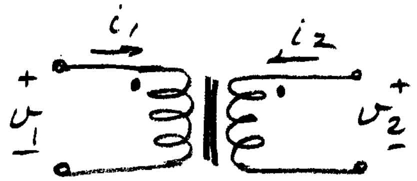
6.4 The Ideal Transformer

DC+L 18.6

- A transformer is a pair of coupled coils with near-unity coupling coeff k, and very high L_1, L_2 and $M \approx \sqrt{L_1 L_2}$. The ideal transformer takes these approximations to the limit. Examine consequences below.

- Unity coupling coefficient $k=1$, $M = \sqrt{L_1 L_2}$.

$$k=1 \Rightarrow \frac{L_1}{M} = \sqrt{\frac{L_1}{L_2}} = \frac{M}{L_2} \doteq a$$



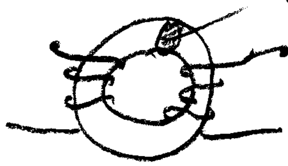
– Text shows that the ratio

$$\frac{v_1(t)}{v_2(t)} = \frac{L_1 \dot{i}_1 + M \dot{i}_2}{M \dot{i}_1 + L_2 \dot{i}_2} = \frac{aM\dot{i}_1 + aL_2\dot{i}_2}{M\dot{i}_1 + L_2\dot{i}_2} \quad \text{if } k=1$$

$$= a \propto \underbrace{\frac{N_1}{N_2}}_{\text{turns ratio}} \quad \text{since } L_1 \propto N_1^2,\ L_2 \propto N_2^2$$

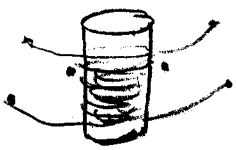
- Physically, $k=1$ corresponds to no flux loss; flux is the same through both coils

e.g.



Φ through x-section same at all points, no leakage outside toroid

or



coils wound on top of each other

If they have a common flux,

$$v_1 = N_1 \frac{d\phi}{dt} \qquad v_2 = N_2 \frac{d\phi}{dt}$$

and

$$\frac{v_1}{v_2} = \frac{N_1}{N_2}, \quad \text{so} \quad a = \frac{N_1}{N_2}$$

- The second idealization is $L_1, L_2 \to \infty$.

  - Text shows

$$\frac{1}{M} v_1(t) = \frac{1}{M}\left(L_1 i_1 + M i_2\right)$$

or

$$\frac{v_1}{\sqrt{L_1 L_2}} = a i_1 + i_2 = \frac{d}{dt}\left(a i_1(t) + i_2(t)\right)$$

For $L_1, L_2 \to \infty$, this is

$$\frac{d}{dt}\left(a i_1 + i_2\right) = 0$$

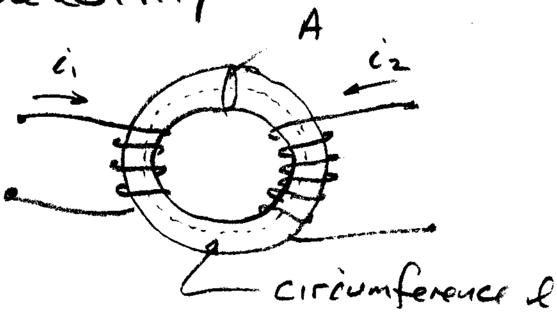
The solution of this DE is

$$a i_1 + i_2 = K$$

Since it holds at all times, including when no current flows,

$$a i_1 + i_2 = 0 \quad \text{or} \quad \frac{i_1(t)}{i_2(t)} = -\frac{1}{a}$$

— Physically, this corresponds to infinite permeability



$$\oint \underline{H} \cdot d\underline{\ell} = N_1 i_1 + N_2 i_2$$

$$H = \frac{N_1 i_1 + N_2 i_2}{\ell}$$

Flux linkage

$$\Phi = AB = A\mu H = \frac{A\mu}{\ell}(N_1 i_1 + N_2 i_2)$$

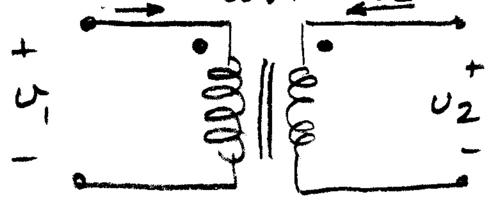
$$N_1 i_1 + N_2 i_2 = \frac{\Phi \ell}{A\mu}$$

Flux Φ is finite, or any change produces infinite voltage. If $\mu \to \infty$, RHS $\to 0$ and

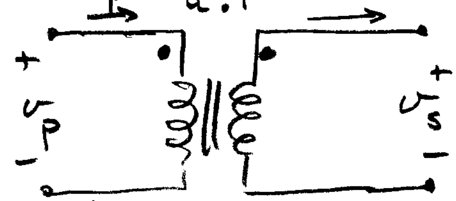
$$\frac{i_1}{i_2} = -\frac{N_2}{N_1} = -\frac{1}{a} \qquad \text{again } a = \frac{N_1}{N_2}$$

turns ratio

- Summary of ideal transformer:



or

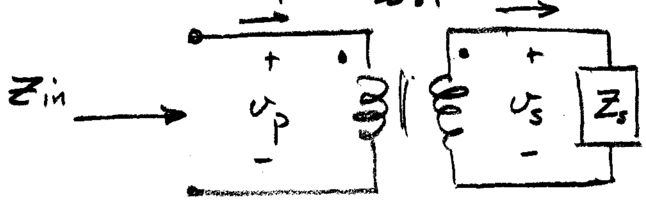


$$\frac{v_1}{v_2} = a, \quad \frac{i_1}{i_2} = -\frac{1}{a} \qquad \frac{v_p}{v_s} = a \qquad \frac{i_p}{i_s} = \frac{1}{a}$$

and $a = \dfrac{N_1}{N_2} = \dfrac{N_p}{N_s}$

Power?*

- Impedance transformation with ideal transformer



$$Z_{in}(s) = \frac{V_p(s)}{I_p(s)} = \frac{a\,V_s(s)}{I_s(s)/a} = a^2 Z_s(s) = \left(\frac{N_1}{N_2}\right)^2 Z_s(s)$$

The impedance is scaled by square of turns ratio.

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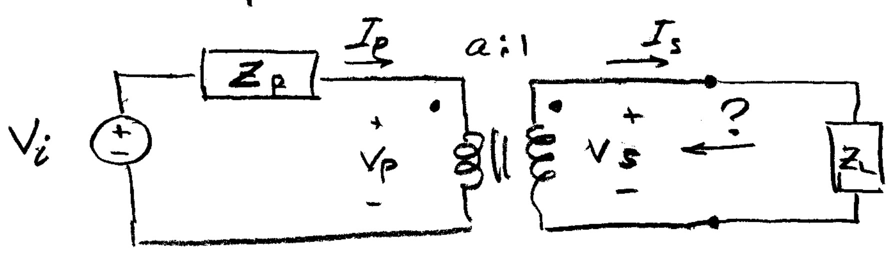
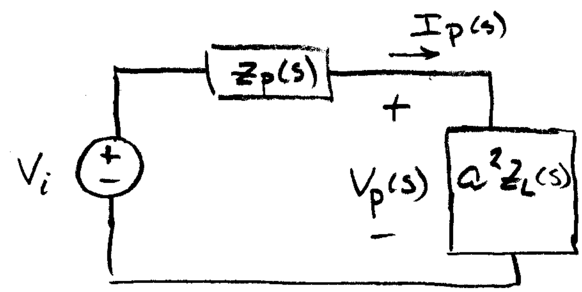
* Power in $= v_p i_p = a v_s \dfrac{i_s}{a} = v_s i_s =$ Power out

– Similarly,



How does the load see the transformer?

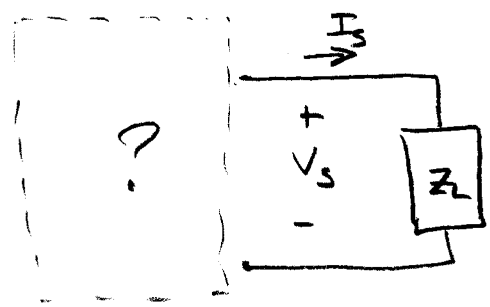
Start with primary, which sees



$$V_p(s) = V_i \frac{a^2 Z_L(s)}{Z_p(s) + a^2 Z_L(s)}$$

$$I_p(s) = \frac{V_i}{Z_p(s) + a^2 Z_L(s)}$$

On the secondary, we have

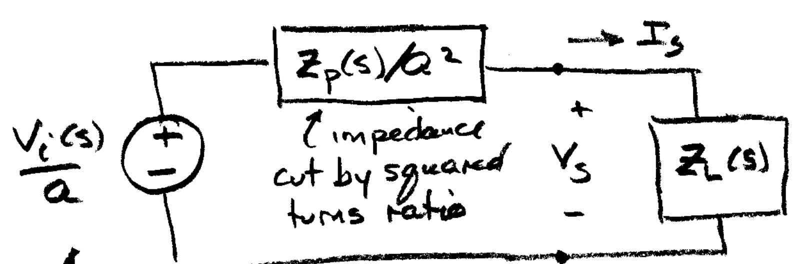


$$I_s = a I_p = a \frac{V_i}{Z_p + a^2 Z_L}$$

$$= \frac{V_i}{a} \frac{1}{\frac{Z_p}{a^2} + Z_L}$$

$$V_s = \frac{V_p}{a} = \frac{V_i}{a} \frac{Z_L}{\frac{Z_p}{a^2} + Z_L}$$

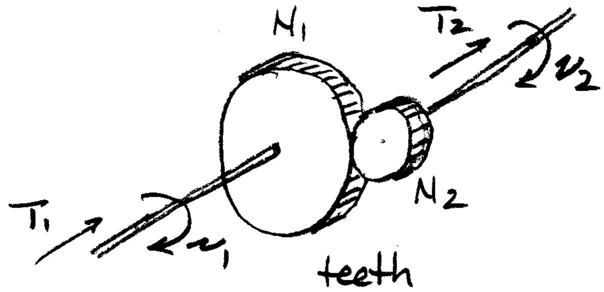
so equiv source:



voltage cut by turns ratio

An unsurprising result, given p. 6.4.4.

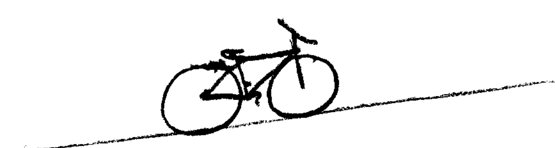
- The ideal transformer has a mechanical analogy — the ideal gearbox.



$$\frac{v_1}{v_2} = \frac{N_2}{N_1}$$

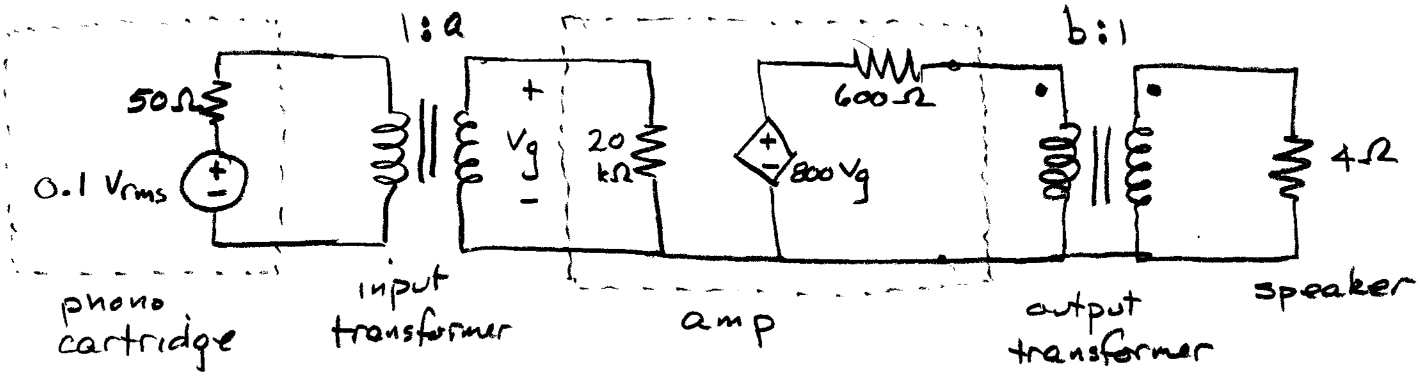
$$\frac{T_1}{T_2} = \frac{N_1}{N_2}$$

Like the ideal transformer, the power in and power out are identical; just the torque and speed factors of $P = Tv$ are varied.



The gears on your bicycle convert the impedance of the road/wind load to one your legs are comfortable with — a speed and force range that works for you.

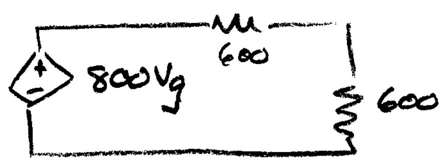
- example DC+L Q 18.53



What values of turns ratios a and b give maximum power to the speaker? What is that max power?

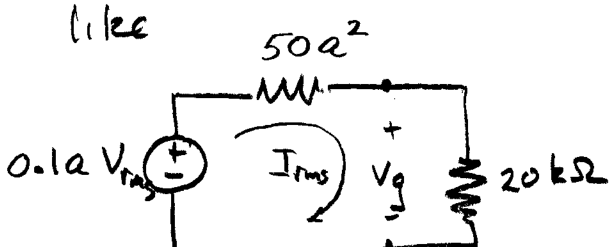
- Start with output transformer. The speaker looks like $4b^2\ \Omega$ to the amp. For max pwr, make it 600Ω, so $b = 12.25$.

  The power to the speaker is



$$P_s = \frac{800^2\ \overline{V_g^2}}{4 \cdot 600} = 266\ \overline{V_g^2}$$

- Now the input transformer. The cartridge looks like



Maximize $\overline{V_g^2}$

equiv, max power to the 20kΩ

The rms voltage is

$$V_{g\,rms} = 0.1\,a \cdot \frac{20k}{50a^2+20k}$$

so

$$\overline{V_g^2} = (2k)^2 \left(\frac{a}{50a^2+20k}\right)^2$$

Set derivative wrt a to zero*

$$50a^2+20k - a(2\cdot 50a) = 0$$

$$50a^2 = 20k$$

$$a = 20$$

– The mean square value

$$\overline{V_g^2} = (2\,k\Omega)^2 \left(\frac{20}{50\cdot 20^2+20k}\right)^2 = 1\ V_{rms}^2$$

and power to the speaker is

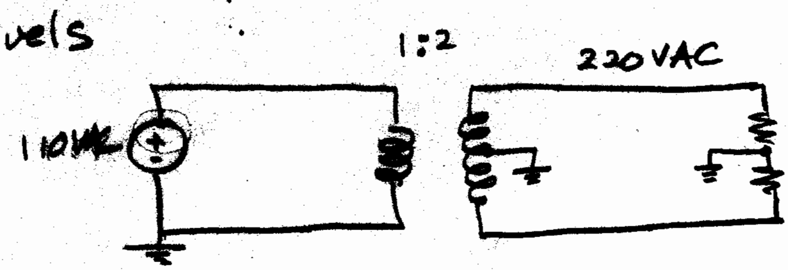
$$P_s = 266\,\overline{V_g^2} = 266\ W.$$

$$* \quad \frac{d\overline{V_g^2}}{da} = (2k)^2\, 2\, \frac{a}{50a^2+20k} \cdot \frac{d}{da}\left(\frac{a}{50a^2+20k}\right)$$

$$= (2k)^2\, 2\, \frac{a}{50a^2+20k} \cdot \frac{50a^2+20k - a(2\cdot 50a)}{(50a^2+20k)^2}$$

Ignore root at $a=0$.

- Centre taps allow symmetry in shifting voltage levels



← 110 VAC each, 180° difference

- Another asymmetric-to-symmetric configuration in this push-pull amp, where only the positive supply needs to carry high current.

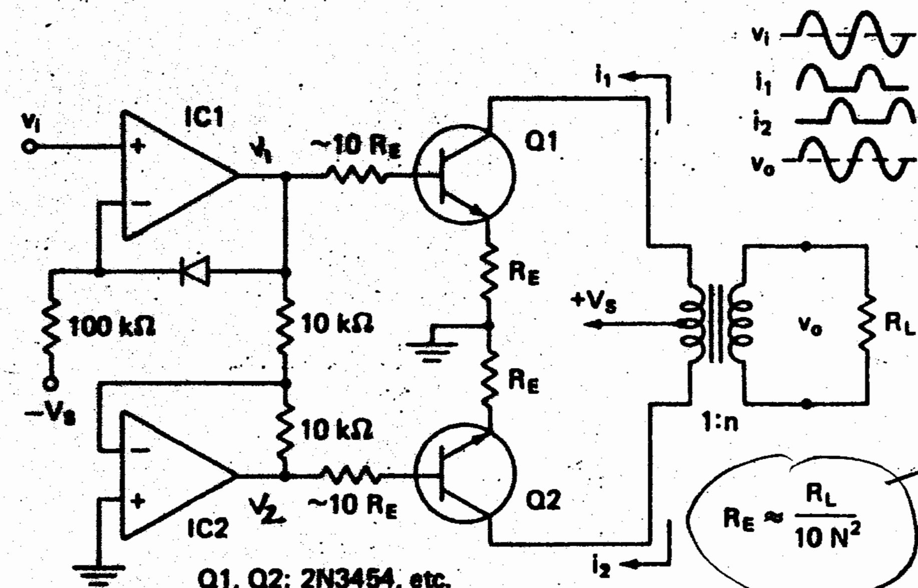


Fig. 22-5 Transformer-coupled ac amplifier.

from D. Wobschall, "Circuit Design for Electronic Instrumentation," McGraw-Hill 1979.

$R_E \approx \dfrac{R_L}{10\,N^2}$

source resistance $\approx 10 R_E$

impedance match thru transformer

Notes: – IC1 non-inverting, IC2 inverting

– During positive half cycle of V_i:

→ $V_1 = V_i + 0.7V > 0$; $V_2 = -(V_i + 0.7V) < 0$

→ Q1 active ; Q2 cutoff

i_1 positive $\frac{1}{2}$ cycle ; $i_2 = 0$

→ V_o positive $\frac{1}{2}$ cycle.

– During negative half cycle of V_i:

→ $V_1 = V_i + 0.7V < 0$; $V_2 = -(V_i + 0.7V) > 0$

→ Q1 cutoff ; Q2 active

$i_1 = 0$; i_2 positive $\frac{1}{2}$ cycle

→ V_o negative half cycle