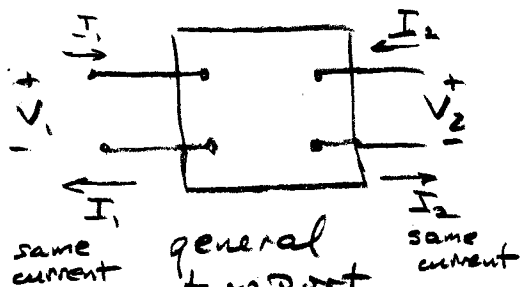
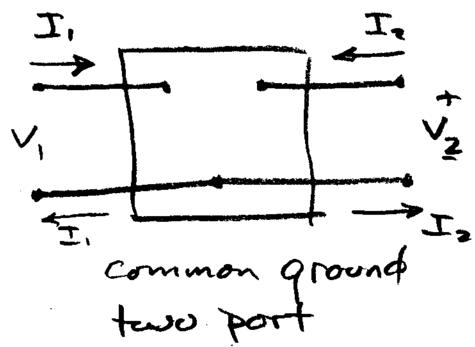
7.4 Elementary Connection of Two Ports

DC+L Chapt 20

- We have already seen two-ports connected to sources and to loads. More generally, we can connect two ports to each other, giving a larger two port — but what are its parameters?

- Distinguish two kinds of two port:

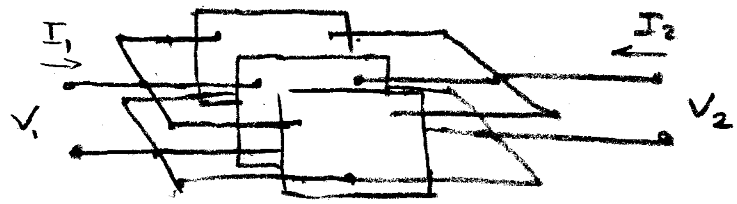




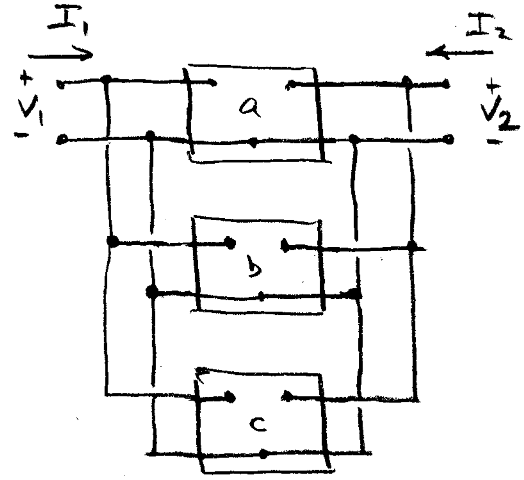
Note that, in both cases, the same current flows in on one connection and out on the other at a given port.

- We'll consider only common ground two ports in this section

- We could connect them in parallel

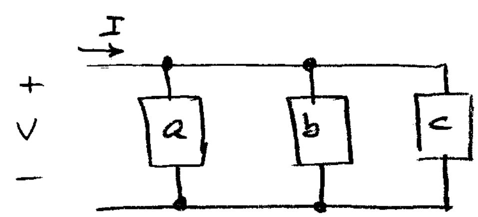


or



They all have the same V_1, V_2 but the currents I_1, I_2 split among the networks.

Like one-ports



Denote the admittance matrices (y-params) as Y_a, Y_b, Y_c, and

$$\underline{I}_a = Y_a \underline{V}_a$$

$$\underline{I}_b = Y_b \underline{V}_b$$

$$\underline{I}_c = Y_c \underline{V}_c$$

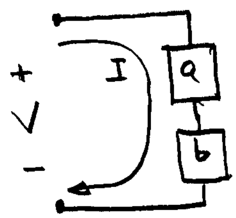
but $\underline{V}_a = \underline{V}_b = \underline{V}_c = \underline{V} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$

and $\underline{I}_a + \underline{I}_b + \underline{I}_c = \underline{I} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$

$$\underline{I} = \underbrace{(Y_a + Y_b + Y_c)}_{\text{overall } Y \text{ of parallel units}} \underline{V}$$

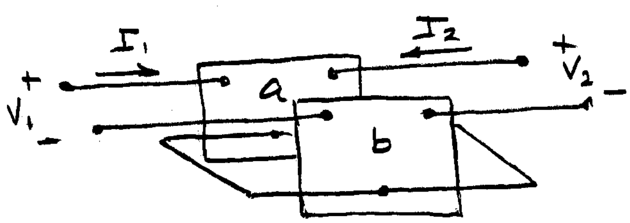
$\longleftarrow$ but not always true for general two-ports

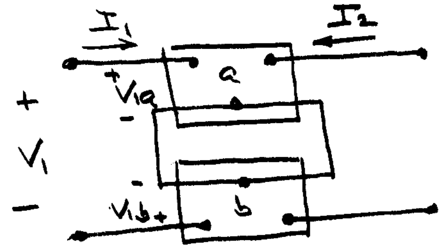
- What about series connections?
  - By analogy with one-ports



They all have the same current, in and out, and the overall voltage is the sum of the individual voltages.

we connect like this





They must be connected ground-to-ground to ensure current out of each port of individual equals the current in. $\boxed{\text{DC+L, ex 20.1}}$. This limits the stack to two modules.

Here, $\underline{V} = \underline{V_a} - \underline{V_b}$

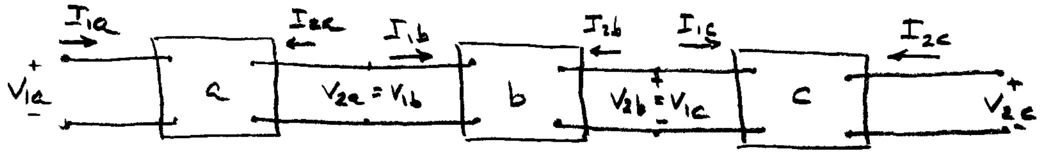
and $\underline{\underline{I}} = \underline{\underline{I_a}} = -\underline{\underline{I_b}}$

But $-\underline{V_b} = Z_b(-\underline{\underline{I_b}})$ since $\underline{V_b} = Z_b \underline{\underline{I_b}}$

Hence $\underline{V} = Z_a \underline{\underline{I}} + Z_b \underline{\underline{I}}$

and $Z = Z_a + Z_b$

- Now for the very common cascade connection.



$$\begin{bmatrix} V_{1a} \\ V_{2a} \end{bmatrix} = Z_a \begin{bmatrix} I_{1a} \\ I_{2a} \end{bmatrix}, \quad \begin{bmatrix} V_{1b} \\ V_{2b} \end{bmatrix} = Z_b \begin{bmatrix} I_{1b} \\ I_{2b} \end{bmatrix}, \quad \begin{bmatrix} V_{1c} \\ V_{2c} \end{bmatrix} = Z_c \begin{bmatrix} I_{1c} \\ I_{2c} \end{bmatrix}$$

The overall Z matrix is... umm...

There is an expression, but neither z params nor y params lend themselves well to cascade.

- Why not? Because each vector contains quantities from both ports — but what adjacent networks have in common are current and voltage at a single port; e.g.

$$\begin{bmatrix} V_{2a} \\ I_{2a} \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} V_{2b} \\ I_{2b} \end{bmatrix}$$

- This motivates the h (hybrid) and t (transmission) parameters, which relate variables at one port to those at another.