7.5 Mixed Variables: the h and t Parameter Sets

$\boxed{\text{DC+L } 19.7, 19.9}$

- Some circuits have neither z nor y parameters.
  Examples: ideal transformer DC+L, ex 19.13, DC+L, ex 19.14.

- The h parameters group the V, I variables at the two ports differently — and they may exist when the others do not, which at least allows a circuit model.

- Here they are:

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

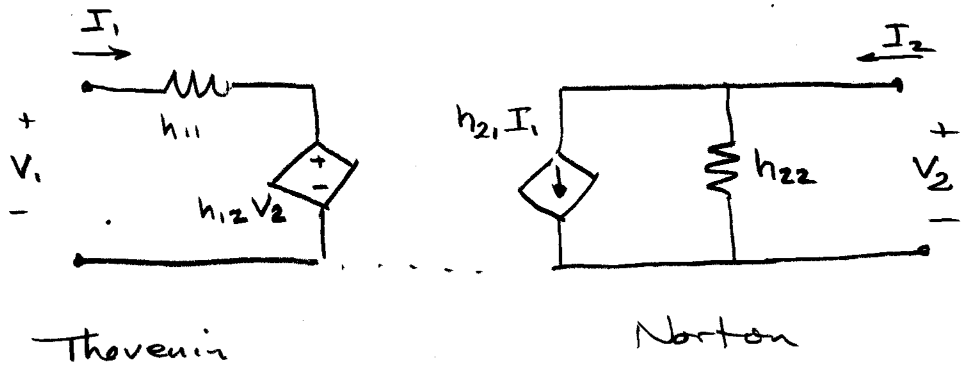
– first component of the vectors is from side 1, as in y, z.

$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}$ short circuit input impedance, ohms

$h_{12} = V_1/V_2\big|_{I_1=0}$ reverse open circ. voltage gain, dimensionless

$h_{21} = I_2/I_1\big|_{V_2=0}$ forward short circuit current gain, dimensionless

$h_{22} = I_2/V_2\big|_{I_1=0}$ open circuit output admittance, Siemens



- Although mixed I, V, the vector components are drawn from different ports, so no progress toward cascade connection
- Now read DC+L ex 19.15, 19.16 for calculation of h parameters for circuits containing transformers.

- Calculation of h-parameters from a circuit diagram can be done in a few ways; two are:
  - work from the definitions; short or open circuit one or the other port; apply circuit reduction or node/loop eq'ns to get each of the four ratios;
  - or, since most circuits have y or z parameters, calculate them and transform to h params

Note DC+L error p 823, 2nd eq'n from bottom, should be $G_2 V_2$

- Conversion of z params to h params shows the pattern for all these conversions

$$\begin{tikzcd} z \arrow[rr, leftrightarrow] \arrow[dr, leftrightarrow] & & y \arrow[dl, leftrightarrow] \\ & h & \end{tikzcd}$$

Start with and rearrange to

$$V_1 = z_{11} I_1 + z_{12} I_2 \qquad \longrightarrow \qquad V_1 - z_{12} I_2 = z_{11} I_1$$

$$V_2 = z_{21} I_1 + z_{22} I_2 \qquad\qquad z_{22} I_2 = -z_{21} I_1 + V_2$$

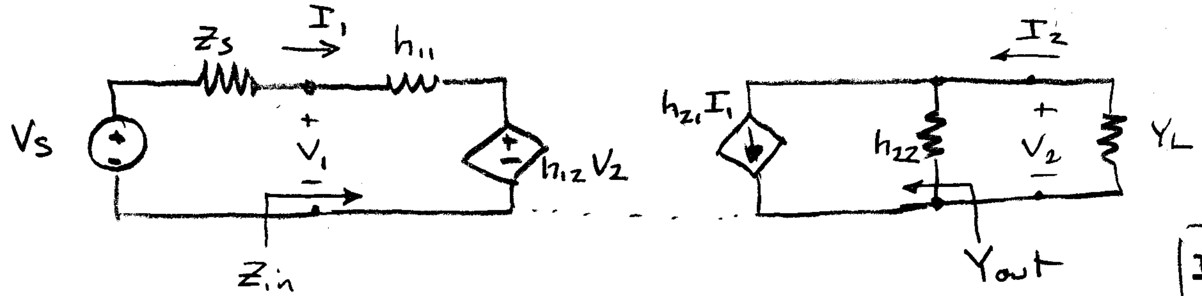
$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & -z_{12} \\ 0 & z_{22} \end{bmatrix}^{-1} \begin{bmatrix} z_{11} & 0 \\ -z_{21} & 1 \end{bmatrix}}_{\text{H matrix}} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} \longleftarrow \begin{bmatrix} 1 & -z_{12} \\ 0 & z_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} z_{11} & 0 \\ -z_{21} & 1 \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \frac{1}{z_{22}} \begin{bmatrix} z_{11} z_{22} - z_{12} z_{21} & z_{12} \\ -z_{21} & 1 \end{bmatrix}$$

TABLE 19.1 Interrelations among Two-Port Parameters, where γ Can Stand for z, y, h, or t in $\Delta\gamma = \gamma_{11}\gamma_{22} - \gamma_{12}\gamma_{21}$

	z-Parameters	*y*-Parameters	*h*-Parameters	*t*-Parameters
z-parameters	$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{y_{22}}{\Delta y} & \frac{-y_{12}}{\Delta y} \\ \frac{-y_{21}}{\Delta y} & \frac{y_{11}}{\Delta y} \end{bmatrix}$	$\begin{bmatrix} \frac{\Delta h}{h_{22}} & \frac{h_{12}}{h_{22}} \\ \frac{-h_{21}}{h_{22}} & \frac{1}{h_{22}} \end{bmatrix}$	$\begin{bmatrix} \frac{t_{11}}{t_{21}} & \frac{\Delta t}{t_{21}} \\ \frac{1}{t_{21}} & \frac{t_{22}}{t_{21}} \end{bmatrix}$
y-parameters	$\begin{bmatrix} \frac{z_{22}}{\Delta z} & \frac{-z_{12}}{\Delta z} \\ \frac{-z_{21}}{\Delta z} & \frac{z_{11}}{\Delta z} \end{bmatrix}$	$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{h_{11}} & \frac{-h_{12}}{h_{11}} \\ \frac{h_{21}}{h_{11}} & \frac{\Delta h}{h_{11}} \end{bmatrix}$	$\begin{bmatrix} \frac{t_{22}}{t_{12}} & \frac{-\Delta t}{t_{12}} \\ \frac{-1}{t_{12}} & \frac{t_{11}}{t_{12}} \end{bmatrix}$
h-parameters	$\begin{bmatrix} \frac{\Delta z}{z_{22}} & \frac{z_{12}}{z_{22}} \\ \frac{-z_{21}}{z_{22}} & \frac{1}{z_{22}} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{y_{11}} & \frac{-y_{12}}{y_{11}} \\ \frac{y_{21}}{y_{11}} & \frac{\Delta y}{y_{11}} \end{bmatrix}$	$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{t_{12}}{t_{22}} & \frac{\Delta t}{t_{22}} \\ \frac{-1}{t_{22}} & \frac{t_{21}}{t_{22}} \end{bmatrix}$
t-parameters	$\begin{bmatrix} \frac{z_{11}}{z_{21}} & \frac{\Delta z}{z_{21}} \\ \frac{1}{z_{21}} & \frac{z_{22}}{z_{21}} \end{bmatrix}$	$\begin{bmatrix} \frac{-y_{22}}{y_{21}} & \frac{-1}{y_{21}} \\ \frac{-\Delta y}{y_{21}} & \frac{-y_{11}}{y_{21}} \end{bmatrix}$	$\begin{bmatrix} \frac{-\Delta h}{h_{21}} & \frac{-h_{11}}{h_{21}} \\ \frac{-h_{22}}{h_{21}} & \frac{-1}{h_{21}} \end{bmatrix}$	$\begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix}$

- When we terminate a two port described by h-parameters, we must be able to calculate input & output impedances, voltage gain.



DC+L p 826

$$Z_{in} = h_{11} - \frac{h_{12}h_{21}}{h_{22} + Y_L} \qquad Y_{out} = h_{22} - \frac{h_{12}h_{21}}{h_{11} + Z_s}$$

They follow the same pattern as that of the immitance generalization p. 7.3.7, and are dimensionally consistent.

Also $\quad G_v = \dfrac{V_2}{V_s} = -\dfrac{1}{Z_{in}+Z_s} \cdot \dfrac{h_{21}}{h_{22}+Y_L}$

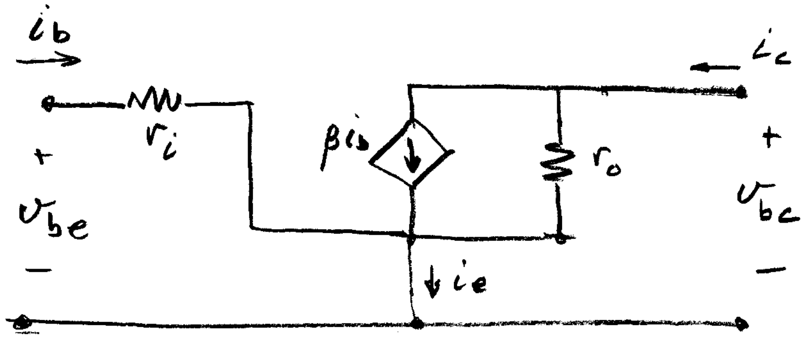
- The h parameters are not useful for any of the two-port connection patterns (parallel, series, cascade). Consequently, circuit simulators, which perform circuit reduction by exploiting the connection patterns, make little use of h-parameters.

So why were they defined?

- The h params arose from analysis of transistors. A fundamental attribute of the BJT is current gain ($i_c = \beta i_b$), so if we want it as one of the four parameters, we have I_2 on the left, I_1 on the right. That leaves V_1, V_2, one on each side – but which side? Put V_2 on the right, since it affects I_2, also.

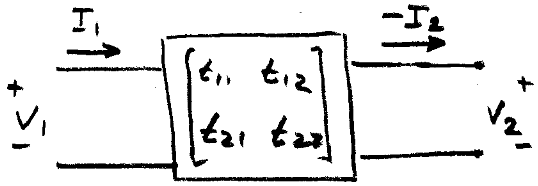
Symmetry gives

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$



small signal NPN BJT.

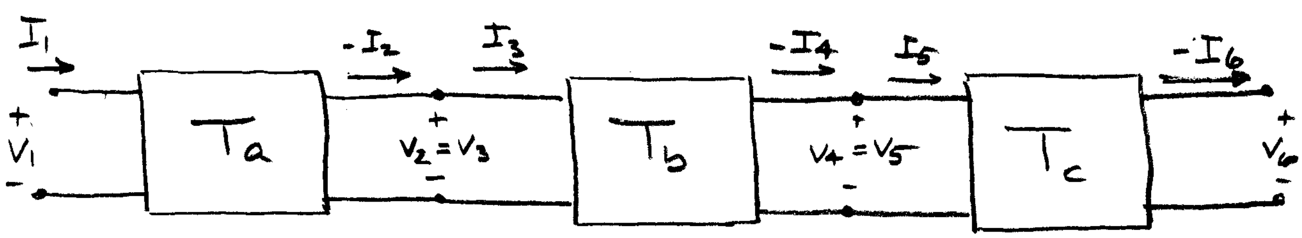
- Our quest for parameters useful in the cascade connection leads us finally to the transmission (t-) parameters. (ABCD params)



$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

each vector contains quantities from a single port

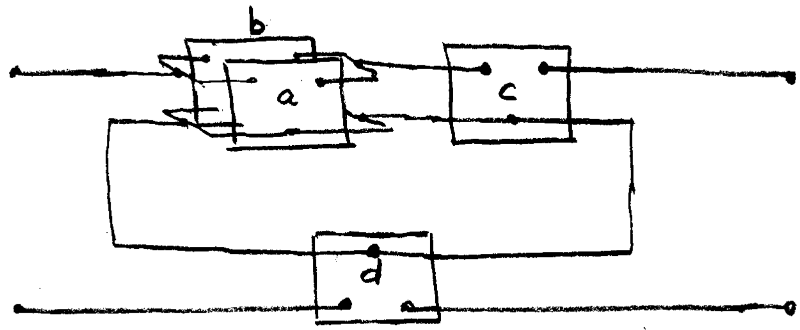
Why the sign reversal on I_2? It's so the two ports can be plugged into each other with a common interface vector.



$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = T_a \underbrace{\begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}}_{= \begin{bmatrix} V_3 \\ I_3 \end{bmatrix}} = T_a T_b \underbrace{\begin{bmatrix} V_4 \\ -I_4 \end{bmatrix}}_{= \begin{bmatrix} V_5 \\ I_5 \end{bmatrix}} = T_a T_b T_c \begin{bmatrix} V_6 \\ -I_6 \end{bmatrix}$$

- Now we have a full set of characterizations useful in circuit reduction:
  - combine parallel two ports by adding their Y (admittance) matrices;
  - combine series two ports by adding their Z (impedance) matrices;
  - combine cascaded two ports by multiplying their T (transmission) matrices

Simulators frequently switch between parameters during circuit reduction:



- Form $Y_a + Y_b = Y_{sum}$
- Convert to T_{sum}
- Form $T_{prod} = T_{sum} T_c$
- Convert to Z_{prod}
- Form $Z_{prod} + Z_d$

- So let's do one of those conversions.

Try T to Z:

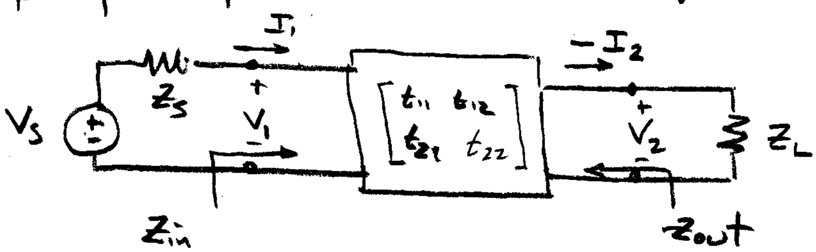
$$\begin{aligned} V_1 &= t_{11} V_2 - t_{12} I_2 \\ I_1 &= t_{21} V_2 - t_{22} I_2 \end{aligned} \longrightarrow \begin{aligned} V_1 - t_{11} V_2 &= -t_{12} I_2 \\ t_{21} V_2 &= I_1 + t_{22} I_2 \end{aligned}$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & -t_{11} \\ 0 & t_{21} \end{bmatrix}^{-1} \begin{bmatrix} 0 & -t_{12} \\ 1 & t_{22} \end{bmatrix}}_{Z} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \longleftarrow \begin{bmatrix} 1 & -t_{11} \\ 0 & t_{21} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 & -t_{12} \\ 1 & t_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$Z = \frac{1}{t_{21}} \begin{bmatrix} t_{11} & -t_{12}t_{21} + t_{11}t_{22} \\ 1 & t_{22} \end{bmatrix}$$

See p. 7.5.3 for others,

- Input, output impedances?



note $V_2 = -Z_L I_2$

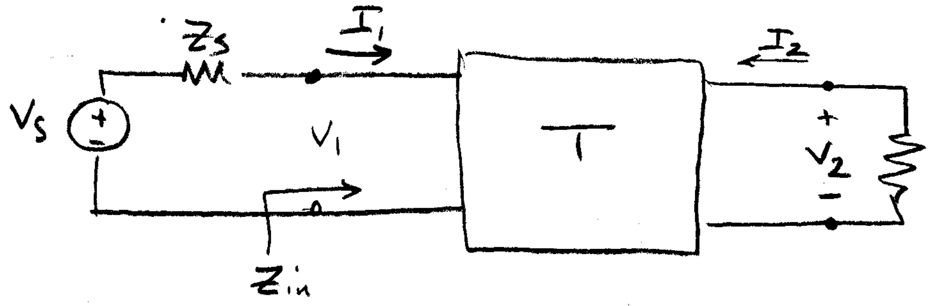
$$V_1 = t_{11} V_2 - t_{12} I_2 = -t_{11} Z_L I_2 - t_{12} I_2$$

$$I_1 = t_{21} V_2 - t_{22} I_2 = \underbrace{-t_{21} Z_L I_2 - t_{22} I_2}$$

So

$$Z_{in} = \frac{V_1}{I_1} = \frac{t_{11} Z_L + t_{12}}{t_{21} Z_L + t_{22}}, \qquad Z_{out} = \frac{t_{22} Z_s + t_{12}}{t_{21} Z_s + t_{11}}$$

- The voltage gain is a little messier than with the other parameters



As usual, $V_1 = \dfrac{Z_{in}}{Z_{in}+Z_s} V_s$, $I_1 = \dfrac{V_1}{Z_{in}}$

so $$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} Z_{in} \\ 1 \end{bmatrix} \frac{V_s}{Z_{in}+Z_s}$$

Next $$\begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} = T^{-1} \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \frac{1}{|T|} \begin{bmatrix} t_{22} & -t_{12} \\ -t_{21} & t_{11} \end{bmatrix} \begin{bmatrix} V_1 \\ I_1 \end{bmatrix}$$

so $$G_v = \frac{V_2}{V_s} = \frac{t_{22} Z_{in} - t_{12}}{|T| \cdot (Z_s + Z_{in})}$$

- And this is as far as we go with t-parameters.