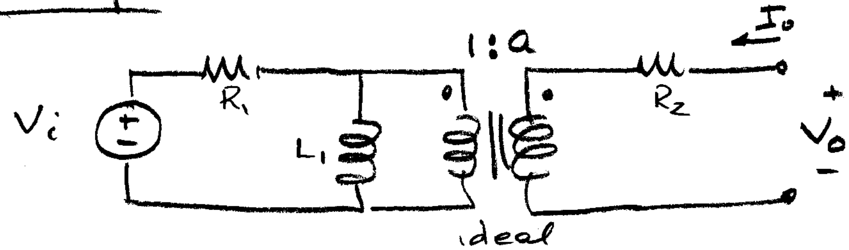
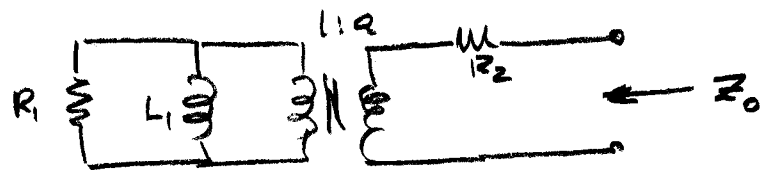
7.1 Review of One-Ports | DC+L 19.2 |

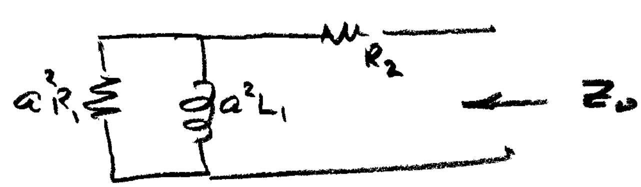
- In addition to basic R, L, C components, circuits may contain independent sources, dependent sources, transformers, etc, and they must all be accounted for in the calculation of Thevenin and Norton models.
- We'll warm up with examples, then look at node or loop analysis.

- example



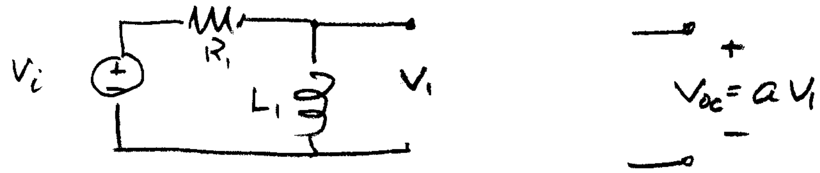
- To obtain the output impedance, set all independent sources to zero.



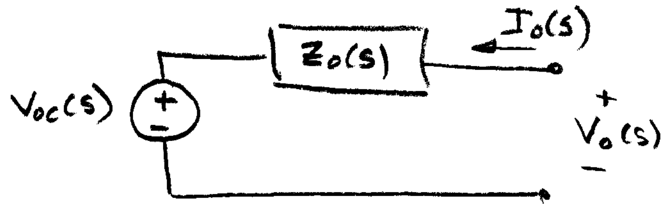


$$Z_o = R_2 + a^2 R_1 \| s a^2 L_1 = \frac{(R_2 + a^2 R_1)s + R_2 R_1 / L_1}{s + R_1 / L_1}$$

– For Thevenin, we need the open circuit voltage. The embedded transformer is ideal, so the primary is an open circuit, too

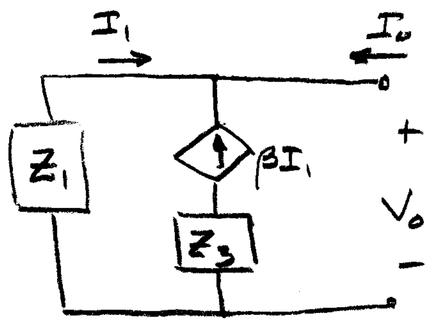


$$V_{oc} = \frac{sL_1}{R_1 + sL_1} V_i \cdot a = \frac{a s}{s + R_1/L_1} V_i(s)$$



• Obtaining the impedance with dependent sources requires some care. Generally it's V_o/I_o when independent sources are set to zero.

example 19.2 from DC+L – output of a common collector (emitter follower) stage



KCL:

$$I_1 + \beta I_1 + I_o = 0$$

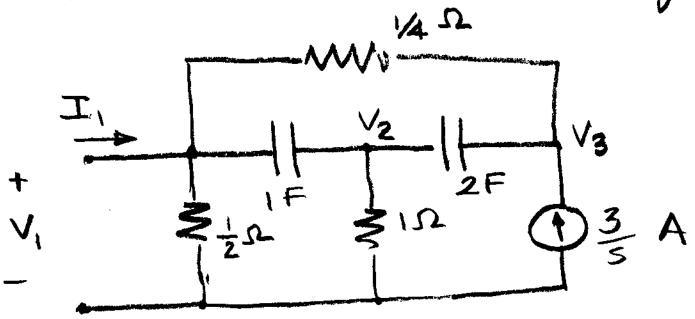
$$V_o = -I_1 Z_1$$

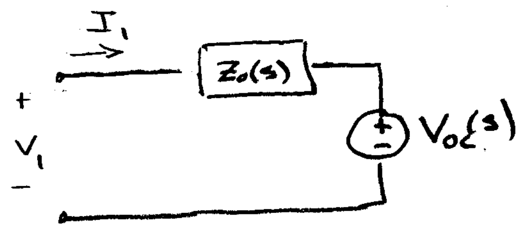
so $I_o = (1+\beta) \dfrac{V_o}{Z_1}$

and $Z_o(s) = \dfrac{Z_1(s)}{1+\beta}$

- For larger circuits, we need to be more systematic, and that means systems of equations and matrix analysis. Partitioned matrix inverses are very useful. Text does <u>example 19.5</u> based on loop equations, so we'll do node equations below.

- <u>example</u> Find a Thevenin equivalent of this circuit.





- Three node equations. Recall KCL is node volts times sum of all admittances impinging, minus each neighbour voltage times sum of connecting admittances.

node 1: $(s+\overset{2+4}{6})V_1 - sV_2 - 4V_3 = I_1$

node 2: $-sV_1 + (\underset{2+1}{3}s+1)V_2 - 2sV_3 = 0$

node 3: $-4V_1 - 2sV_2 + (2s+4)V_3 = 3/s$

$$\underbrace{\begin{bmatrix} s+6 & -s & -4 \\ -s & 3s+1 & -2s \\ -4 & -2s & 2s+4 \end{bmatrix}}_{W} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} I_1 \\ 0 \\ 3/s \end{bmatrix}$$

W symmetric
– no internal sources.

This is the matrix equation

$$W(s)\, \underline{V}(s) = \underline{I}(s)$$

Note $W(s)$ is symmetric.

Now what? All we want is the $V_1(s)$, $I_1(s)$ relationship, and we have a big matrix instead.

— We could approach it numerically. For any given ω, the arrays $W(j\omega)$ $\underline{I}(j\omega)$, $V(j\omega)$ just contain numbers, so solve by inversion (or Cramer's rule, or whatever...)

$$\underline{V}(j\omega) = W^{-1}(j\omega)\, I(j\omega) \qquad \text{define } U(s) = W^{-1}(s)$$

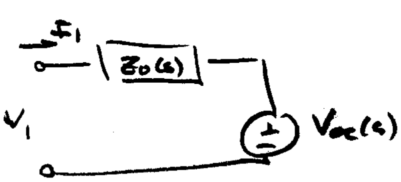
or

$$\begin{bmatrix} V_1(j\omega) \\ V_2(j\omega) \\ V_3(j\omega) \end{bmatrix} = \begin{bmatrix} U_{11}(j\omega) & U_{12}(j\omega) & U_{13}(j\omega) \\ U_{21}(j\omega) & U_{22}(j\omega) & U_{23}(j\omega) \\ U_{31}(j\omega) & U_{32}(j\omega) & U_{33}(j\omega) \end{bmatrix} \begin{bmatrix} I_1(j\omega) \\ 0 \\ 3/j\omega \end{bmatrix}$$

or

$$V_1(j\omega) = \underbrace{U_{11}(j\omega)}_{Z_0(j\omega)} I_1(j\omega) + \underbrace{U_{13}(j\omega)\, 3/j\omega}_{V_{oc}(j\omega)}$$

by comparison with



This lets us produce graphs of $Z_0(j\omega)$, $V_{oc}(j\omega)$ vs. ω, and it's not a bad way to go.

- Or, we could approach it symbolically, finding $W^{-1}(s)$ as a function of s.

  eg. if $W(s)$ were $\begin{bmatrix} s+1 & -s \\ -s & s \end{bmatrix}$ then $W^{-1}(s) = \begin{bmatrix} 1 & 1 \\ 1 & \frac{s+1}{s} \end{bmatrix}$

  This is <u>very</u> tedious by hand, even for 3×3. Even symbolic math packages are taxed for larger matrices, and the expressions they produce shed little light in this case.

- Even numerical inversion of an N×N matrix has computation load proportional to N^3. There is value in methods that reduce the number of equations or the size of the matrix being inverted.

– For example, sometimes we have a system of equations in several variables, but we are interested only in the solution for one of them.

See how it works for a simple 2×2 set, want V_1:

(a) $$w_{11}V_1 + w_{12}V_2 = I_1$$
$$w_{21}V_1 + w_{22}V_2 = I_2$$

$$\begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

(b) solve one equation (say, the second one) for the unwanted variable V_2:

$$V_2 = w_{22}^{-1}(I_2 - w_{21}V_1)$$
$$= -w_{22}^{-1}w_{21}V_1 + w_{22}^{-1}I_2$$

$$w_{22}^{-1} = \frac{1}{w_{22}}$$

(c) substitute the unwanted variable in the remaining equation:

$$w_{11}V_1 + w_{12}(-w_{22}^{-1}w_{21}V_1 + w_{22}^{-1}I_2) = I_1$$

(d) and solve for the desired variable V_1:

$$(w_{11} - w_{12}w_{22}^{-1}w_{21})V_1 = I_1 - w_{12}w_{22}^{-1}I_2$$

so
$$V_1 = (w_{11} - w_{12}w_{22}^{-1}w_{21})^{-1}(I_1 - w_{12}w_{22}^{-1}I_2)$$

or, more simply,

$$V_1 = \frac{I_1 - \frac{w_{12}}{w_{22}}I_2}{w_{11} - \frac{w_{12}w_{21}}{w_{22}}}$$

– We can apply the same method to larger sets of equations, in which we want one set of variables, and don't care about others. In the case at hand (top of p.7.1.4), we have $W(s)\underline{V}(s) = \underline{I}(s)$ but all we want is the $V_1(s)$, $I_1(s)$ relationship.

So write the equations in partitioned form, to eliminate V_2, V_3:

$$\left[\begin{array}{c|c} W_{11} & W_{12} \\ \hline W_{21} & W_{22} \end{array}\right] \left[\begin{array}{c} V_1 \\ \hline V_2 \\ V_3 \end{array}\right] = \left[\begin{array}{c} I_1 \\ \hline 0 \\ 3/s \end{array}\right] = \left[\begin{array}{c} I_1 \\ \hline I_2 \\ I_3 \end{array}\right]$$

compare with p. 7.1.3

Now it's

$$W_{11} V_1 + W_{12} \begin{bmatrix} V_2 \\ V_3 \end{bmatrix} = I_1$$

$$W_{21} V_1 + W_{22} \begin{bmatrix} V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} I_2 \\ I_3 \end{bmatrix}$$

<u>dimensions</u>

W_{11}

W_{12}

W_{21}

W_{22}

For nonsingular W_{22}, its inverse exists. Solve for $\begin{bmatrix} V_2 \\ V_3 \end{bmatrix}$ and substitute into first equation – and it's gone!

$$W_{22}\begin{bmatrix} V_2 \\ V_3 \end{bmatrix} = -W_{21} V_1 + \begin{bmatrix} I_2 \\ I_3 \end{bmatrix}$$

$$\begin{bmatrix} V_2 \\ V_3 \end{bmatrix} = -W_{22}^{-1} W_{21} V_1 + W_{22}^{-1} \begin{bmatrix} I_2 \\ I_3 \end{bmatrix}$$

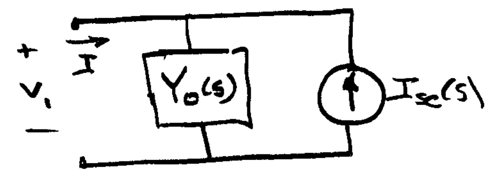
Substitute

$$W_{11} V_1 + W_{12}\left(-W_{22}^{-1} W_{21} V_1 + W_{22}^{-1}\begin{bmatrix} I_2 \\ I_3 \end{bmatrix}\right) = I_1$$

or

$$I_1(s) = \underbrace{\left(W_{11} - W_{12}W_{22}^{-1}W_{21}\right)}_{Y_o(s)} V_1 + \underbrace{W_{12}W_{22}^{-1}\begin{bmatrix} I_2 \\ I_3 \end{bmatrix}}_{-I_{sc}(s)}$$

Norton model



or

$$V_1(s) = \underbrace{\frac{1}{W_{11} - W_{12}W_{22}^{-1}W_{21}}}_{Z_o(s)} I(s) - \underbrace{\frac{W_{12}W_{22}^{-1}\begin{bmatrix} I_2 \\ I_3 \end{bmatrix}}{W_{11} - W_{12}W_{22}^{-1}W_{21}}}_{V_{oc}(s)}$$

Thevenin

- Now let's do it and see how bad it gets.

$$W_{11} = s + 6 \qquad W_{12} = \begin{bmatrix} -s & -4 \end{bmatrix}$$

$$W_{21} = \begin{bmatrix} -s \\ -4 \end{bmatrix} \qquad W_{22} = \begin{bmatrix} 3s+1 & -2s \\ -2s & 2s+4 \end{bmatrix}$$

The sole inversion is W_{22}, and it is

$$W_{22}^{-1} = \frac{1}{s^2+7s+2}\begin{bmatrix} s+2 & s \\ s & \frac{3s+1}{2} \end{bmatrix}$$

This makes the input admittance

$$Y_o(s) = s+6 - \begin{bmatrix} -s & -4 \end{bmatrix} \begin{bmatrix} s+2 & s \\ s & \frac{3s+1}{2} \end{bmatrix} \begin{bmatrix} -s \\ -4 \end{bmatrix} \cdot \frac{1}{s^2+7s+2}$$

$$= \frac{3s^2+20s+4}{s^2+7s+2}$$

and impedance

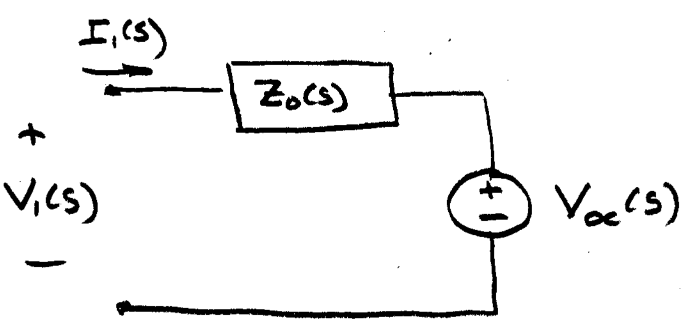
$$Z_o(s) = \frac{s^2+7s+2}{3s^2+20s+4}$$

As for the source,

$$V_{oc}(s) = -\frac{W_{12} W_{22}^{-1}}{Y_o(s)} \begin{bmatrix} I_2 \\ I_3 \end{bmatrix}$$

$$= -\frac{1}{3s^2+20s+4} \cdot \begin{bmatrix} -s & -4 \end{bmatrix} \begin{bmatrix} s+2 & s \\ s & \frac{3s+1}{2} \end{bmatrix} \begin{bmatrix} 0 \\ 3/s \end{bmatrix}$$

$$V_{oc}(s) = \frac{3(s^2+6s+2)}{s(3s^2+20s+4)}$$

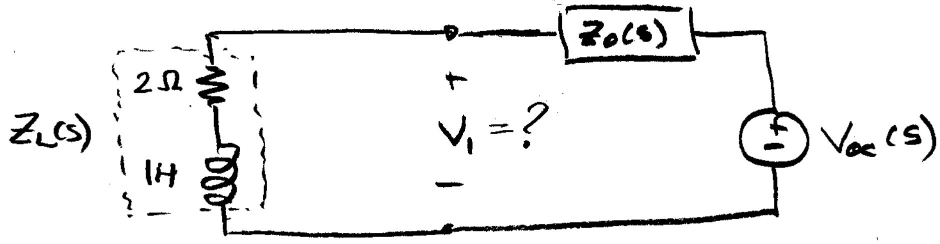


$\boxed{\text{Now read DC+L example 19.5}}$

- The manipulations are cumbersome, but typical of larger circuits — and we didn't even use R, L, C symbols! They illustrate the value of:
  - symbolic math packages
  - circuit analysis and simulation packages.

- However, once we have obtained the expressions for $Z_o(s)$, $V_{oc}(s)$ or $Y_o(s)$, $I_{sc}(s)$, we don't have to re-solve the circuit when we connect something to it.

For example, connect a load with $Z_L = s+2$



$$V_1(s) = \frac{Z_L(s)}{Z_L(s) + Z_o(s)} \cdot V_{oc}(s)$$

The $Z_o(s)$ and $V_{oc}(s)$ expressions <u>summarize</u> a reasonably complex circuit.