golden mean values.

$Q_3 = \frac{1}{Q_2}$

or from Table, p. 5.2.2

– Normalized Butterworth filter:



K=1 Sallen-Key
$Q_2 = 0.618$

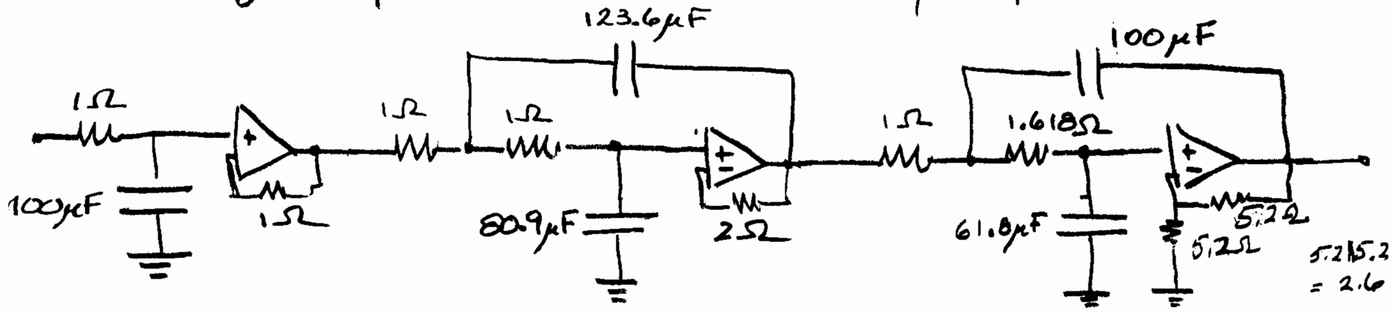
K=2 design
$Q_3 = 1.618$

p_1

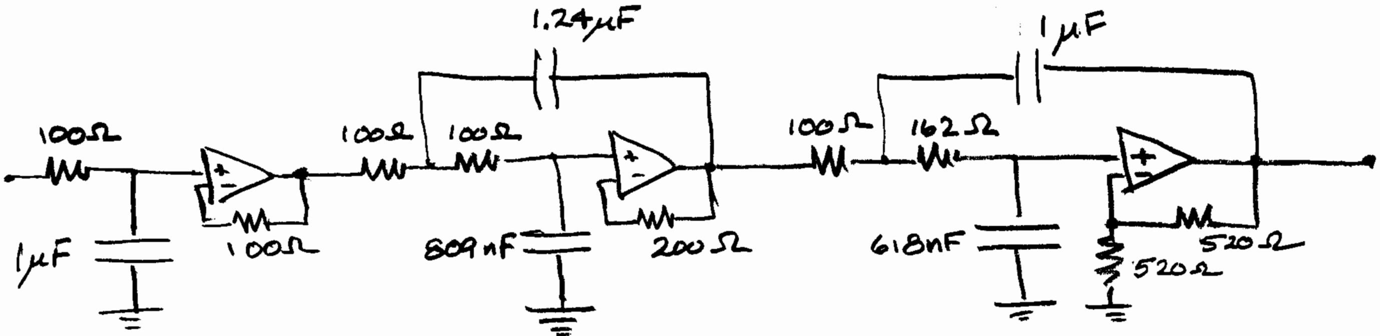
p_2, p_2^*

p_3, p_3^*

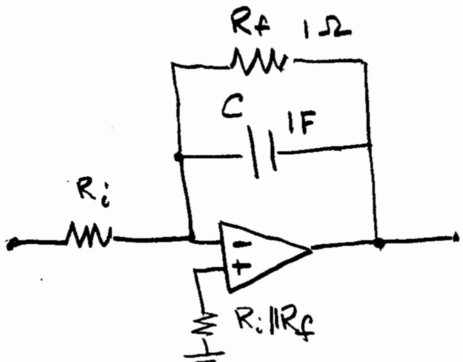
- Next, frequency scale all sections by $K_f = 10^4$.



- And impedance scale by, say, 100:



- Note an alternative design for first-order stage

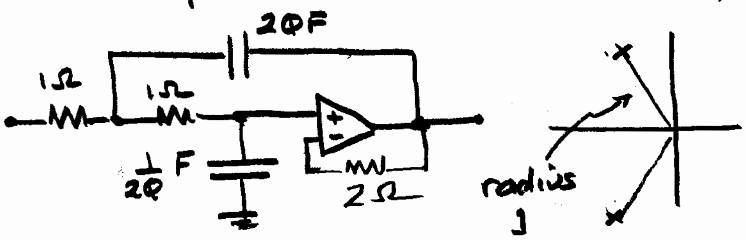


$$H(s) = -\frac{1/R_iC}{s + 1/R_fC}$$

shown for $\omega_c = 1$ rad/s

Uses one more resistor, but DC gain $H(0) = R_f/R_i$ is selectable.

- High order Chebyshev filters are a little more complicated. Here's why:
  - Our portfolio of prototype filters is set up for unit radius poles. For example,

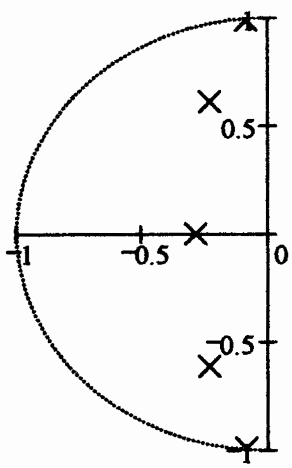


$$H(s) = \frac{K}{s^2 + \frac{1}{Q}s + 1}$$

a special case of

$$H(s) = \frac{K\omega_o^2}{s^2 + \frac{\omega_o}{Q}s + \omega_o^2}$$

  - But the Chebyshev poles don't all have the same radius.



Chebyshev I filter, 1 dB ripple, 5th order. Pole locations, Q values and radii.

$$p = \begin{pmatrix} -0.089 + 0.99j \\ -0.234 + 0.612j \\ -0.289 \\ -0.234 - 0.612j \\ -0.089 - 0.99j \end{pmatrix} \qquad Q = \begin{pmatrix} 5.589 \\ 1.4 \\ \text{[illegible]} \\ \text{[illegible]} \\ \text{[illegible]} \end{pmatrix} \qquad \omega_0 = \begin{pmatrix} 0.994 \\ 0.655 \\ 0.289 \\ 0.655 \\ 0.994 \end{pmatrix}$$

Even though overall filter $\omega_p = 1$, individual sections have their own values for radius ω_o

and this means a different frequency scale ~~factor for~~ each pole when we implement

$$H(s) = \frac{0.123}{(s+0.289)(s^2+0.468s+0.429)(s^2+0.179s+0.988)}$$

5^{th} order, 1dB ripple

- Here's how to handle implementation from the H(s) polynomials without confusion:

  - For each quadratic factor
    $$s^2 + \frac{\omega_0}{Q}s + \omega_0^2,$$
    calculate the Q. This number does not change with frequency scaling, which changes only the radius.

    eg $s^2 + 0.468\,s + 0.429$
    $$Q = \frac{\sqrt{0.429}}{0.468} = 1.4$$

  - For each denominator factor, linear or quadratic, calculate the pole radius, for use in frequency scaling.

    eg $s + 0.289$ radius $\omega_0 = 0.289$

    eg $s^2 + 0.468\,s + 0.429$
    $$\omega_0 = \sqrt{0.429} = 0.655$$

- We <u>could</u> design it like this
  - → sketch each unit-radius prototype filter with its own Q.
  - → frequency scale each prototype section down to its correct radius ω_0; the cascade of sections is now a Cheby filter for a unit passband
  - → frequency scale all sections by ω_p, the desired passband frequency.

<u>but</u> it is a lot easier to do one stage of scaling:

  - → sketch each unit-radius prototype filter with its own Q
  - → frequency scale each section by $\omega_{0k}\omega_p$, $k = 1 \ldots$ # of sections

- Each stage has dc gain equal to the gain of the embedded VCVS (i.e., 1, 2 or 4/3). The correct normalized gain of the cascade (if this is important) is

$$\begin{cases} 1, & n \text{ odd} \\ \dfrac{1}{\sqrt{1+\epsilon^2}}, & n \text{ even} \end{cases}$$

- example. Design a 5^{th} order Chebyshev I filter with 1dB passband ripple and a bandwidth $\omega_p = 10$ krad/s.

  - From the table p. 5.3.7

$$H(s) = \frac{0.123}{\underbrace{(s+0.289)}_{\text{stage 1}}\underbrace{(s^2+0.468s+0.429)}_{\text{stage 2}}\underbrace{(s^2+0.179s+0.988)}_{\text{stage 3}}}$$

Note dc gain $H(0) = 1$.

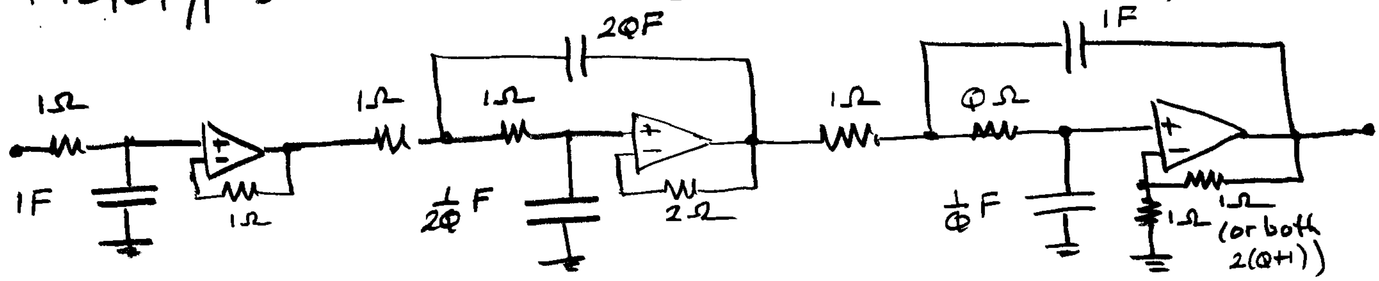
  - Find ω_o, Q of each stage:

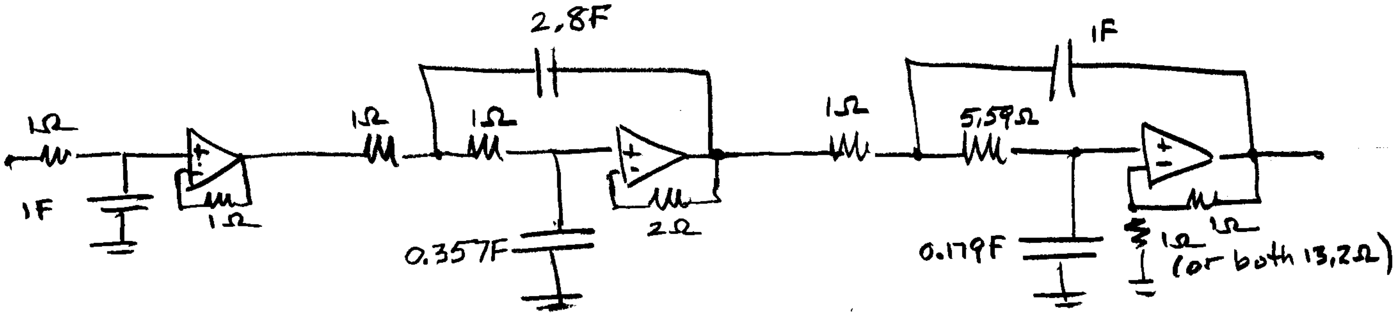
	stage 1	stage 2	stage 3
ω_o	0.289	$\sqrt{0.429} = 0.655$	$\sqrt{0.988} = 0.994$
Q		$\frac{\omega_o}{0.468} = 1.4$	$\frac{\omega_o}{0.179} = 5.59$

  - Frequency scale factors $K_{f_k} = \omega_{oK}\,\omega_p$:

	stage 1	stage 2	stage 3
K_f	2,890.	6,550	9,940

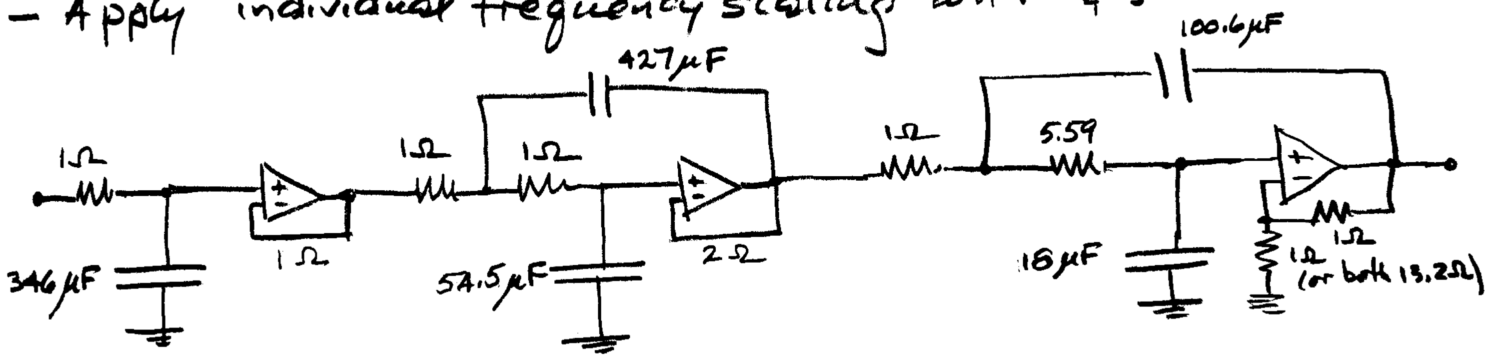
– Prototypes with correct Q's but incorrect K_f's, K_i :





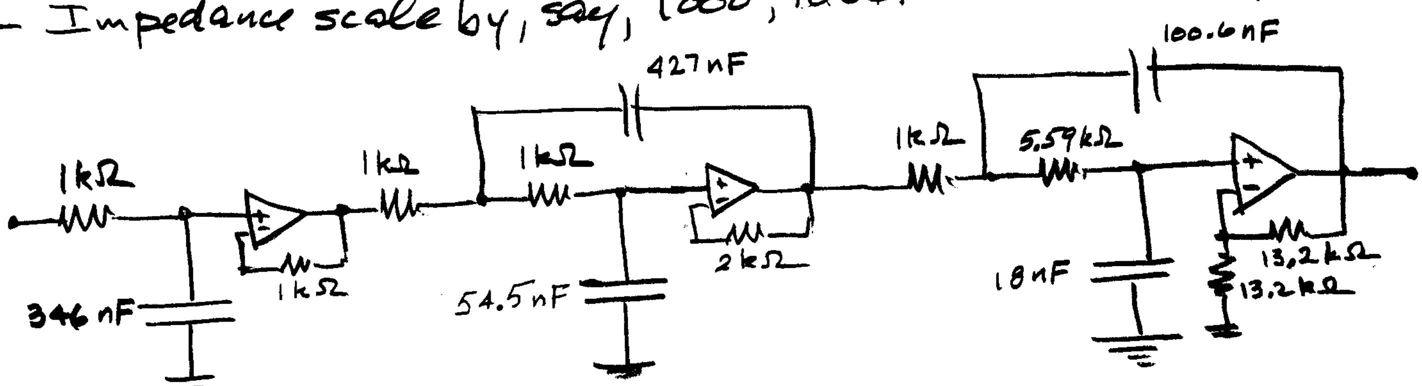
Note - this is not yet a Cheby filter

– Apply individual frequency scaling with K_f's



This is Chebyshev now.

– Impedance scale by, say, 1000, to obtain the final filter:



Done. DC gain is 2, contributed by 3rd stage.
Attenuate if you don't like it.