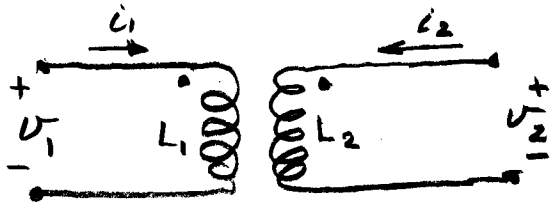
6.3 Coupled Coils as Circuit Elements

- Describing equations



$$v_1 = L_1 \dot{i}_1 + M \dot{i}_2$$

$$v_2 = M \dot{i}_1 + L_2 \dot{i}_2$$

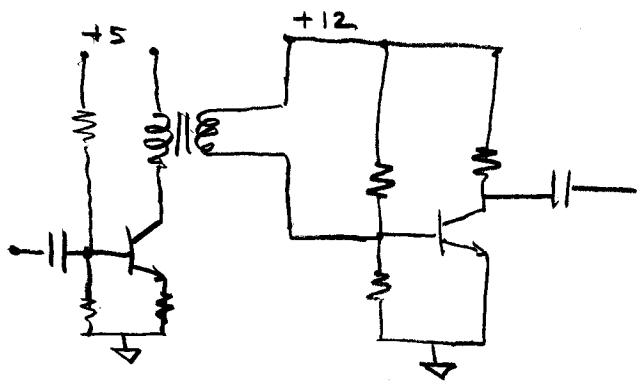
$$V_1(s) = sL_1 I_1(s) + sM I_2(s)$$

$$V_2(s) = sM I_1(s) + sL_2 I_2(s)$$

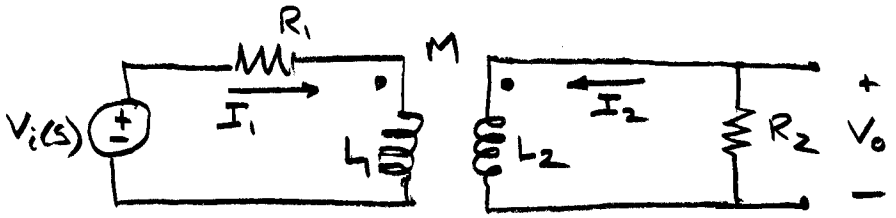
$$\begin{bmatrix} V_1(s) \\ V_2(s) \end{bmatrix} = \begin{bmatrix} sL_1 & sM \\ sM & sL_2 \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix}$$

Notes: – DC doesn't get through

– The two sides can be connected to different circuits, different power supplies, different grounds



• **example** Example 18.6 done with symbols



- Response in secondary coil to input $V_i(s)$?

$$I_1 R_1 + sL_1 I_1 + sMI_2 = V_i$$

$$sMI_1 + sL_2 I_2 = -I_2 R_2 \qquad V_o = -R_2 I_2$$

or

$$(R_1 + sL_1) I_1 + sMI_2 = V_i$$

$$sMI_1 + (R_2 + sL_2) I_2 = 0$$

- Solve for $I_2(s)$ (eliminate I_1 or use Cramer's rule)

$$I_2(s) = \frac{-sM}{(L_1L_2 - M^2)s^2 + (R_1L_2 + R_2L_1)s + R_1R_2} \cdot V_i(s)$$

This is negative, by the dot sign convention.

It has a zero, so dc gain is zero
(as expected)

- The output voltage

$$V_o(s) = -R_2 I_2(s) = \frac{sMR_2}{(L_1L_2 - M^2)s^2 + (R_1L_2 + R_2L_1)s + R_1R_2} V_i(s)$$

is positive.

– Use $M = k\sqrt{L_1 L_2}$ for more insight

$$V_o = \frac{R_2 k}{(1-k^2)\sqrt{L_1 L_2}} \cdot \frac{s}{s^2 + \frac{1}{1-k^2}\left(\frac{R_1}{L_1} + \frac{R_2}{L_2}\right)s + \frac{1}{1-k^2}\frac{R_1}{L_1}\frac{R_2}{L_2}} V_i(s)$$

As $k \to 1$, second two denom terms dominate and it's effectively 1st order.

- The dot convention

(increasing i_1 induces positive component of voltage in v_2)
might cause us to expect that v_1, v_2 are in phase for sinusoidal excitation; But it depends on the attached circuit elements, since (for example) v_2 is a sum $v_2 = M\dot{i}_1 + L_2\dot{i}_2$

– Even the example above shows a phase shift between $v_i(t)$, $v_o(t)$

$V_o(s) = H(s)\,V_i(s)$ $\quad \angle H(j\omega)$ is ph. shift at ω

– DC+L example 18.7 is extreme, with $v_o(t)$, $v_i(t)$ in antiphase