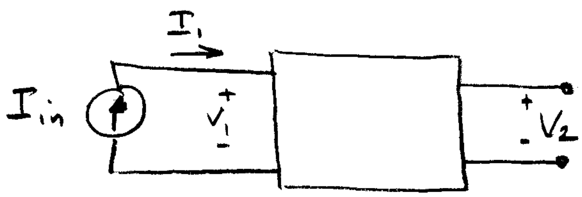
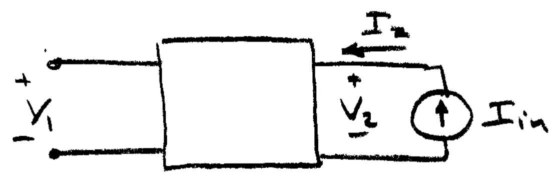
7.6 Reciprocal Networks

DC+L 19.10

- Recall that passive networks have symmetric y- and z-parameter matrices.
- This produces interesting properties.





the open circuit voltage response V_2 to current source I_{in} as I_1 is the same a the open circuit voltage response V_1 to current source I_{in} as I_2

Why? $$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Since $z_{12} = z_{21}$,

$$\left.\frac{V_2}{I_1}\right|_{I_2=0} = z_{21} = z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$

Interchange current source and voltmeter, no change to voltmeter reading.

It's no longer true if V_2 is terminated.

- Similarly,

The short circuit current response to a voltage source on the other port is unaffected by an interchange.

Use y-pameters to see it.

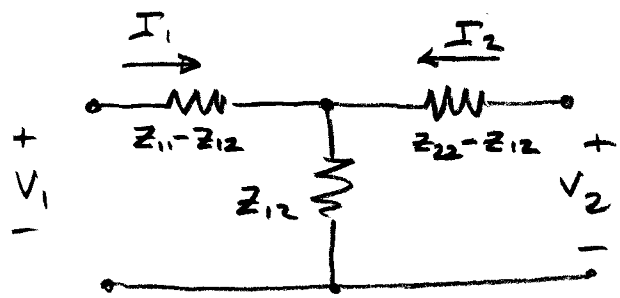
- Reciprocal networks have equivalent circuits with no dependent voltage sources.

  - First, for z-parameters,

$$V_1 = z_{11} I_1 + z_{12} I_2 = (z_{11} - z_{12}) I_1 + z_{12}(I_1 + I_2)$$

$$V_2 = z_{21} I_1 + z_{22} I_2 = z_{21}(I_1 + I_2) + (z_{22} - z_{21}) I_2$$

$$= z_{12}(I_1 + I_2) + (z_{22} - z_{12}) I_2$$



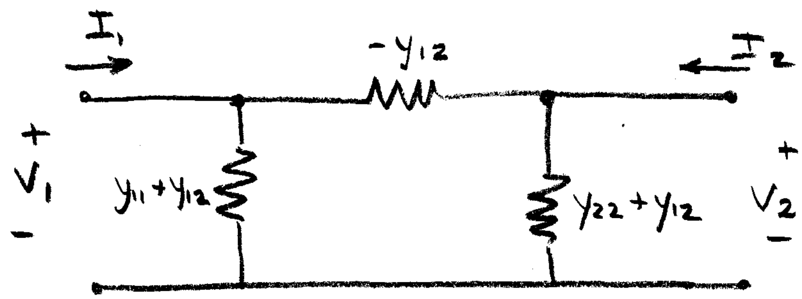
T-equivalent for symmetric z params

— Next, y-parameters.

$$I_1 = y_{11} V_1 + y_{12} V_2 = (y_{11} + y_{12}) V_1 - y_{12}(V_1 - V_2)$$

$$I_2 = y_{21} V_1 + y_{22} V_2 = -y_{21}(V_2 - V_1) + (y_{22} + y_{21}) V_2$$

$$= -y_{12}(V_2 - V_1) + (y_{22} + y_{12}) V_2$$



Pi-equivalent circuit for symmetric y-parameters.

Note DC+L error p836

— So any passive network can be reduced to a T- or Pi-equivalent network.