8 THREE-PHASE CIRCUITS

DC+L Chapts 11,12

- In this section, we shift our attention from filters and electronics to electric power:
  - how to calculate it
  - how to generate it
  - how to transmit it

  We will deal solely with sinusoidal signals.

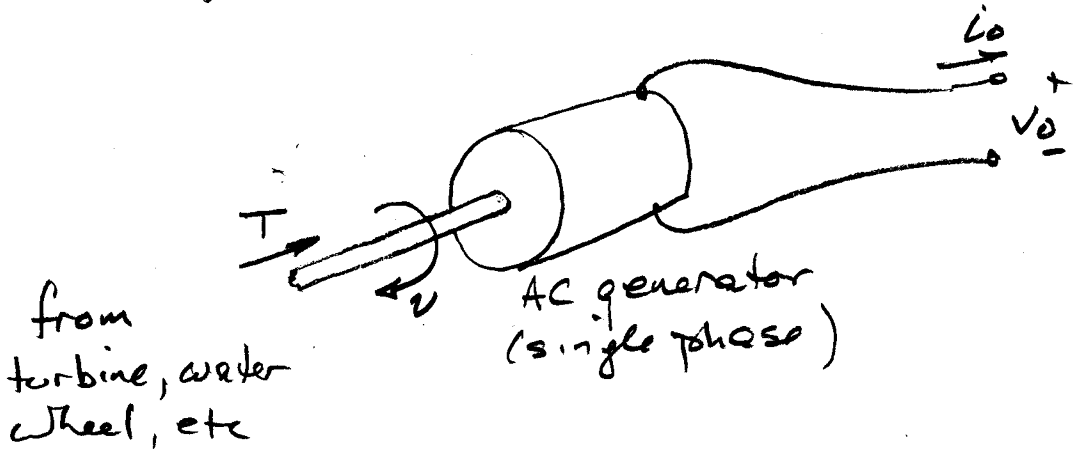
- Most of the topics in AC power apply equally to milliwatt electronics in the audio range, radio frequency circuits and megawatt 60 Hz power circuits — and almost all the topics will be familiar to you.

- Three phase power will be new to most of you, although you see examples of 3-φ systems almost every day. They are just three ordinary circuits with mutual phase angles of 120° ganged together. There are good reasons for doing so.

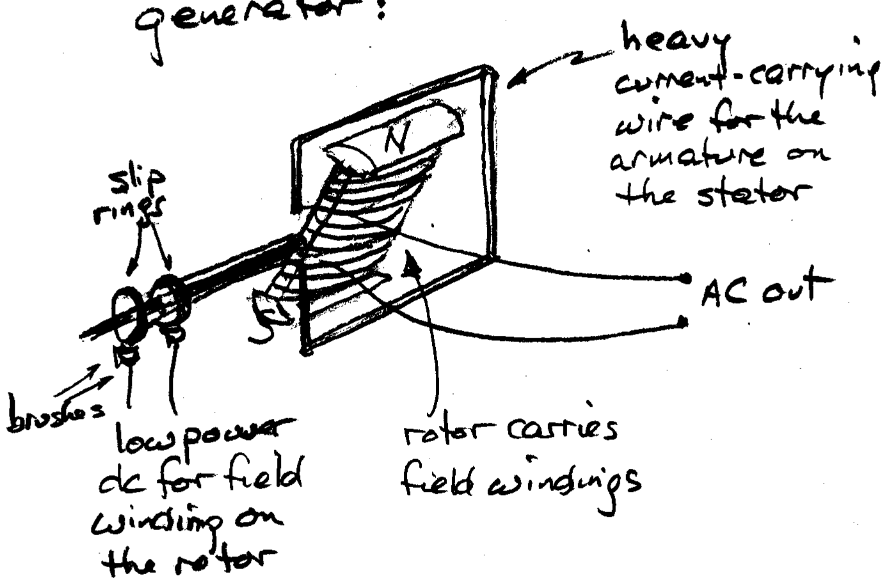
8.1 Review of AC Power

- This is familiar stuff — instantaneous power, average power, power factor, rms quantities, impedance matching, etc. That's why it's a review.

- But first — where does AC power come from?



– Cross section of elementary two-pole, single phase generator:

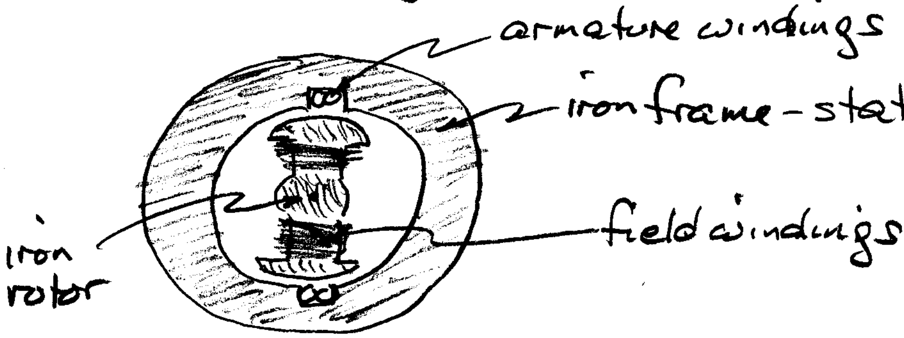


Basic idea:

– Spin a magnet (the field) to induce voltage in a coil (the armature)

– Use an electromagnet powered through brushes and slip rings.

– Cross section again, showing iron:

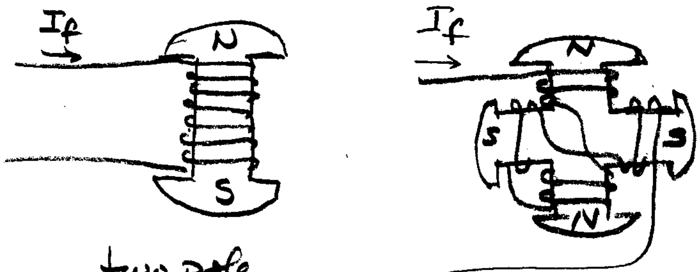


– Basic relations

– Generated voltage on the armature is proportional to rotor speed, field current (establishes B field), number of armature turns (wires).

– Torque experienced by turbine is proportional to armature current (depends on electrical load), field current, number of armature turns

– Generated frequency is proportional to rotor speed, number of poles.

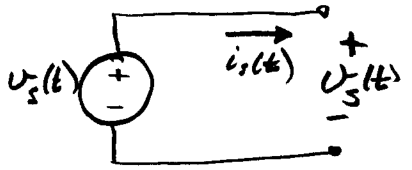


two pole rotor

four pole rotor

just imagine six! (I won't draw it). Quite common.

- This is an AC source



60 Hz in North America

50 Hz in UK, Aus, NZ

400 Hz in avionics

If the voltage is sinusoidal and the load is a linear circuit, then the current is sinusoidal with the same frequency.

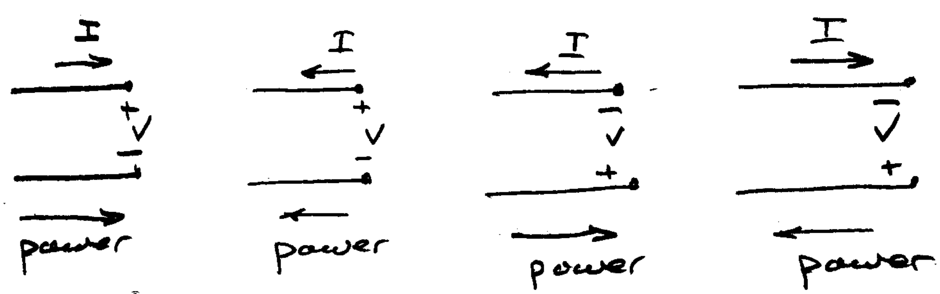
$$v(t) = V_m \cos(\omega t + \theta_v) = \mathrm{Re}\left[V e^{j\omega t}\right]$$

$\omega = 377$ rad/s here

phasor $V = V_m e^{j\theta_v}$

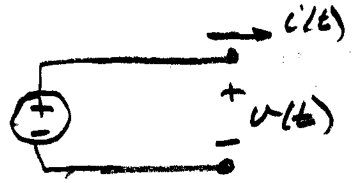
$$i(t) = I_m \cos(\omega t + \theta_i) = \mathrm{Re}\left[I e^{j\omega t}\right]$$

- Instantaneous power varies across the cycle.
  - If it were dc, the direction of power flow would be like this:



but the voltage and current in AC vary over a cycle.

– Instantaneous power out of the source:



$$P(t) = v(t)\, i(t) = V_m I_m \cos(\omega t + \theta_v)\cos(\omega t + \theta_i)$$

$$= \tfrac{1}{2} V_m I_m \cos(\theta_v - \theta_i) \leftarrow \text{dc}$$

$$+ \tfrac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)$$

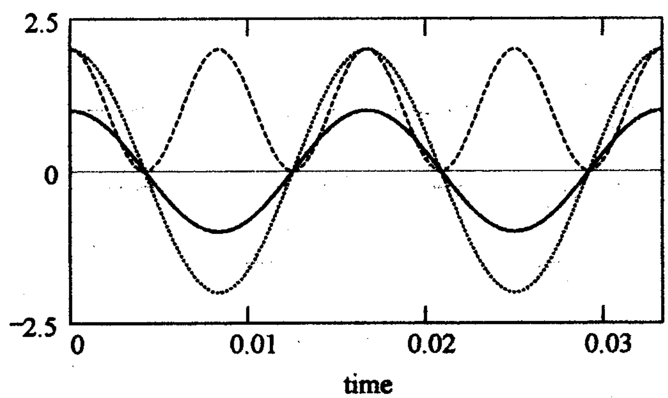
oscillates at 2ω

$$\boxed{\begin{aligned} &\cos(a)\cos(b) \\ &= \tfrac{1}{2}\cos(a-b) \\ &\quad + \tfrac{1}{2}\cos(a+b) \end{aligned}}$$

– First, consider current and voltage in phase

$(\theta_i = \theta_v)$ $\quad P(t) = \frac{1}{2} V_m I_m + \frac{1}{2} V_m I_m \cos(2\omega t + 2\theta_v)$

$= V I \cos^2(\omega t + \theta_v)$



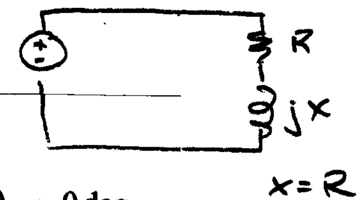
voltage
current
power

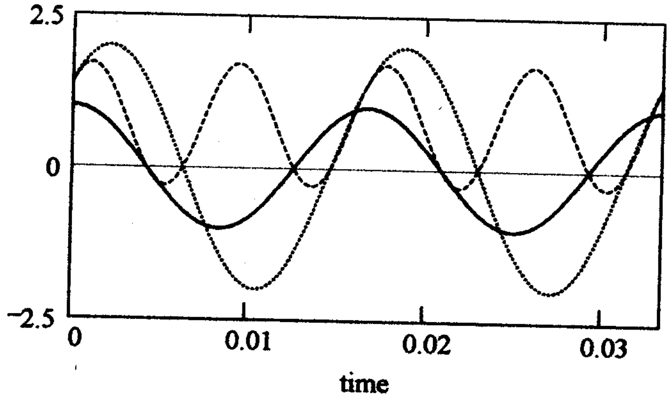
$\theta_v - \theta_i = 0\,\text{deg}$ lagging

→ Power fluctuates – it's not steady – 2ω

→ The average value of $\cos^2$ is $\frac{1}{2}$.

— Next, current lags voltage by 45°. Keep the same amplitudes





$\theta_v = 0\,\mathrm{deg}$

$\theta_i = -45\,\mathrm{deg}$

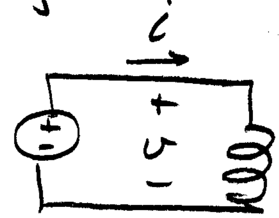
— voltage
······ current
------ power

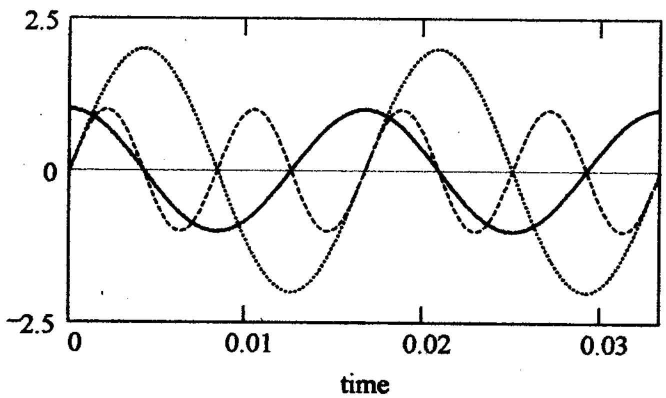
$\theta_v - \theta_i = 45\,\mathrm{deg}$ lagging

→ Same peak-to-peak power swing

→ Power flows back <u>into</u> source during part of the cycle!

— Now let current lag voltage by 90°.





$\theta_v = 0\,\mathrm{deg}$

$\theta_i = -90\,\mathrm{deg}$

— voltage
······ current
------ power

$\theta_v - \theta_i = 90\,\mathrm{deg}$ lagging

→ Power flows out of source for $\frac{1}{2}$ cycle, back into it for $\frac{1}{2}$ cycle.

→ No <u>net</u> power flow either way.

- Lags of more than 90° cause net power flow into the source. The generator starts behaving like a motor (this can happen if the generator is connected to the power grid as a "load")
- Net (average) power doesn't depend on whether current lags or leads the voltage.

- Average power over a cycle is easy:

$$P(t) = \underbrace{\tfrac{1}{2} V_m I_m \cos(\theta_v - \theta_i)}_{\text{a constant, the average value}} + \underbrace{\tfrac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)}_{\text{oscillatory, averages to zero}}$$

$$P_{av} = \tfrac{1}{2} V_m I_m \underbrace{\cos(\theta_v - \theta_i)}_{\text{the power factor (pf)}}$$

a nuisance

lagging pf if current lags voltage

$$= V_{rms} I_{rms} \, \text{p.f.}$$

Note — the p.f. can be positive (power to load) or negative (power back into "generator") or zero (if completely reactive load, 90°)

- A phasor interpretation:

$$v(t) = Re\left[V e^{j\omega t}\right] \quad i(t) = Re\left[I e^{j\omega t}\right]$$

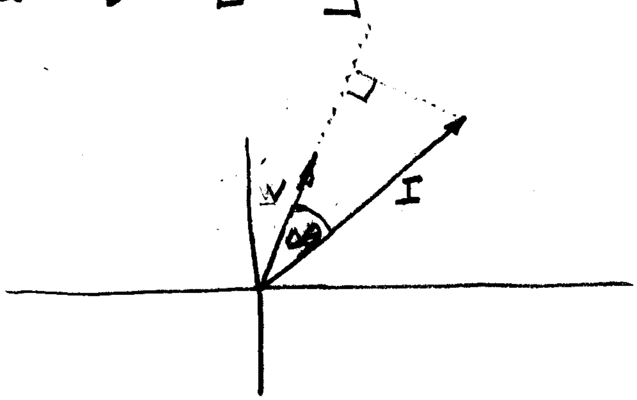
$$P(t) = Re\left[V e^{j\omega t}\right] Re\left[I e^{j\omega t}\right]$$

$$= \frac{1}{2} Re\left[V I^* + V I e^{j2\omega t}\right]$$

$$\boxed{Re[\alpha]\, Re[\beta] = \frac{1}{2} Re\left[\alpha\beta^* + \alpha\beta\right]}$$

covers many trig identities

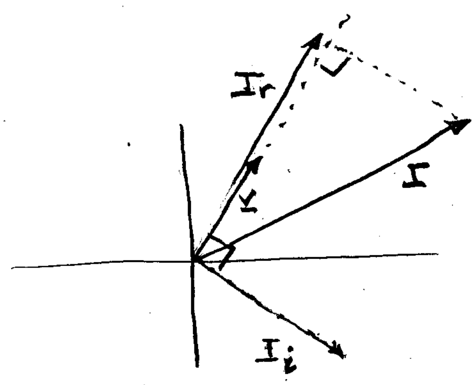
$$P_{av} = \frac{1}{2} Re\left[V I^*\right] = \frac{1}{2} V_m I_m \cos(\Delta\theta)$$



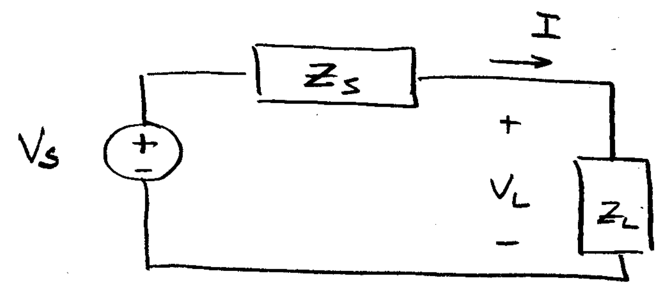
$I_m \cos(\Delta\theta)$ is the length of the projection of I onto V

- A complex interpretation:

$$P = \frac{1}{2} V I^* = \underbrace{\frac{1}{2} Re\left[V I^*\right]}_{\text{real power, watts}} + \underbrace{\frac{j}{2} Im\left[V I^*\right]}_{\text{reactive, imaginary, wattless power VAR}}$$

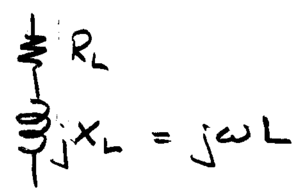


- The nature of the load impedance or admittance has a significant effect on the efficiency of power delivery.



Typical loads in power systems have an inductive component, because of transformers and motors. This causes I to lag V_L:

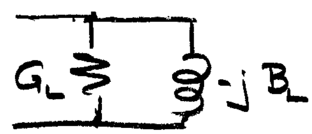
$$Z_L = R_L + jX_L$$



$= |Z_L| \angle \theta_L$

or

$$Y_L = G_L - jB_L$$



$= |Y_L| \angle -\theta_L$

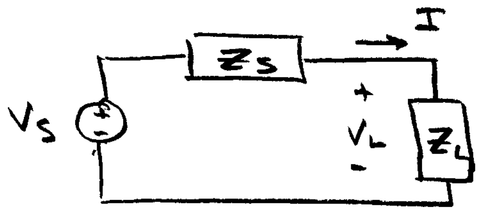
$$\underline{I} = \frac{V_L}{Z_L} = \frac{|V_L|}{|Z_L|} \angle -\theta_L$$

A lagging power factor, less than 1.

The power to the load is less than $\frac{1}{2} V_m I_m$.

Next, some optimizations.

- Effect of source impedance on power transferred.
  - If we could choose source impedance, how could we maximize power transferred to the load, given Z_L?



$$I = \frac{V_s}{Z_s + Z_L}$$

$$V_L = Z_L I = \frac{Z_L V_s}{Z_s + Z_L}$$

$$P_L = \frac{1}{2} \mathrm{Re}[V_L I^*] = \frac{1}{2} \frac{|V_s|^2 R_L}{|Z_s + Z_L|^2}$$

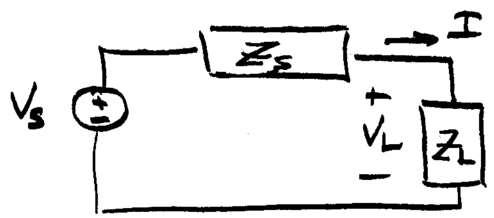
  - Maximize with $Z_s = 0$ (a no-brainer)
  - For non-zero Z_s, we can't add negative R_s, but we can change the reactive part of Z_s (i.e. X_s) by putting L or C in series or parallel.

$$P_L = \frac{1}{2} \frac{|V_s|^2 R_L}{|(R_s + R_L) + j(X_s + X_L)|^2}$$

Maximize with $X_s = -X_L$, so $Z_s + Z_L = R_s + R_L$, real

  - This is the same condition as for resonance. Most circuits would not actually resonate, since resistance typically dominates reactance — but $X_s = -X_L$ does "tune" the system to ω, in a way.

- Effect of load impedance on power transferred.
  - If we can choose load impedance, how can we maximize power transferred to the load, given Z_s?



$$I = \frac{V_s}{Z_s + Z_L}$$

$$V_L = Z_L I = \frac{Z_L V_s}{Z_s + Z_L}$$

$$P_L = \frac{1}{2} \frac{|V_s|^2 R_L}{|Z_s + Z_L|^2}$$

- First, choose $X_L = -X_s$ to minimize the denominator

  Then $P_L = \frac{1}{2} \frac{|V_s|^2 R_L}{(R_s + R_L)^2} = \frac{1}{2} |V_L| |I|$

- A familiar optimization: R_L too large makes I small without continued increase in V_L; R_L too small makes V_L too small without continued increase in I. Optimum, by $dP_L / dR_L = 0$, is

$$R_L = R_s$$

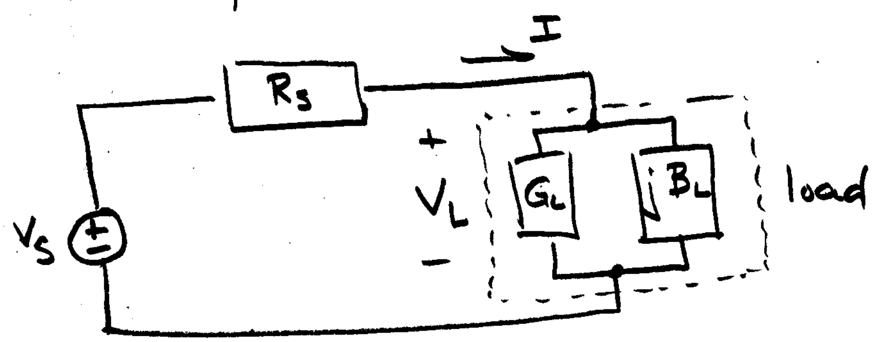
So $Z_L = Z_s^*$. Also $I = \frac{V_s}{Z_s + Z_L} = \frac{V_s}{R_s + R_L} = \frac{V_s}{2R_s}$.

Current I is in phase with source V_s, so

$$\text{source } P_s = \frac{1}{2} \mathrm{Re}[V I^*] = \frac{1}{2} \frac{V_s^2}{2R_s} = \frac{1}{2} \frac{V_{s\,rms}^2}{R_s}$$

- But would you want to operate a power system with matched load?
  - → source and load each dissipate $\frac{1}{4}\frac{V_{s\,rms}^2}{R_s}$
  - → so the same power as the load (an entire suburb?) would be heating up the generator!
  - → normally the load resistance is <u>much</u> higher than the source resistance in power systems
  - → but in low power systems (sensors, antennas) the matching condition is common.

• Another optimization: power factor correction.



- The load consumes power in G_L
- The susceptance B_L allows additional current to flow, without increasing load power.
- The extra current wastes more power in R_s — so cancel B_L by shunting $-B_L$ across the load.