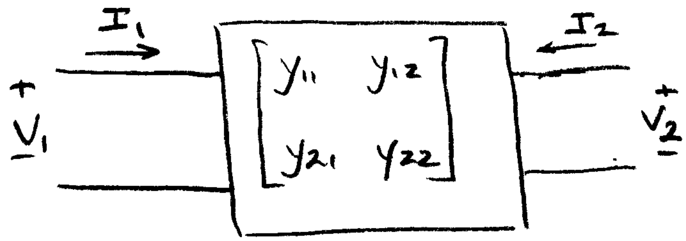
7.2 Admittance Parameter Analysis

DC+L 19.4, 19.5

- One way to characterize a two-port is with admittance parameters ("y-parameters")



$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \quad \text{or} \quad \begin{aligned} I_1 &= y_{11} V_1 + y_{12} V_2 \\ I_2 &= y_{21} V_1 + y_{22} V_2 \end{aligned}$$

where the elements are "short circuit admittances":

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \qquad y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$

units: Siemens

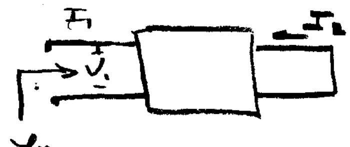
$$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} \qquad y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$

- We'll examine:
  - how to obtain y params from circuit diagram
  - how to calculate input impedance if two port is connected to load
  - how to calculate output impedance if two port is connected to source
  - how to calculate voltage gain if connected to source and load.

- Obtaining y-parameters from circuit diagram

— We can approach it in a couple of ways:

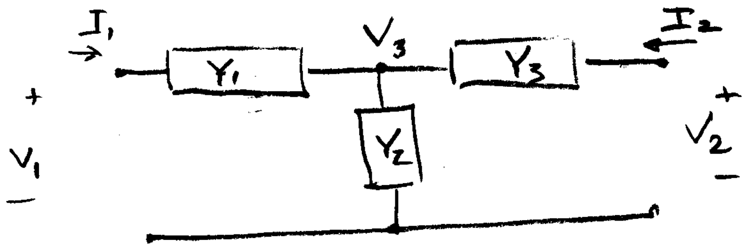
→ Work with the "short circuit" definitions explicitly; e.g.



and solve by circuit reduction or node, loop equations.

→ Or for a little more work, do all at once with a set of nodal equations.

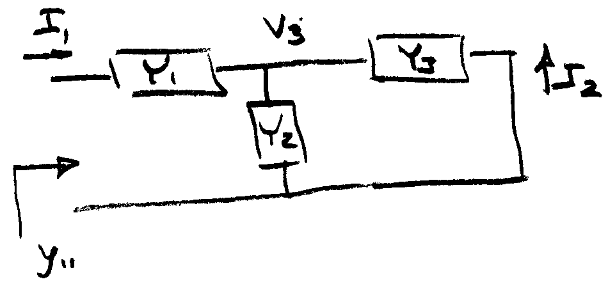
— Simple example — passive circuit



Y's are functions of s

By circuit reduction:

$$y_{11} = \frac{Y_1(Y_2+Y_3)}{Y_1+Y_2+Y_3}$$



$$y_{21} = \frac{I_2}{V_1} = -\frac{I_2}{V_3}\cdot\frac{V_3}{V_1} = -Y_3 \cdot \frac{\frac{1}{Y_2+Y_3}}{\frac{1}{Y_1}+\frac{1}{Y_2+Y_3}} = \frac{-Y_1 Y_3}{Y_1+Y_2+Y_3}$$

and so on.

– By nodal analysis, same circuit

$$I_1 = Y_1 V_1 \qquad\qquad - Y_1 V_3$$

$$I_2 = \qquad\quad Y_3 V_2 - Y_3 V_3$$

$$0 = -Y_1 V_1 - Y_3 V_2 + (Y_1+Y_2+Y_3) V_3$$

Note nodal equations lend themselves to admittance analysis

or

$$\begin{bmatrix} I_1 \\ I_2 \\ 0 \end{bmatrix} = \begin{bmatrix} Y_1 & 0 & -Y_1 \\ 0 & Y_3 & -Y_3 \\ -Y_1 & -Y_3 & Y_1+Y_2+Y_3 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

All we want is equations in $\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$, $\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$

so we should eliminate V_3 (and any other interior nodes, if it's a more complicated circuit).

Again, partition the matrix, for two sets of equations

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_1 & 0 \\ 0 & Y_3 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} + \begin{bmatrix} -Y_1 \\ -Y_3 \end{bmatrix} V_3$$

$$0 = \begin{bmatrix} -Y_1 & -Y_3 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} + \begin{bmatrix} Y_1+Y_2+Y_3 \end{bmatrix} V_3$$

← solve for V_3, subst into first eq'n.

Solve 2nd equation for unwanted variable(s):

$$V_3 = -(Y_1+Y_2+Y_3)^{-1} \begin{bmatrix} -Y_1 & -Y_3 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

and substitute:

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_1 & 0 \\ 0 & Y_3 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} - \begin{bmatrix} -Y_1 \\ -Y_3 \end{bmatrix}(Y_1+Y_2+Y_3)^{-1}\begin{bmatrix} -Y_1 & -Y_3 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$= \left(\begin{bmatrix} Y_1 & 0 \\ 0 & Y_3 \end{bmatrix} - \frac{1}{Y_1+Y_2+Y_3}\begin{bmatrix} Y_1^2 & Y_1Y_3 \\ Y_3Y_1 & Y_3^2 \end{bmatrix} \right)\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

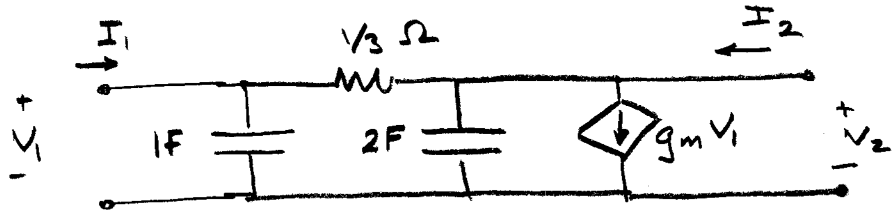
$$= \frac{1}{Y_1+Y_2+Y_3}\begin{bmatrix} Y_1Y_2+Y_1Y_3 & -Y_1Y_3 \\ -Y_3Y_1 & Y_1Y_3+Y_2Y_3 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$= \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Done. It agrees with calculation on previous page.

> Now read DC+L example 19.7. Erratum: fourth eq'n p.809 should not contain [s+3][s+3].

- example with controlled source (like DC+L ex 19.6)



an easier example

Nodal equations again:

$$I_1 = sV_1 + 3(V_1 - V_2) = (s+3)V_1 - 3V_2$$

$$I_2 = g_m V_1 + 2s V_2 + 3(V_2 - V_1) = (g_m - 3)V_1 + (2s+3)V_2$$

or

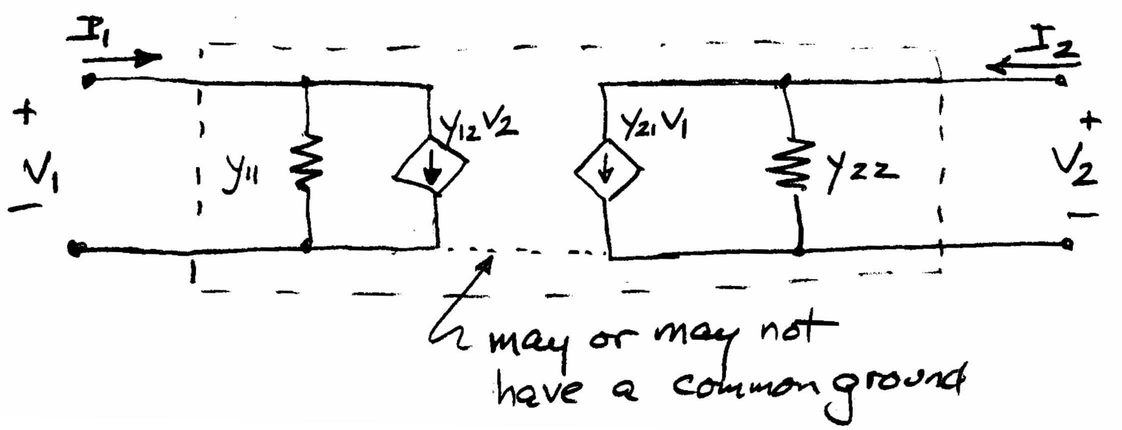
$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \underbrace{\begin{bmatrix} s+3 & -3 \\ g_m-3 & 2s+3 \end{bmatrix}}_{\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Now read DC+L ex 19.8, with transformer.

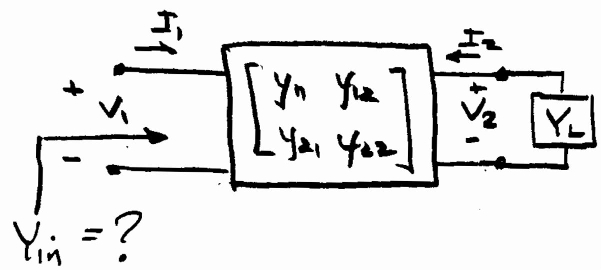
– At this point, we have the tools to reduce any two-port circuit to the y parameters.

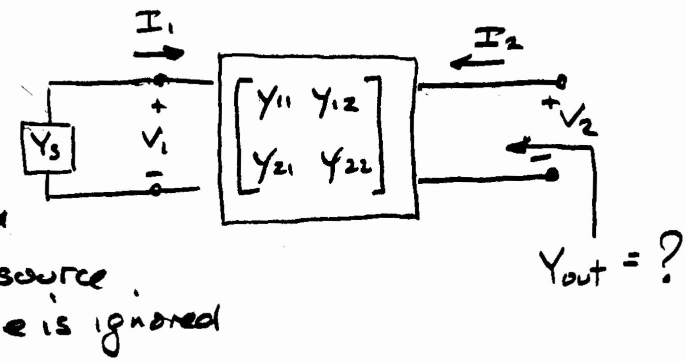
- A dependent-source model of a two-port circuit:

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \quad \text{or} \quad \begin{aligned} I_1 &= y_{11}V_1 + y_{12}V_2 \\ I_2 &= y_{21}V_1 + y_{22}V_2 \end{aligned}$$



- Calculating input and output admittances



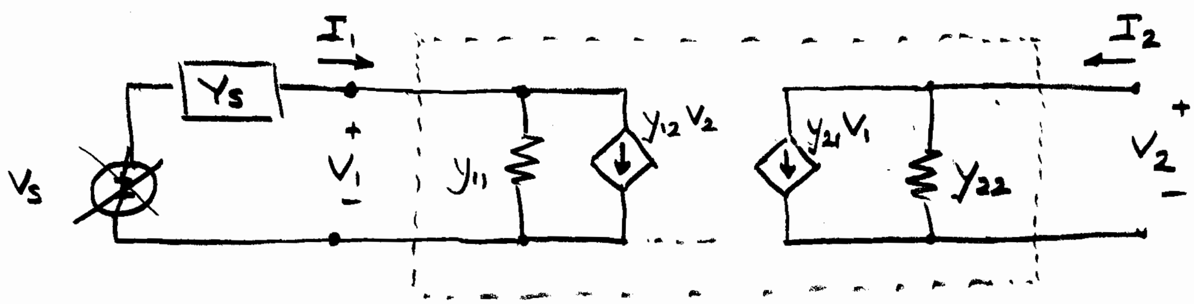


– DC+L shows input admittance

$$Y_{in} = y_{11} - \frac{y_{12}\, y_{21}}{y_{22} + Y_L}$$

note pattern of subscripts

– We can use this and symmetry to obtain Y_{out}. Instead, do it ground-up, for the I_2, V_2 relation.



① Find V_1 on input side

$$y_{12} V_2 = -V_1 \left(y_{11} + Y_s \right)$$

or
$$V_1 = -V_2 \frac{y_{12}}{y_{11} + Y_s}$$

② Eliminate V_1 on output side

$$I_2 = y_{22} V_2 + y_{21} V_1$$

$$= y_{22} V_2 - \frac{y_{12} y_{21}}{y_{11} + Y_s} V_2$$

$$= \left(y_{22} - \frac{y_{12} y_{21}}{y_{11} + Y_s} \right) V_2$$

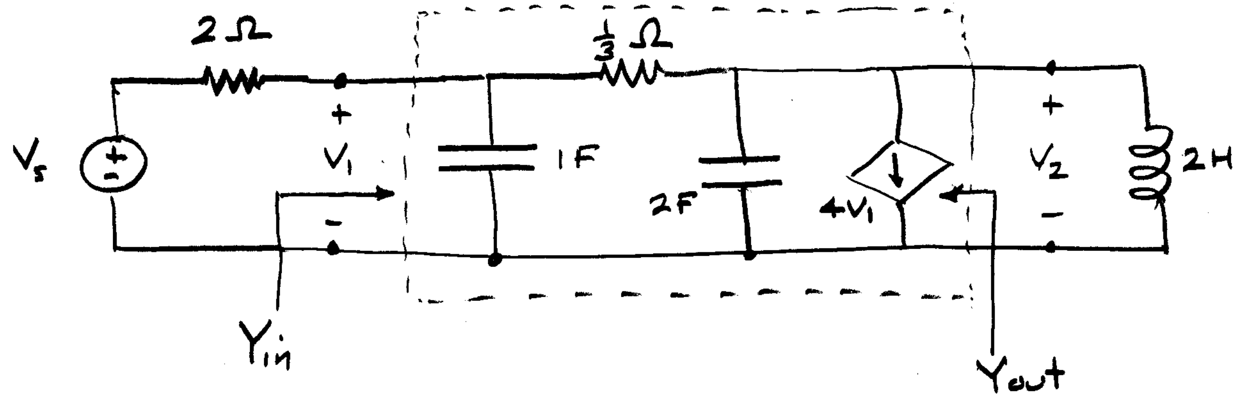
③

$$Y_{out} = y_{22} - \frac{y_{12} y_{21}}{y_{11} + Y_s}$$

note pattern of subscripts.

– <u>example</u> – the circuit of p. 7.2.4 with $g_m = 4$

Add a source and a load.



$Y_s = \frac{1}{2}$ $\qquad \begin{bmatrix} s+3 & -3 \\ 1 & 2s+3 \end{bmatrix}$ $\qquad Y_L = \frac{1}{2s}$

$$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$$

Having done the work to obtain the admittance parameters of the two port, we don't have to solve it again.

$$Y_{in}(s) = y_{11} - \frac{y_{12}\,y_{21}}{y_{22} + Y_L} = s+3 - \frac{-3}{2s+3+\frac{1}{2s}}$$

indep of Y_s !

$$= \frac{4s^3 + 18s^2 + 25s + 3}{4s^2 + 6s + 1}$$

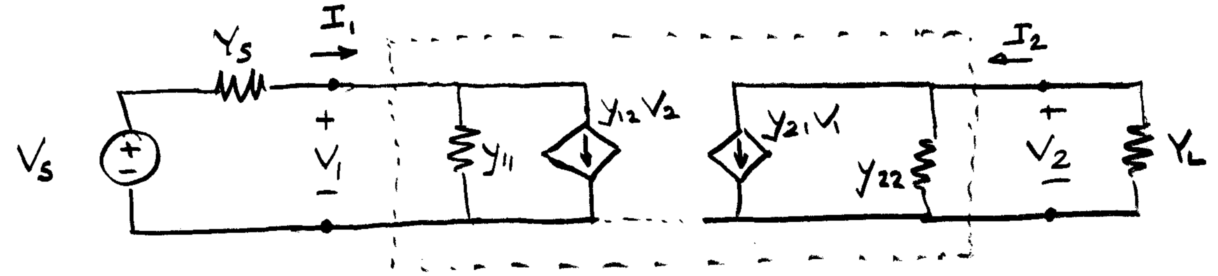
$$Y_{out}(s) = y_{22} - \frac{y_{12}\,y_{21}}{y_{11} + Y_s} = 2s+3 - \frac{-3}{s+3+\frac{1}{2}}$$

indep of Y_L !

$$= \frac{4s^2 + 20s + 27}{2s+7}$$

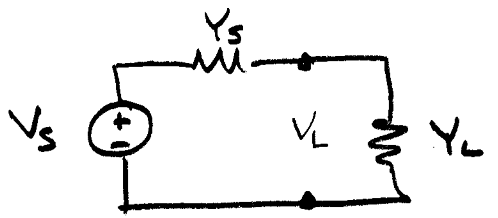
- Calculating the voltage gain

– Attach a source and a load.



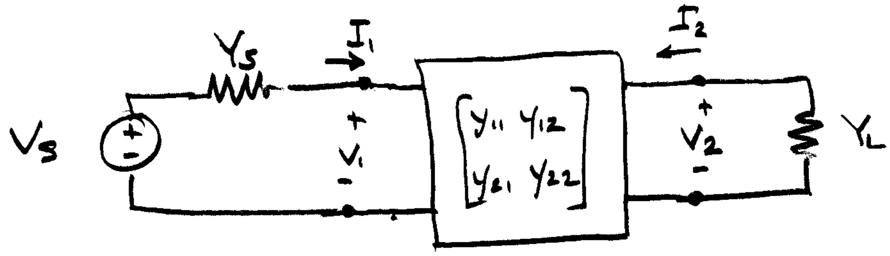
– What is the voltage gain $G_v = \frac{V_2}{V_s}$?

– This is a reasonable question. We are used to asking it when we connect a source to a load:



$$V_L = \frac{Y_s}{Y_s + Y_L} V_s$$

Now we have a two port (e.g. amplifier, filter, etc) inserted between them



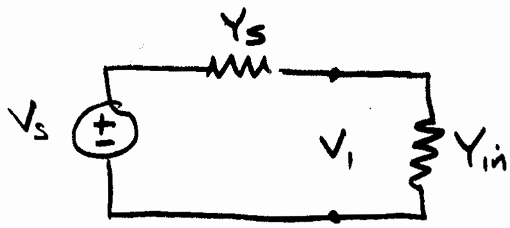
$$G_v = \frac{V_2}{V_s} = ?$$

– Following DC+L p. 812:

$$G_v = \underbrace{\frac{V_2}{V_1}}_{G_{v_2}} \cdot \underbrace{\frac{V_1}{V_s}}_{G_{v_1}}$$ and calculate them separately.

DC+L erratum: p. 813, first equation should be $G_{v_1} = \frac{V_1}{V_s}$

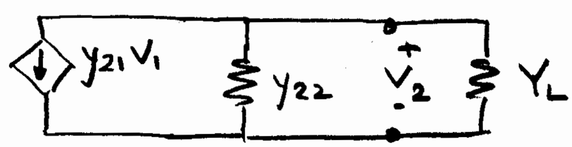
First, $\frac{V_1}{V_s}$. Source sees



We have this, p. 7.2.7

$$G_{v_1} = \frac{V_1}{V_s} = \frac{Y_s}{Y_s + Y_{in}}$$ and Y_{in} depends on Y_L

Next, $\frac{V_2}{V_1}$. Load sees



So $V_2 = -\frac{y_{21}V_1}{y_{22}+Y_L}$, $G_{v_2} = \frac{-y_{21}}{y_{22}+Y_L}$

Finally

$$G_v = G_{v_1} \cdot G_{v_2} = \left(\frac{Y_s}{Y_s+Y_{in}}\right) \cdot \left(\frac{-y_{21}}{y_{22}+Y_L}\right)$$

and $Y_{in} = y_{11} - \frac{y_{12}y_{21}}{y_{22}+Y_L}$

$$= \frac{-y_{21}Y_s}{(y_{22}Y_s + Y_sY_L + y_{11}Y_L + y_{11}y_{22} - y_{12}y_{21}}$$

- These are intricate equations, and we might reasonably ask why we should bother with them.

  Answer: the four parameters and the source & load admittances (all functions of s) are a high-level summary of what can be elaborate circuits. They provide a "birds-eye" view of what's going on.