6 MUTUAL INDUCTANCE AND TRANSFORMERS

$\boxed{\text{DC+L Chapt 18}}$

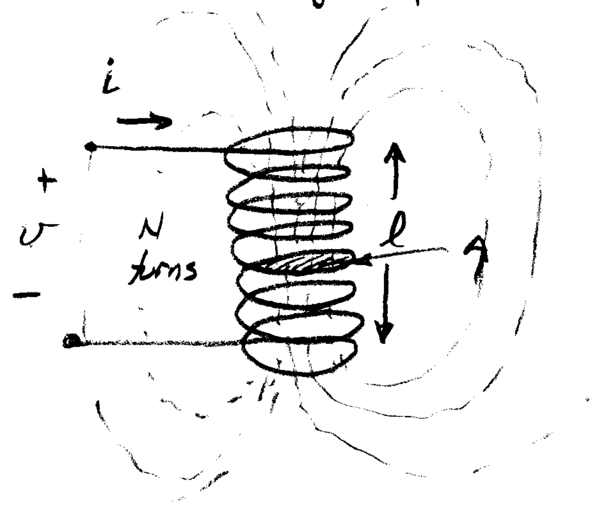
- Magnetically coupled coils are common in electronics and EE in general:
  - transformers to step up/down voltages
  - transformers to electrically isolate sections of a circuit
  - transformers for impedance conversion.
  - inadvertent coupling

- We will focus on coupled coils as circuit elements and gloss over the physics.

  Treatment:
  - single coil (inductor) review;
  - time and frequency domain characterization of general coupled coils;
  - transformers

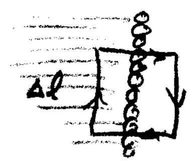
6.1 Review of Inductors

- Consider tightly wound coil, air core



N turns, length l, uniform,
Area A

In the middle,



$$\oint \underline{H} \cdot d\underline{l} = N \frac{\Delta l}{l} i$$

$$H \Delta l = N \frac{\Delta l}{l} i$$

$H = \frac{N}{l} i$ at all points

- The magnetic field and flux in the middle are

$$B = \mu_0 H \qquad \mu_0 = 4\pi \times 10^{-7} \frac{\text{web}}{\text{amp-m}}$$

$$\Phi = AB = \frac{\mu_0 A N}{l} i$$

Flux is almost the same through every turn.

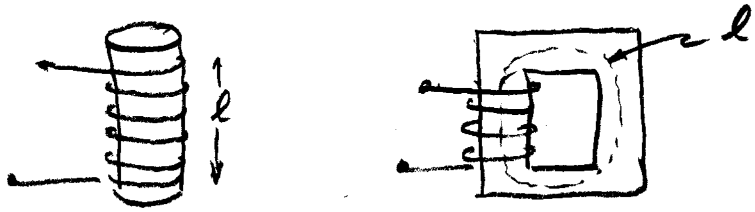
- Induced voltage due to flux changes:

$$v = N \frac{d\Phi}{dt} = \frac{\mu_0 A N^2}{l} \frac{di}{dt} = L \frac{di}{dt}$$

Note − L proportional to N^2

− L proportional to permeability μ_0

- Coil with core of magnetic material



Similar argument, except μ of material is hundreds or thousands of times higher than μ_0. Also, for closed path of the material, l is total length.

$$B = \mu H \quad \text{an approx}$$

$$v = \mu \frac{A N^2}{l} \frac{di}{dt}$$

for $B(H)$

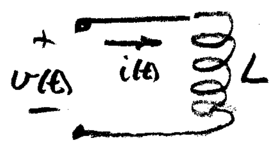
$$V = NA \frac{dB}{dt}$$

$$= NA \frac{dB}{dH} \frac{dH}{dt}$$

$$= NA \frac{dB}{dH} \frac{N}{l} \frac{di}{dt}$$

Inductance is much higher; still proportional to N^2

- Energy stored in the magnetic field:
  - Use the linear model for simplicity



$$v(t) = L\frac{di}{dt} \qquad V(s) = sL\,I(s)$$

  - Increase current from 0 to some i_o

Instantaneous power into the inductor

$$i(t)\,v(t) = P(t)$$

Cumulative energy

$$E = \int_0^{t_o} P(t)\,dt = L\int_0^{t_o} i(t)\frac{di}{dt}\,dt$$

$$= L\int_0^{i_o} i\,di = \frac{1}{2}L\,i_o^2$$

Proportional to i_o^2 and to L (so N^2), independent of trajectory $i(t)$