6.5 Less-Than-Ideal Transformers | DC+L 18.7

- Real-world transformers differ from the ideal model in several ways, such as:
  - (a) coupling coefficient $k < 1$
  - (b) finite permeability
  - (c) winding resistance and capacitance
  - (d) nonlinear ferromagnetic effects

The first three are linear, so we can model them using the methods of this course.

- Our models use an embedded ideal transformer. But we already have coupled coil models for $k<1$ and finite μ — why bother with new ones?
  - Coupled coil analysis usually has systems of equations
  - Ideal transformers allow impedance, voltage and current transformations, so simpler analysis.

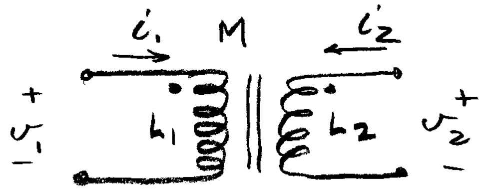
- Modeling Finite Permeability

  - Focus on finite μ, so $N_1 i_1 \neq -N_2 i_2$ and L_1, L_2, M are all finite.

  - To keep it simple, retain $k=1$, so

$$\frac{v_1}{v_2} = \frac{N_1}{N_2}$$

  - The coupled coil model, as usual, is



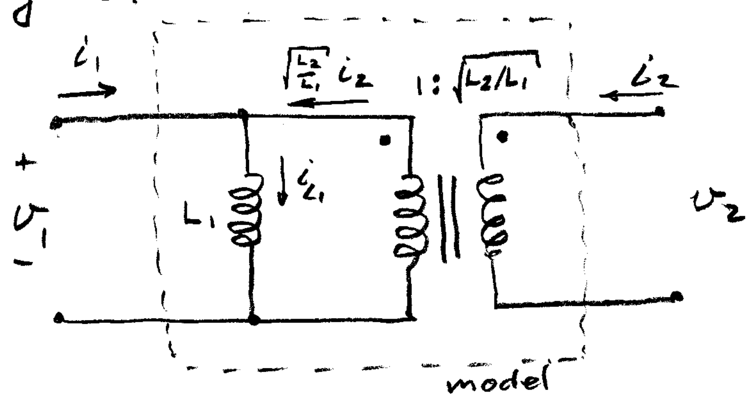
$M = \sqrt{L_1 L_2}$ for $k=1$

$$v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} = L_1 \frac{di_1}{dt} + \sqrt{L_1 L_2}\, \frac{di_2}{dt}$$

$$= L_1 \frac{d}{dt}\left(i_1 + \sqrt{\frac{L_2}{L_1}}\, i_2 \right)$$

looks like KCL with a current transformation

which gives:



since $v_1 = L_1 \frac{di_{L_1}}{dt} = L_1 \frac{d}{dt}\left(i_1 + \sqrt{\frac{L_2}{L_1}}\, i_2\right)$

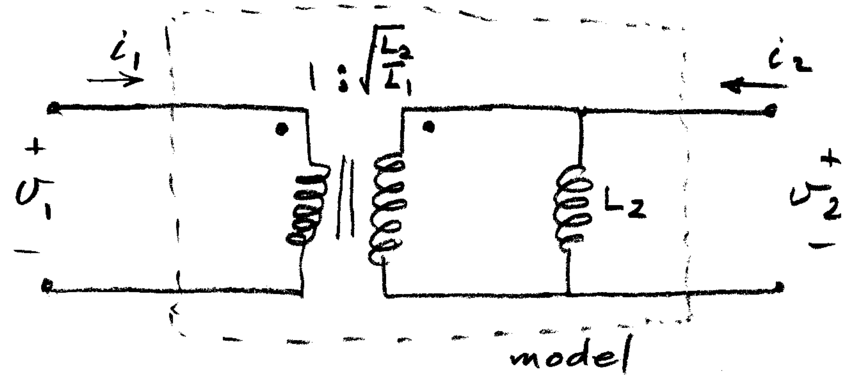
The currents i_1, i_2 are no longer linked by $N_1 i_1 + N_2 i_2 = 0$

– A variation moves the inductor to the secondary, with impedance transformation.

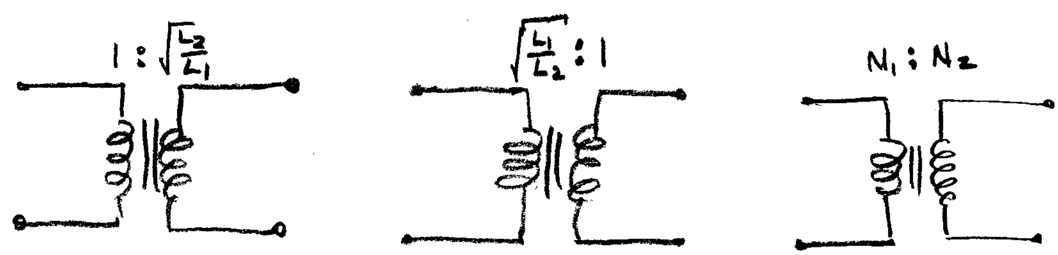
Seen from the secondary, L_1 looks like

$$\underbrace{\left(\sqrt{\frac{L_2}{L_1}}\right)}_{N_2/N_1}^{2} L_1 = L_2$$

so

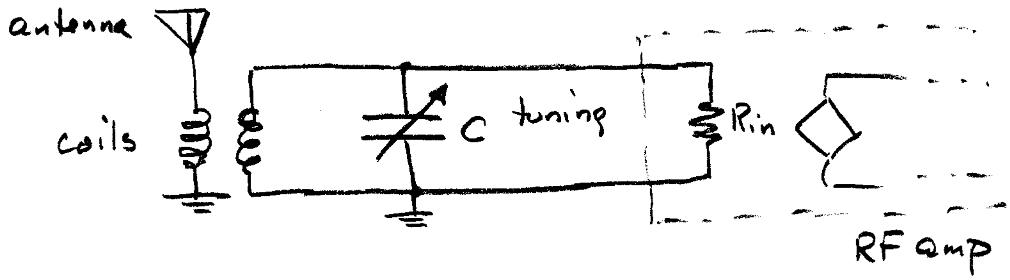


Note that these ideal transformers are equivalent:

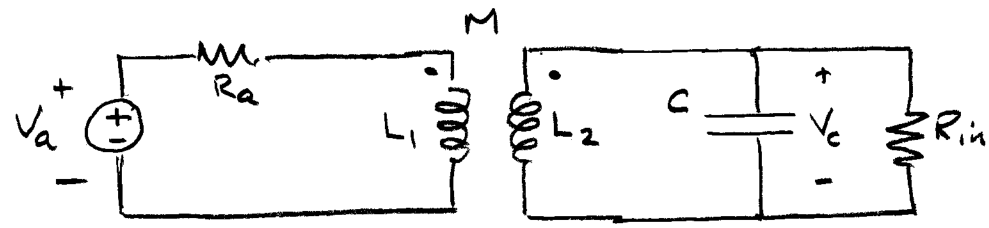


- <u>example</u> 18.15 from DC+L (resting on example 18.9)

  - Radio front end:



  - Model with coils



In example 18.9, values:

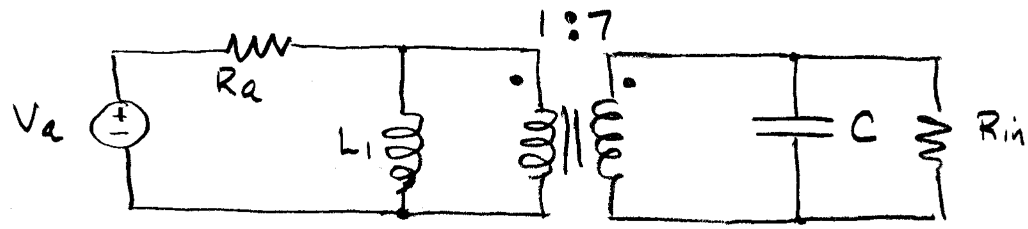
$R_a = 300\,\Omega$

$L_1 = 50\,nH \quad L_2 = 2450\,nH \quad M = 350\,nH = \sqrt{L_1 L_2}$

$C = 104.5\,pF \quad R_{in} = 14.7\,k\Omega.$

  - Use of coupled coil analysis gives coupled equations on the two sides which must be solved jointly.

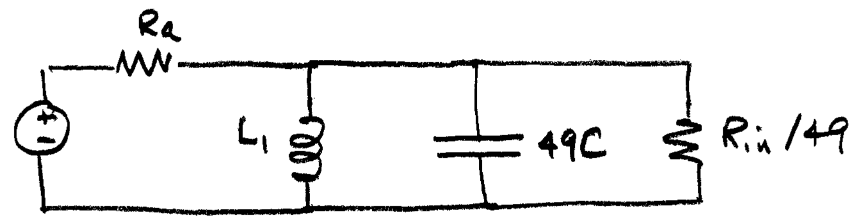
– New alternative: model coils with embedded ideal transformer



since $M = \sqrt{L_1 L_2} \Rightarrow k = 1$

and $\sqrt{\frac{L_2}{L_1}} = \sqrt{\frac{2450}{50}} = 7$, $a = \frac{1}{7}$

Just refer secondary to the primary:



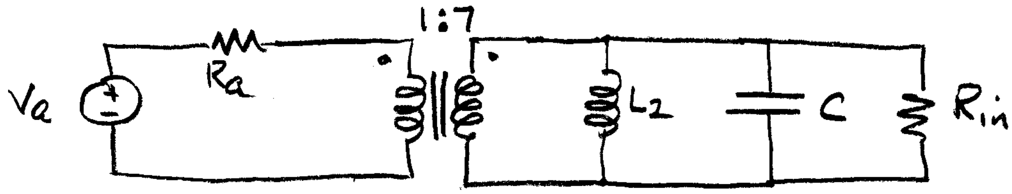
impedances scaled by a^2, or $\frac{1}{49}$

Now it is a straightforward analysis.

Note that the cap looks bigger, so resonance frequency is 7 times lower than $\frac{1}{\sqrt{L_1 C}}$

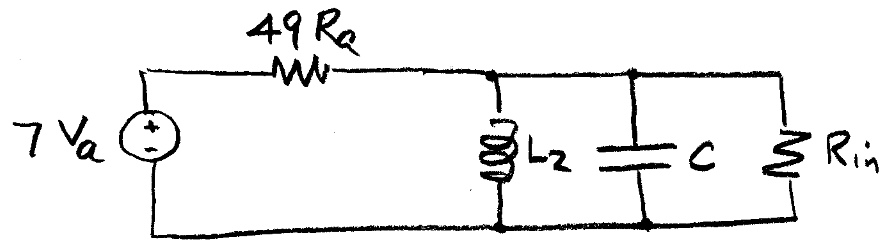
Also, R_{in} appears as $R_{in}/49 = 14.7k\Omega/49 = 300\Omega$, so it equals R_a — so max power transfer at resonance, where the parallel LC looks like an open circuit.

– Even easier: use the model with inductor on the secondary



L_2 is $49 L_1$ since unity coupling.

Transform the antenna:



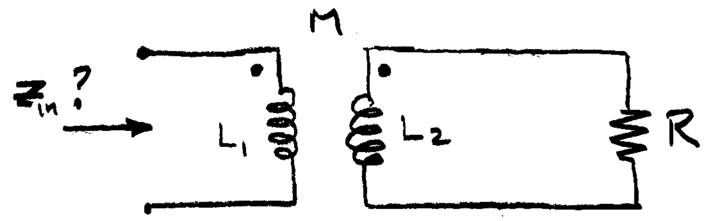
$49 R_a = 49 \cdot 300 = R_{in}$, so matched at resonance

Voltage gain at resonance: $7 \times \frac{R_{in}}{R_{in} + 49 R_a} = 3.5$

although no power gain in the transformer.

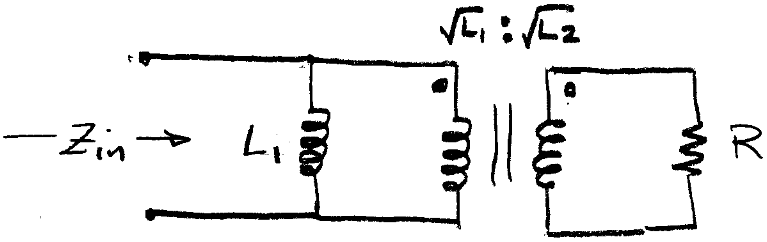
- A consequence of non-ideality is that impedance transformations with real transformers are only approximations of what you could obtain with ideal transformers (if you could obtain ideal transformers).

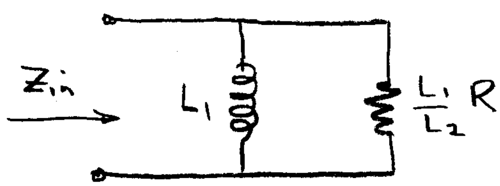
For example,



Ideally, $Z_{in} = \left(\frac{N_1}{N_2}\right)^2 R = \frac{L_1}{L_2} R$

More realistically,





$$Z_{in}(s) = \frac{\frac{L_1}{L_2} R \cdot sL_1}{\frac{L_1}{L_2} R + sL_1} = \underbrace{\frac{L_1}{L_2} R}_{\text{ideal}} \cdot \underbrace{\frac{s}{s + \frac{R}{L_2}}}_{\approx 1 \text{ for } \omega \gg \frac{R}{L_2}}$$

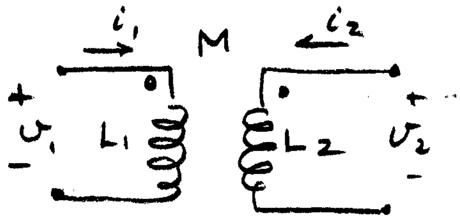
If the frequency is too low, Z_{in} becomes small, since shunt L_1 has decreasing impedance.

Transformers don't work for DC.

• Modeling Finite μ and Coupling $k<1$

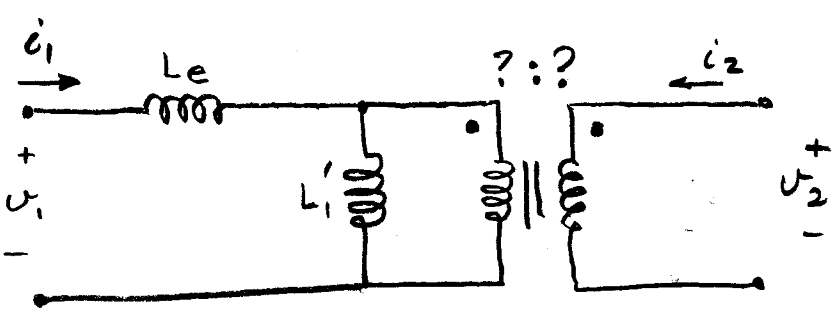
— We can't quite reach $k=1$ in reality. If we have to account for $k<1$, we could use coupled coil analysis, but it's tedious.

— There is a model with an embedded ideal transformer. Start with coupled coils



$$M^2 = k^2 L_1 L_2$$

Since $L_1 L_2 > M^2$, L_1 is larger than unity coupling with M, L_2 requires. Split L_1 into a unity coupling portion L_1' and the extra, L_e



based on unity coupling with M, L_2.

- For the unity coupling subcircuit

$$L_1' L_2 = M^2 = k^2 L_1 L_2$$

So

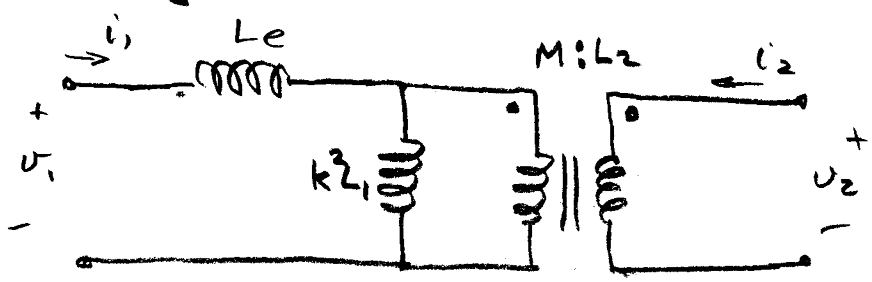
$$L_1' = k^2 L_1$$

and the turns ratio is

$$1 : \sqrt{\frac{L_2}{L_1'}} \quad \text{or} \quad 1 : \sqrt{\frac{L_2}{M^2/L_2}}$$

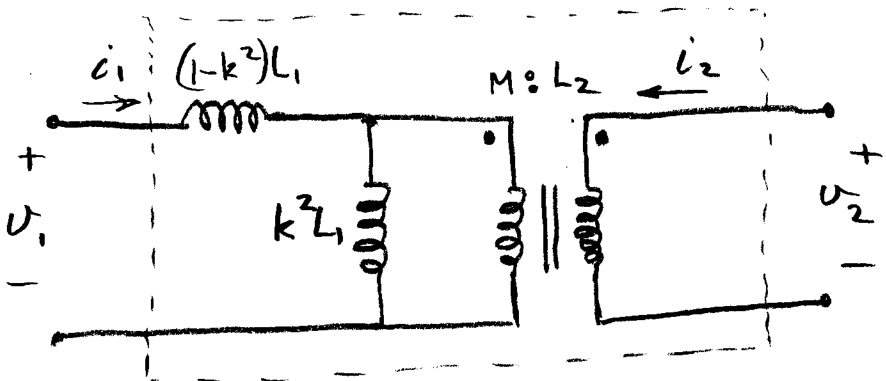
$$\text{or} \quad 1 : \frac{L_2}{M} \quad \text{or} \quad M : L_2 \quad \text{or} \quad \frac{M}{L_2} : 1 \quad \text{etc, etc.}$$

which gives

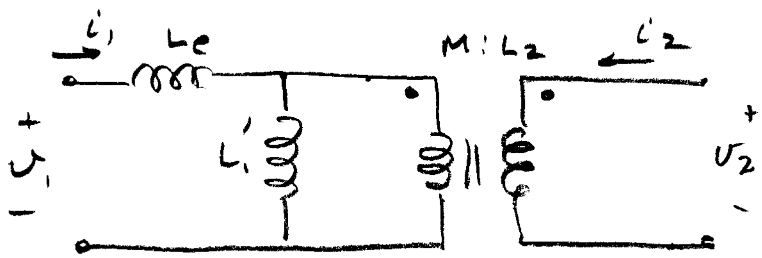


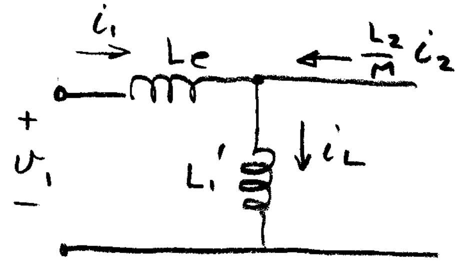
- The extra inductance $L_e = L_1 - k^2 L_1$
$= (1-k^2) L_1$

so final model is



- Quick confirmation that the model works: reduce it to coupled coils.



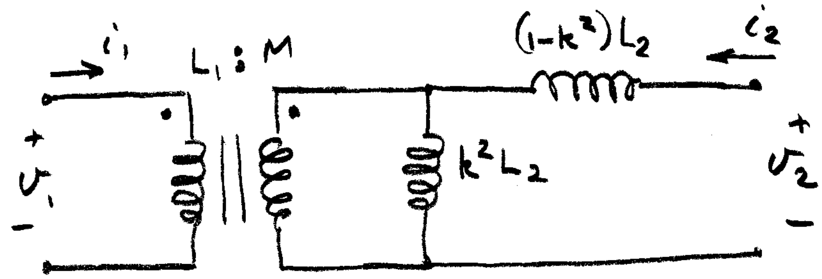


$$v_1 = L_e \dot{i}_1 + L_1' \left(\dot{i}_1 + \frac{L_2}{M} \dot{i}_2 \right)$$

$$= (1-k^2) L_1 \dot{i}_1 + k^2 L_1 \dot{i}_1 + \frac{L_1' L_2}{M} \dot{i}_2$$

$$= L_1 \dot{i}_1 + M \dot{i}_2$$

- This model development could equally have broken up L_2



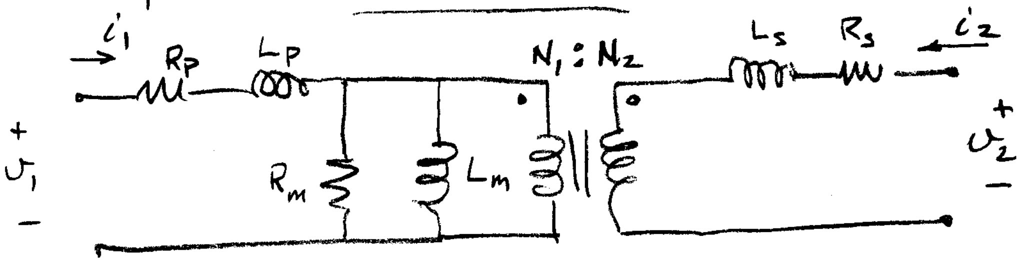
- And either of them could have referred the shunt inductor to the other side of the ideal transformer — four models

• Models for Some Other Non Idealities

- So far, even with $k<1$ and finite μ, transformer models have been lossless – just inductors and ideal transformers.

- Real transformers have winding resistance and power loss due to hysteresis in the core. We can model them as resistors (approximately, in the case of hysteresis).

- Text presents $\boxed{\text{DC+L 18.8}}$



  - We have already seen L_m (from finite μ) and L_p, L_s (from $k<1$, but a more symmetric form).
  - Resistors R_p, R_s are for windings, R_m for core loss.

- Then there is capacitance in the windings...

- But enough! Consult a more advance text if necessary