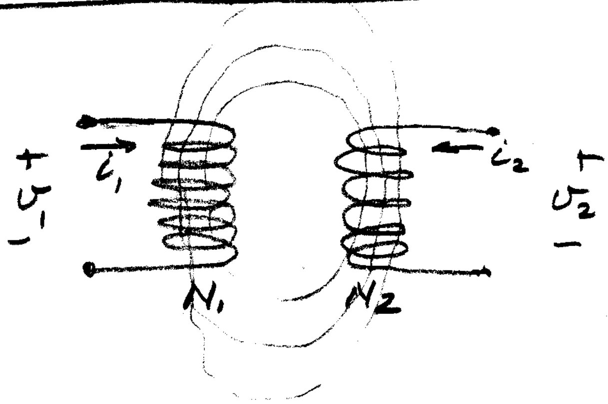
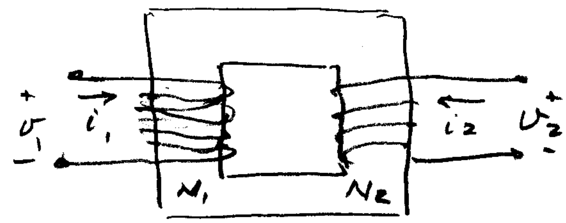
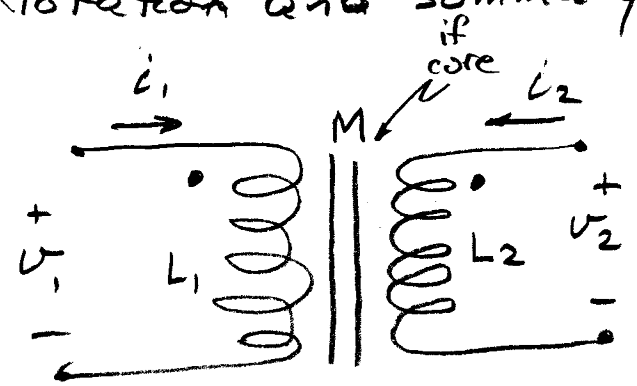
6.2 Coupled Coils





- Each coil acts as an inductor, but flux change due to changes in $i_1(t)$ induces voltage in coil 2, and vice versa.

- Notation and summary



dot convention

increasing current into the dotted terminal induces positive voltage at dotted terminal of other side (wrt its undotted terminal)

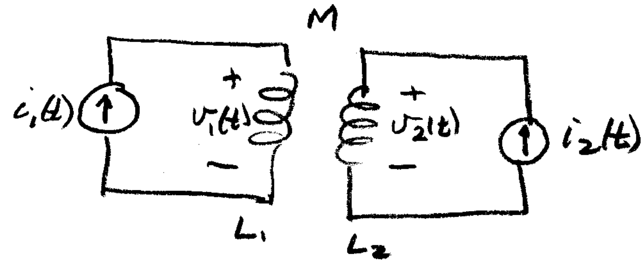
$$v_1(t) = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$v_2(t) = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$

Notes:
- symmetry of mutual inductance $\boxed{\text{DC+L 18.5}}$
- $M \geq 0$ from sign convention (dots)
- $M \leq \sqrt{L_1 L_2}$, $M = k\sqrt{L_1 L_2}$ $\boxed{\text{DC+L 18.5}}$

  k coupling coefficient, $k \leq 1$

- Energy storage in coupled coils. $\boxed{\text{DC+L 18.5}}$
  - Increase currents from 0 to i_{10}, i_{20}



  - Instantaneous powers

$$P_1(t) = i_1(t)\,v_1(t) = L_1\, i_1(t) \frac{d i_1(t)}{dt} + M\, i_1(t) \frac{d i_2(t)}{dt}$$

$$P_2(t) = L_2\, i_2(t) \frac{d i_2(t)}{dt} + M\, i_2(t) \frac{d i_1}{dt}$$

  - Accumulated energy:

$$E_1 = \int_0^{t_0} P_1(t)\,dt = \frac{1}{2} L_1 i_{10}^2 + M \int_0^{t_0} i_1(t) \frac{d i_2(t)}{dt}\,dt$$

$$= \frac{1}{2} L_1 i_{10}^2 + \underbrace{M\, i_{10}\, i_{20} - M \int_0^{t_0} i_2(t) \frac{d i_1(t)}{dt}\,dt}_{\text{by parts}}$$

$$E_2 = \int P_2(t)\,dt = \frac{1}{2} L_2 i_{20}^2 + M \int_0^{t_0} i_2(t) \frac{d i_1(t)}{dt}\,dt$$

$$E = E_1 + E_2 = \frac{1}{2} L_1 i_{10}^2 + \frac{1}{2} L_2 i_{20}^2 + M\, i_{10}\, i_{20}$$

Note – independent of trajectory
  - text uses symmetric argument
  - $L_1 \propto N_1^2$, $L_2 \propto N_2^2$, $M \propto N_1 N_2$