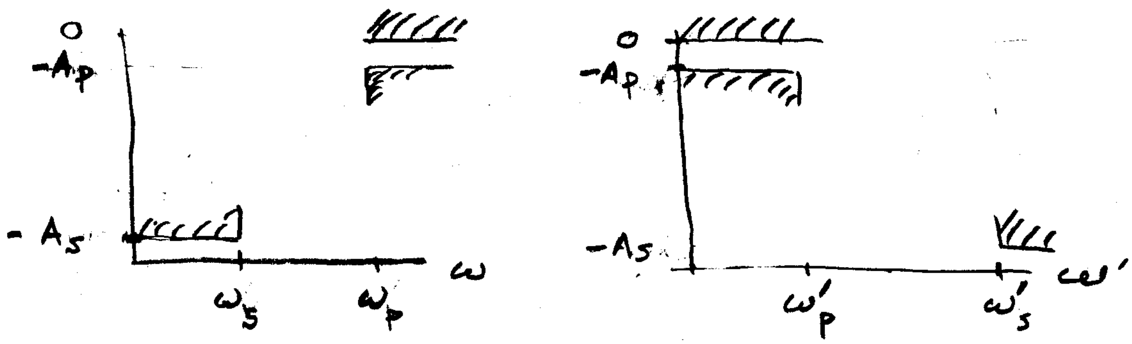
5.7 Lowpass to Highpass/Bandpass Transformation

- To this point, we have focused almost exclusively on lowpass design. What about highpass, bandpass? Fortunately, they can be obtained by transformation from lowpass designs.

- We'll do a little of it here, but read a book on active filter design if you need more.

- <u>From Lowpass to Highpass</u>

  – Compare specs



  – An inverse transformation

$$\omega \propto \frac{1}{\omega'}$$

interchanges passbands, stop bands.

The proportionality constant is a choice, but normally it is done by passband edges:

$$\frac{\omega'}{\omega_p'} = \frac{\omega_p}{\omega} \qquad \omega' < \omega_p' \longleftrightarrow \omega > \omega_p$$

so $\quad \omega' = \omega_p' \dfrac{\omega_p}{\omega}$

This makes the lowpass stopband edge corresponding to highpass edge ω_s equal to

$$\omega_s' = \omega_p' \frac{\omega_p}{\omega_s}.$$

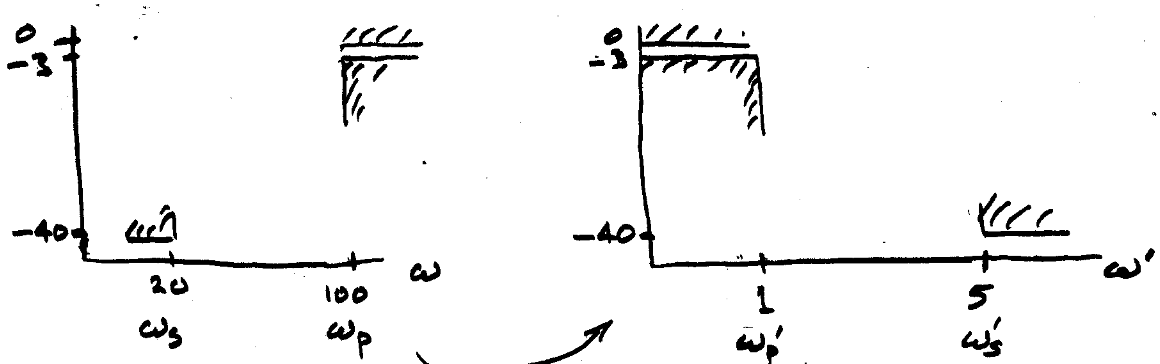
Usually we like normalized lowpass design, so $\omega_p' = 1$ rad/s, and

$$s' = \frac{\omega_p}{s} \qquad \omega_s' = \frac{\omega_p}{\omega_s}.$$

<u>example</u> Highpass spec:

stopband $\quad A_s = 40\text{ dB} \quad \omega_s = 20$ rad/s

passband $\quad A_p = 3\text{ dB} \quad \omega_p = 100$ rad/s.



$$\omega' = \frac{100}{\omega}$$

$$s' = \frac{100}{s}$$

— Design a lowpass to the equivalent lowpass specs in s'. This is $H'(s')$.

<u>example</u>: 3^{rd} order Butterworth meets lowpass specs on previous page. For $\omega_p' = 1$ rad/s, p 5.2.10 gives

$$H'(s') = \frac{1}{s'^3 + 2s'^2 + 2s' + 1}$$

— Substitute $s' = \frac{100}{s}$ to obtain the desired hipass

<u>ex</u> $$H(s) = H'\left(\frac{100}{s}\right) = \frac{s^3}{s^3 + 200s^2 + 20{,}000s + 1{,}000{,}000}$$

— Hmm — we picked up some zeros. Why?

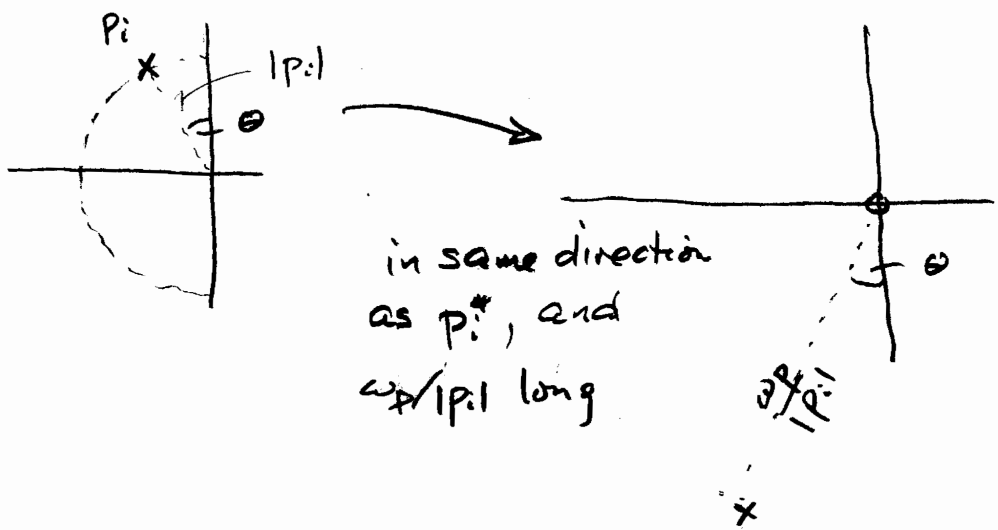
Pole movement $\boxed{\text{DC+L 21.9}}$

A typical pole

$$\frac{1}{s'-p_i'} \rightarrow \frac{1}{\frac{\omega_p}{s} - p_i'} = \frac{-s/p_i'}{s - \frac{\omega_p}{p_i'}}$$

← fresh zero

← new pole location



- <u>Implementation</u>
  - As usual, it is easier to transform an existing normalized prototype.
  - For passive circuits, the transformation $s' = \omega_p / s$ does this to caps and inductors

$\boxed{\text{DC+L } 21.8}$

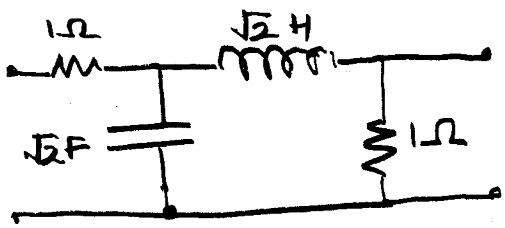
<u>lowpass proto</u> | <u>hipass</u>

$s'L \longrightarrow \frac{\omega_p}{s} L \qquad C = \frac{1}{\omega_p L}$

$\frac{1}{s'C} \longrightarrow \frac{s}{\omega_p C} \qquad L = \frac{1}{\omega_p C}$

and it accomplishes frequency scaling on the way.

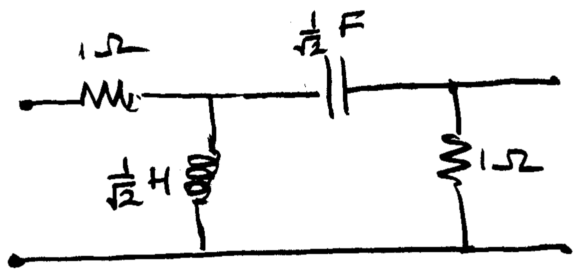
<u>lowpass (p. 5.4.2)</u>



Butterworth values, normalized for $\omega_p' = 1$

$\xrightarrow[\omega_p' = 1]{s' = \frac{1}{s}}$

<u>highpass proto</u>



Butterworth, normalized for $\omega_p = 1$

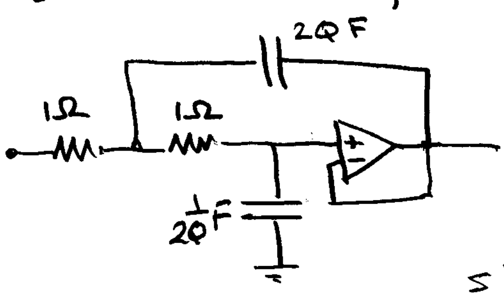
Then frequency scale, impedance scale as usual.

- For active filters, state without proof that cap/resistor interchange performs the transformation:

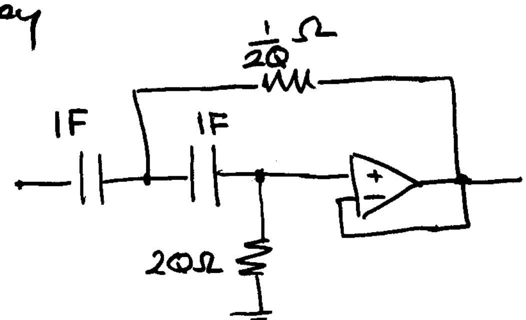
lowpass	hipass
C	$R = \frac{1}{\omega_p' C}$
R	$C = \frac{1}{\omega_p' R}$

and leave VCVS alone

- In particular, $\omega_p' = 1$ transforms lowpass protos to highpass protos. Example: 2nd order section, Sallen-Key



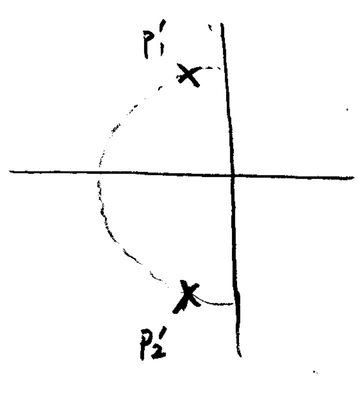
→

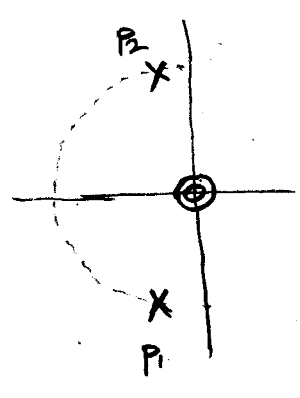


$s' = \frac{1}{s}$

$\omega_p = 1$

$$H'(s') = \frac{1}{s'^2 + \frac{1}{Q}s' + 1} \qquad H(s) = \frac{s^2}{s^2 + \frac{1}{Q}s + 1}$$





unit radius, so poles stay on the circle, but interchange (p 5.7.3)

– Chebyshev filters require some care, because pole radii differ. To design the highpass

→ transform specs to normalized lowpass, s'

→ select Chebyshev lowpass filter $H'(s)$

→ design filter, section k with caps frequency scaled by ω_{0k}, C/ω_{0k}

→ convert to highpass using $\omega_p' = 1$ rad/s, so

$$C\text{'s} \longrightarrow R = \frac{1}{C}$$

$$R\text{'s} \longrightarrow C = \frac{1}{R}$$

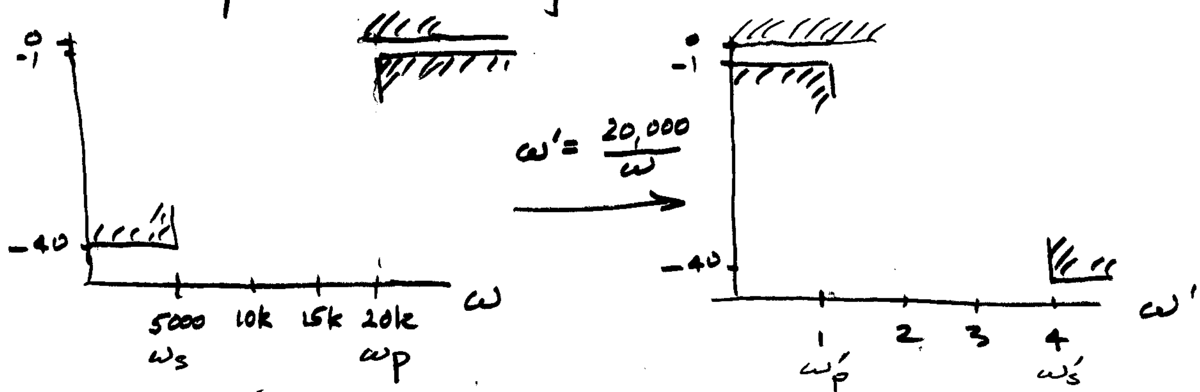
→ frequency scale to original spec. with K_f

Or combine the last three steps. For each section, a Sallen Key prototype with the right Q, but unit natural frequency is transformed by

unit proto	Cheby LP	Cheby HP (norm)	Cheby HP	Final
C	$\frac{C}{\omega_{0k}}$	$\frac{\omega_{0k}}{C}$	$\frac{\omega_{0k}}{C}$	$\frac{\omega_{0k} K_i}{C}$
R	R	$\frac{1}{R}$	$\frac{1}{R K_f}$	$\frac{1}{R K_f K_i}$

– *example* Design a Chebyshev highpass filter with at least 40 dB attenuation in the stopband ($\omega_s = 5$ krad/s) and 1 dB ripple in the passband ($\omega_p = 20$ krad/s).

→ start by transforming to normalized LP:

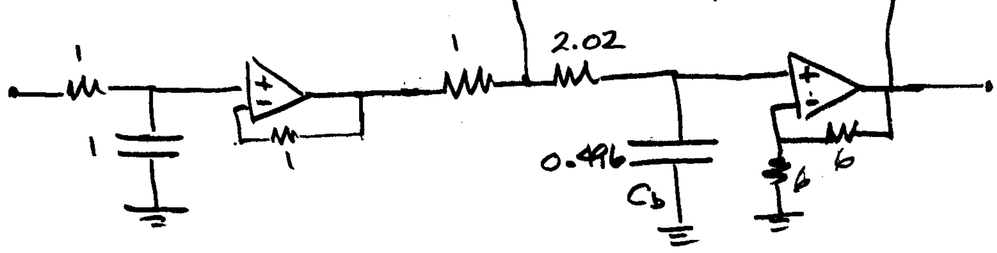


→ Comparison with example on p. 5.3.8 shows that a 3rd order Chebyshev will meet the spec. Here is the normalized LP transfer function (p 5.3.7)

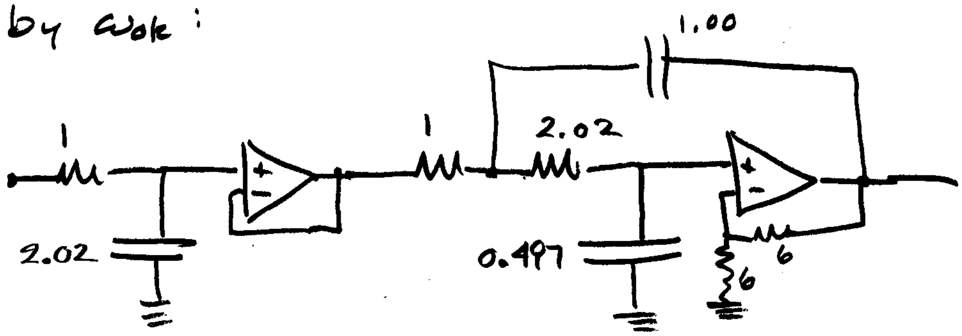
$$H'(s') = \frac{0.4913}{(s'+0.494)(s'^2+0.4942s'+0.9942)}$$

	section 1	section 2
ω_0	0.494	0.997
Q		2.02

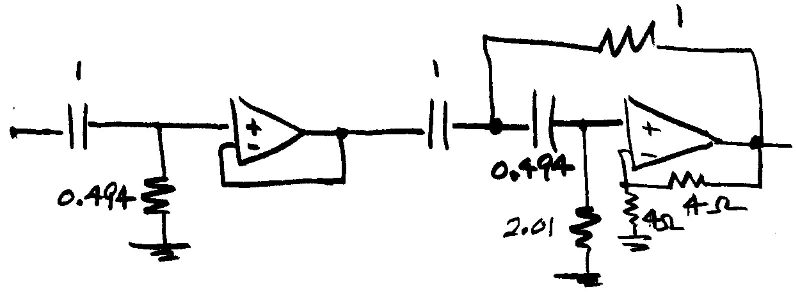
→ Non-Cheby normalized lowpass (p 5.5.4)



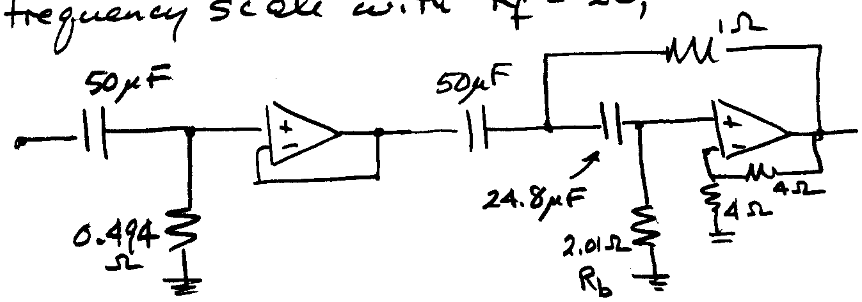
→ make it normalized ($\omega_p = 1$) Cheby lowpass, scale C's by ω_{0k}:



→ make it normalized ($\omega_p' = 1$) Cheby highpass.



→ frequency scale with $K_f = 20{,}000$



then impedance scale.

→ to do the transformation of circuit, bottom of p 5.7.7, use the one step method on p. 5.7.6. For example,

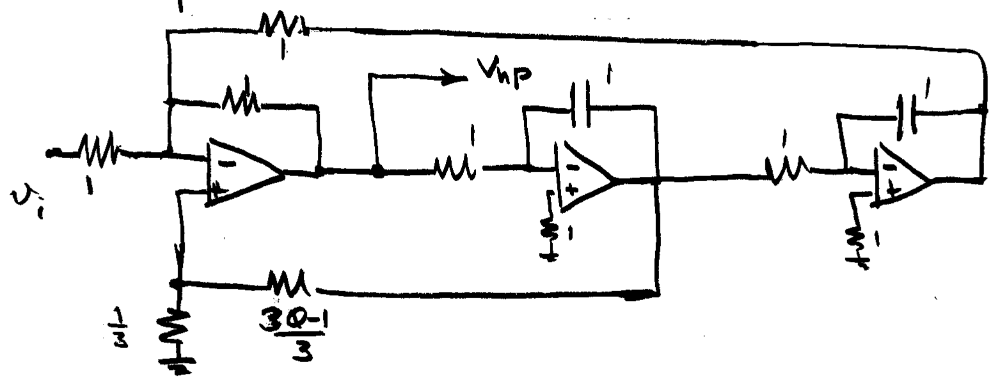
C_b on p 5.7.7 becomes $R_b = \dfrac{\omega_{02}}{C_b} = \dfrac{\omega_{02}}{1/Q_2} = 0.997 \cdot 2.02$

$= 2.01\ \Omega$

– With state variable filters, it's much easier, since they inherently provide the highpass transfer function

$$H(s) = \frac{s^2}{s^2 + \frac{1}{Q}s + 1} \quad \text{(normalized to } \omega_0 = 1\text{)}$$

See p. 5.6.11:



So do this:

→ transform HP spec to normalized LP

→ select $H'(s')$ polynomials, normalized, $\omega_p' = 1$

→ calculate Q_k, ω_{0k} for each section

→ for each section, select components in proto circuit above by setting Q_k, then frequency scaling by ω_p/ω_{0k} (ω_p the original HP passband). Justification: pole movement p 5.7.3

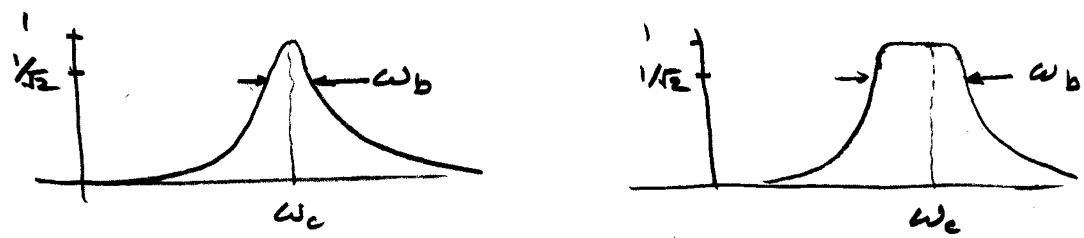
The procedure needs a little tweak for elliptic filters, which have zeros, too.

- **Lowpass to Bandpass**

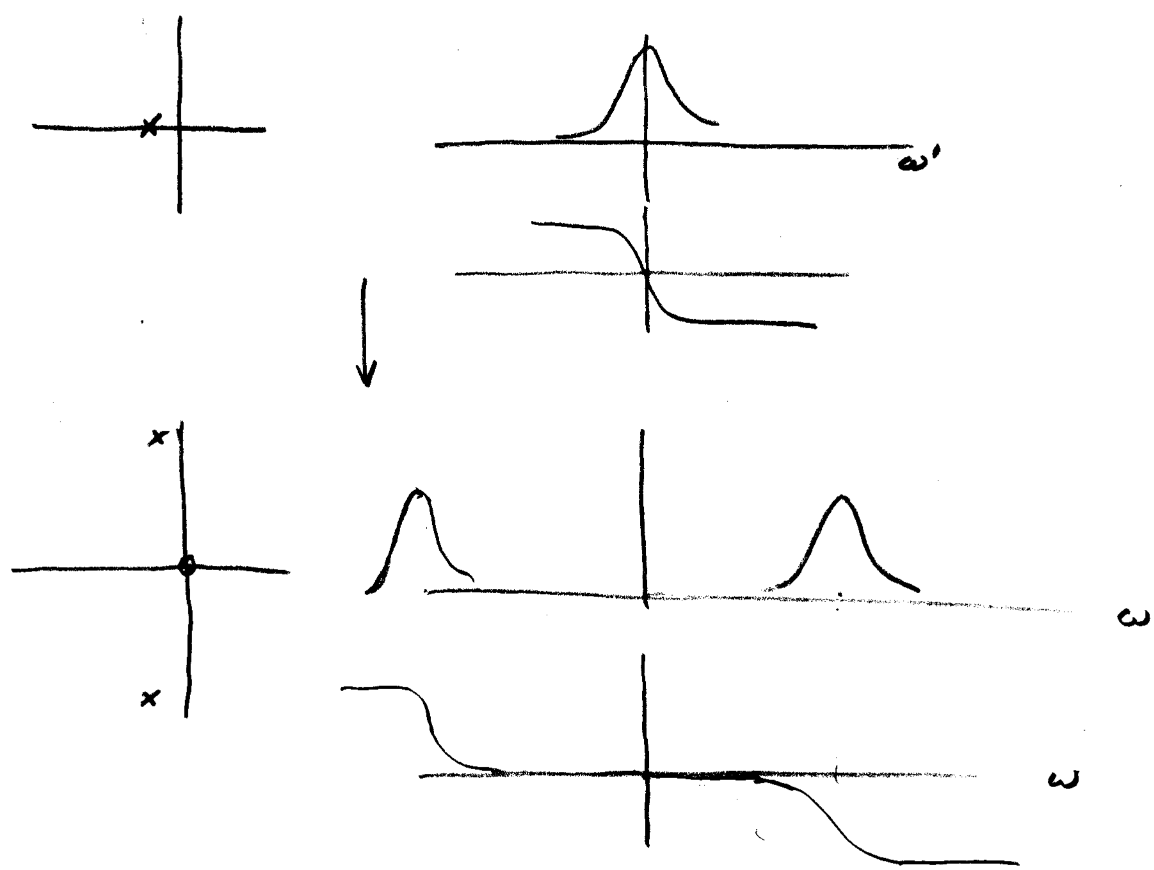
  - If we have a lowpass transfer function normalized to unit bandwidth $H'(s')$, substitute

$$s' = \frac{s^2 + \omega_c^2}{s\omega_b}$$

ω_c centre freq (geometric)
ω_b 3 dB bw

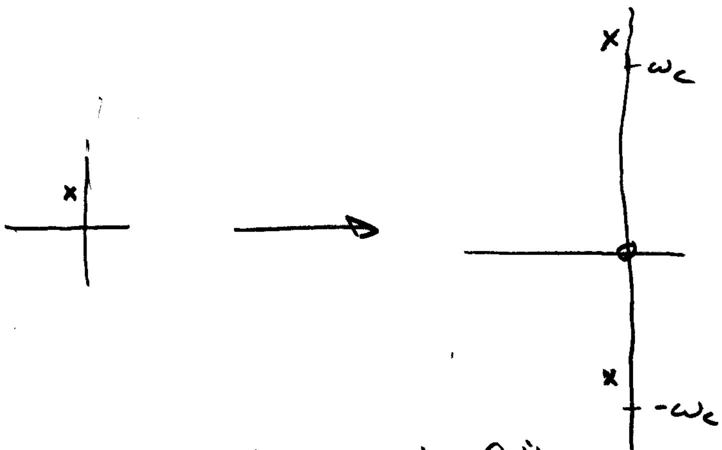


  - We saw this on p 4.2.9, taking 1st order lowpass to 2nd order bpass

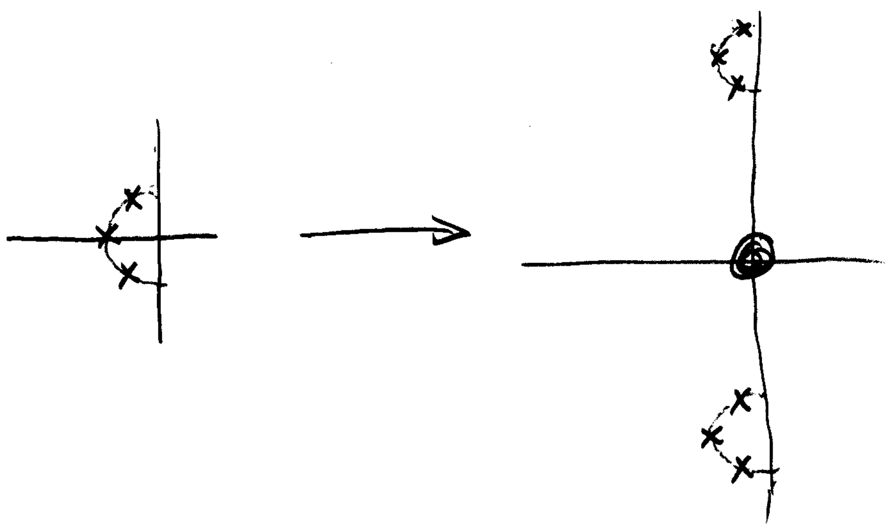


- Each pole of a lowpass splits in two and picks up a zero:

$$\frac{-p_i}{s'-p_i} = \frac{-p_i}{\dfrac{s^2+\omega_c^2}{s\,\omega_b} - p_i} = \frac{-p_i\omega_b s}{s^2 - p_i\omega_b s + \omega_c^2}$$



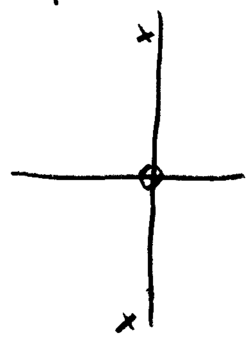
and more elaborate filters do this:



So <u>filter order doubles</u>

- we won't get into bandpass circuit design in this course (consult a reference).

However, you can see from the pole zero diagram on the previous page that
  - it can be decomposed to quadratic bandpass sections



  - and that numerical accuracy and component tolerances are tight, in order to place poles correctly

not