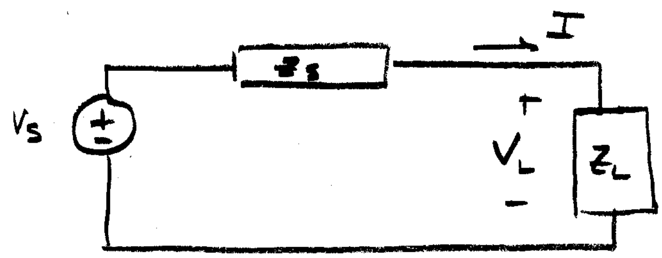
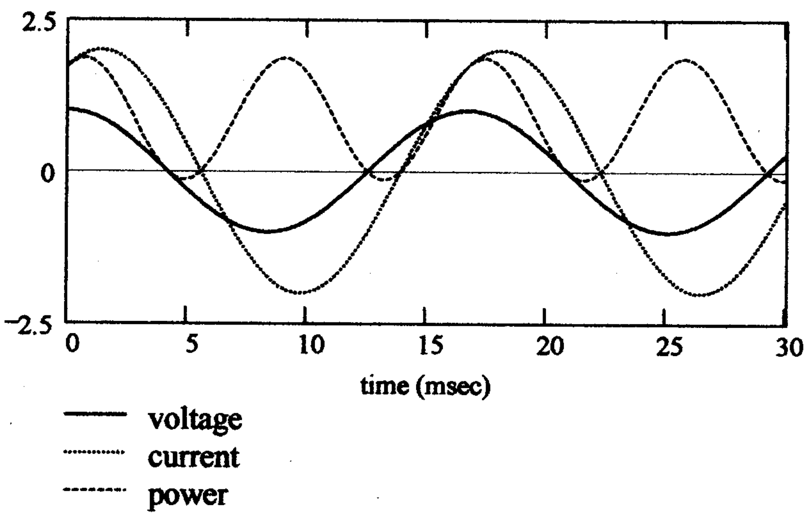
8.3 Three-Phase Systems in Y Format

- We are used to the idea of power generation, transmission and consumption in this format:



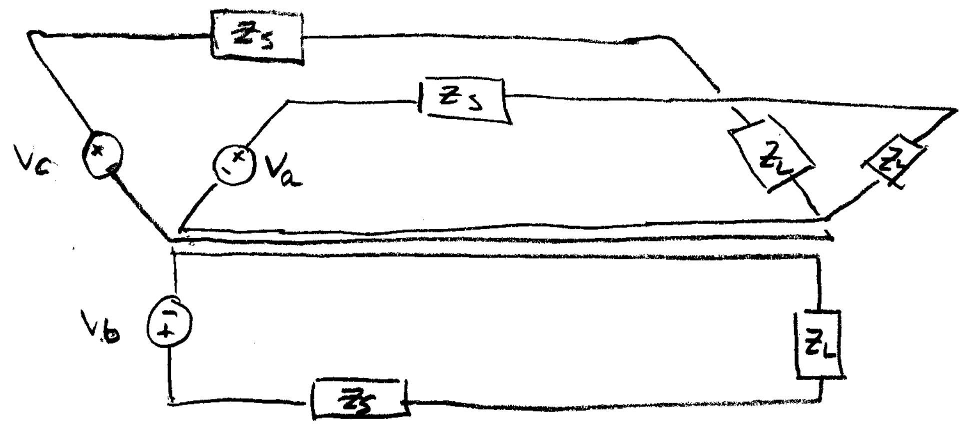
Recall that the power oscillates at 120 Hz.



This is inconvenient for some applications, particularly those involving high power motors. For them, a steady torque – especially one created by a rotating magnetic field — is preferable.

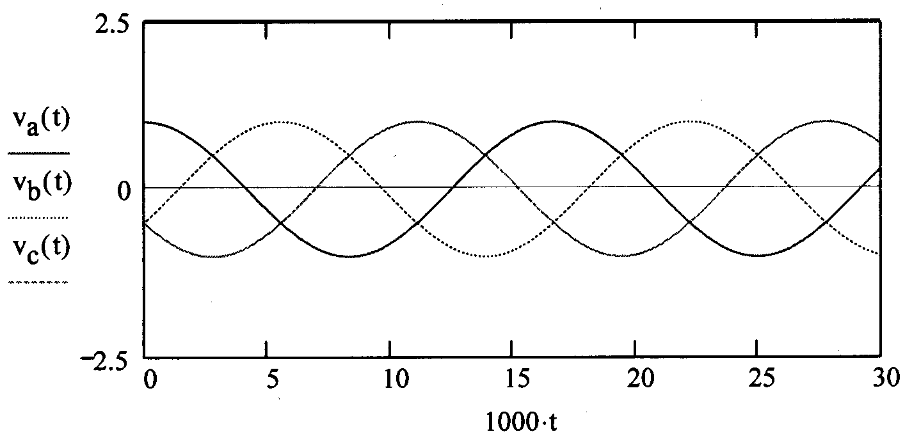
Note also that transmission of large power requires high voltage levels.

- These observations motivate polyphase systems — normally three phases.
  - Use three voltage sources with mutual phase angles of 120°.



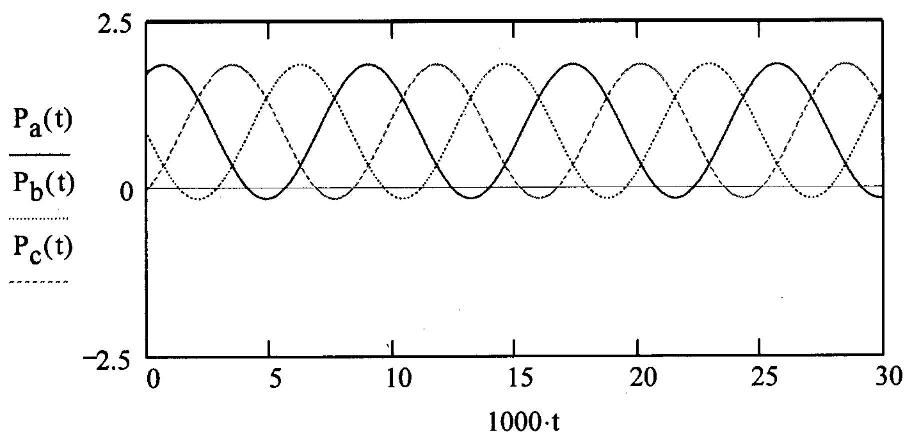
$V_a = V_m \angle 0 \qquad V_b = V_m \angle -120 \qquad V_c = V_m \angle -240$

  - The voltages and powers look like this:



V_b lags V_a by 120°

V_c lags V_b by 120°



The powers sum to 3 times the average power of a single phase, and it's constant.

– With three phases, each needs to carry only $\frac{1}{3}$ of the total power demand, so the voltage is $\frac{1}{\sqrt{3}} = 0.577$ that of a single-phase system

• How can we generate three-phase power?

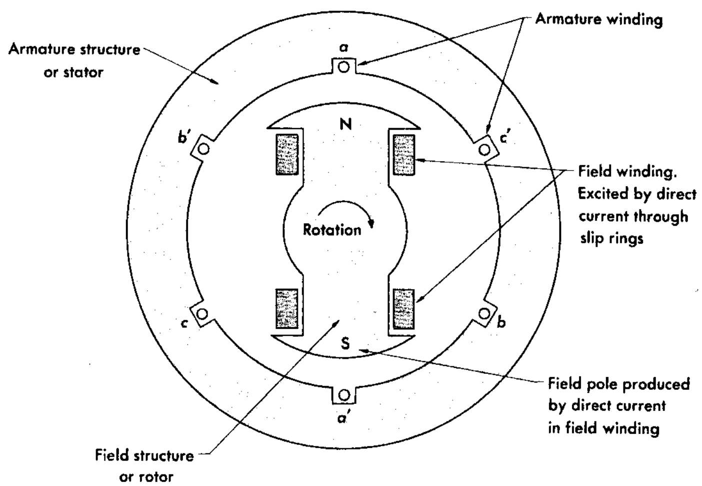


FIGURE 14-10 Elementary three-phase two-pole generator.

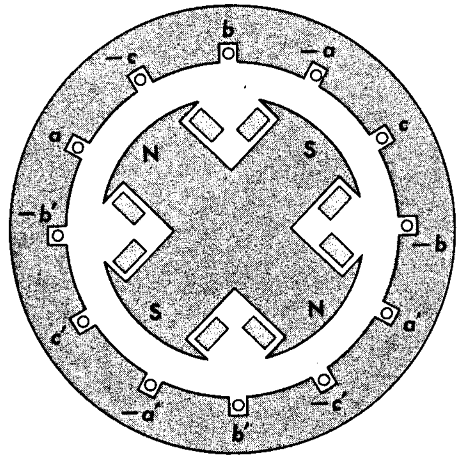
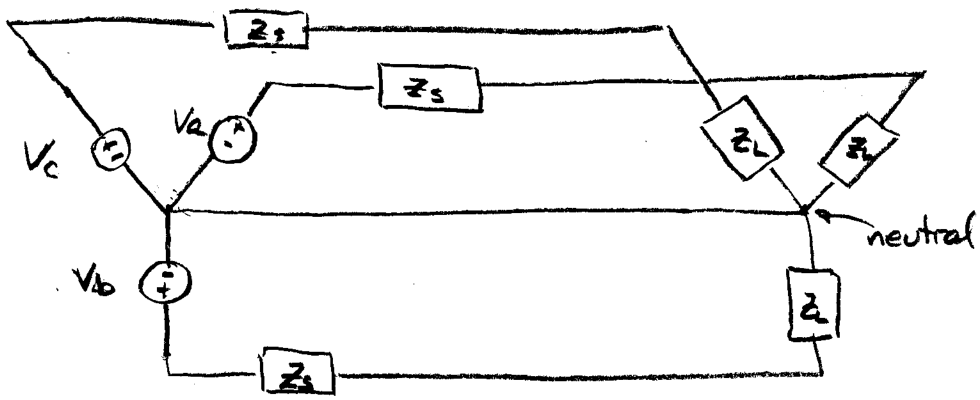


FIGURE 14-12 Elementary four-pole three-phase alternator.

– Three armature windings, physically separated by an amount that gives 120 electrical degrees.

– Four-pole generators ("alternators" for AC) can be turned half as fast as two pole. Note same number of poles on stator and rotor.

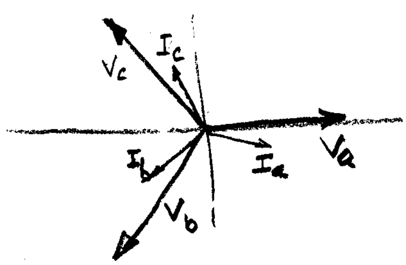
- How can we transmit 3-φ power?
  - We could give the three sources a common ground, unlike p. 8.3.2



4 wires in the transmission lines.

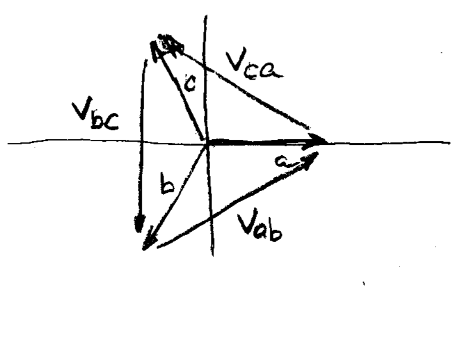
This is a Y configuration.

- The phasor diagram is that of a single phase, replicated 3 times with 120° sep'n.



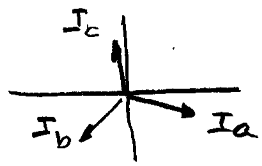
V_a is the phase voltage (i.e. to neutral) of phase a.

The line-to-line voltages have amplitudes $\sqrt{3}$ times phase voltage amplitudes and they, too have a mutual separation of 120°.

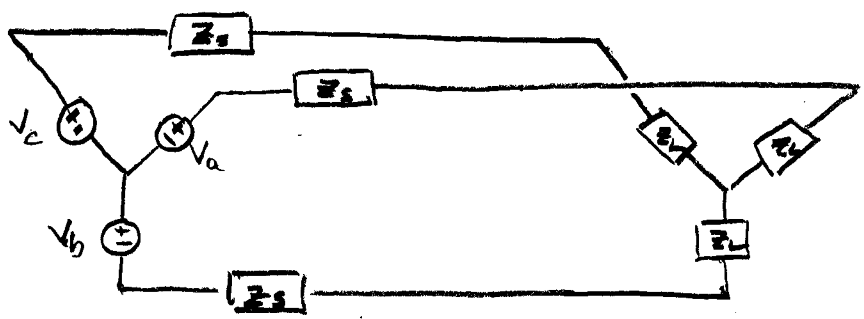


$$\left.\begin{aligned} V_{ca} &= \sqrt{3}\,|V_a| \measuredangle 150^\circ \\ V_{bc} &= \sqrt{3}\,|V_a| \measuredangle 270^\circ \\ V_{ab} &= \sqrt{3}\,|V_a| \measuredangle 30^\circ \end{aligned}\right\} \text{30° rotation wrt phase voltages}$$

— Now look at the current in the neutral line. It carries $I_a + I_b + I_c = 0$!



Looks as though we can remove it, and transmit 3ϕ power with just 3 lines:

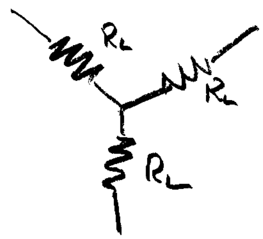


This works if Z_s, Z_L are identical in all three phases, ie a balanced system.

More generally, be prepared for some imbalance and keep a neutral line that is lighter than the others.

— Now look at transmission lines as you ride home tonight, or in to the university tomorrow.

- How can we consume 3-ϕ power?
  - All the usual ways, including resistive heating.



  - Three phase motors are interesting (and are one of the motivations for 3ϕ). The armature currents produce a rotating magnetic field.

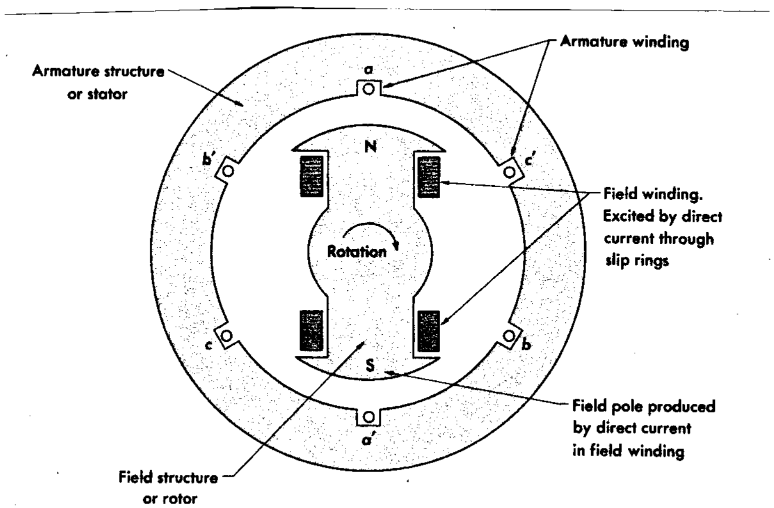
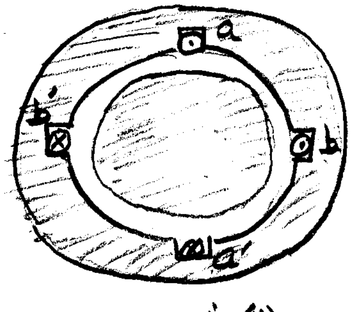


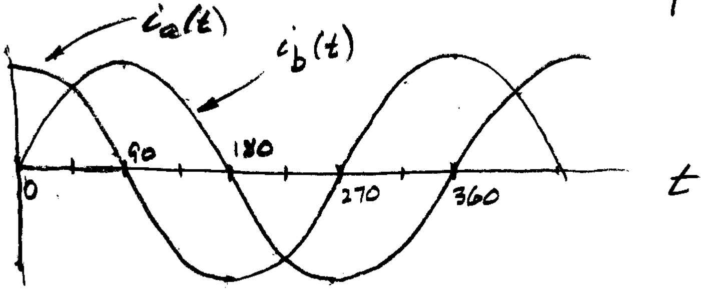
FIGURE 14-10 Elementary three-phase two-pole generator.

Plausibility: A rotating field set up the pattern of armature voltages and currents in the first place. If they are applied to a motor armature, they will reproduce a rotating field.

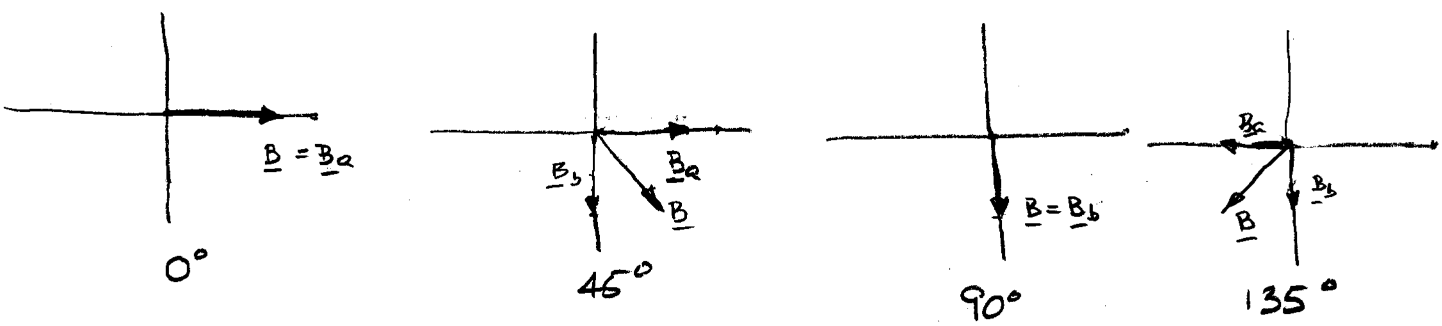
Demonstration (easier to see with two phases, 90°)



Current in the a phase windings creates a B field component horizontally; b phase current sets up B vertically.



Spatial snapshots of field vectors:



etc.

The spatial separation combined with phase difference creates a rotating magnetic field.

The point? In a motor, the rotating magnetic field drags the rotor around with it. Steady torque.* Harder to do with single phase (only low power applications)

* No vibration.

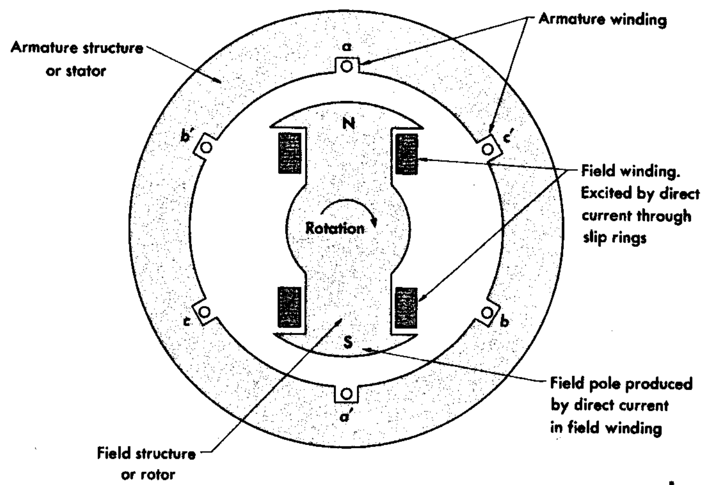
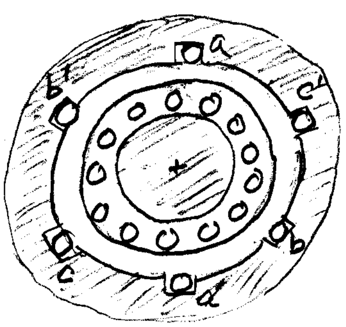


FIGURE 14-10 Elementary three-phase two-pole ~~generator~~ motor.

– A synchronous motor has the same construction as an alternator (3-φ generator), with a separately excited (powered) rotor.

The rotor turns at a fixed speed determined by the supply frequency and number of poles. Good for high power, high torque.

– An induction motor has no external excitation of the rotor. Instead, voltages and currents are induced in it when rotor turns more slowly than the field. The current-field interaction gives torque.



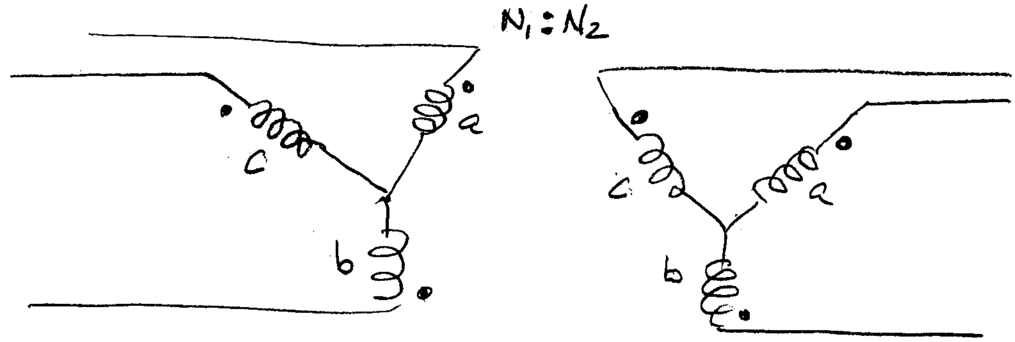
Rotor has several wires or copper rods short circuited at each end.

Rotation speed depends on the mechanical load.

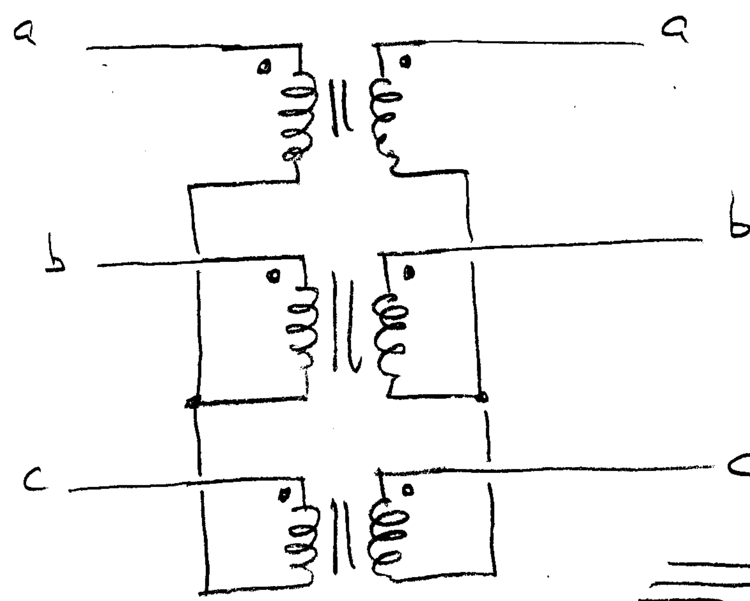
Cheaper to make than synch. motor, very widely used.

– Both motors appear as an electrical load that combines resistive (from conversion to mech'l power) and reactive (from the windings) components.

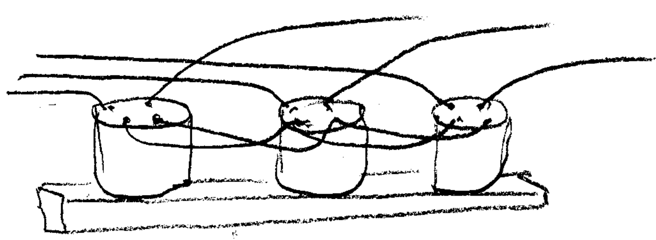
- How can we transform 3-ϕ power?
  - With a 3-ϕ transformer:



  - Three single phase transformers

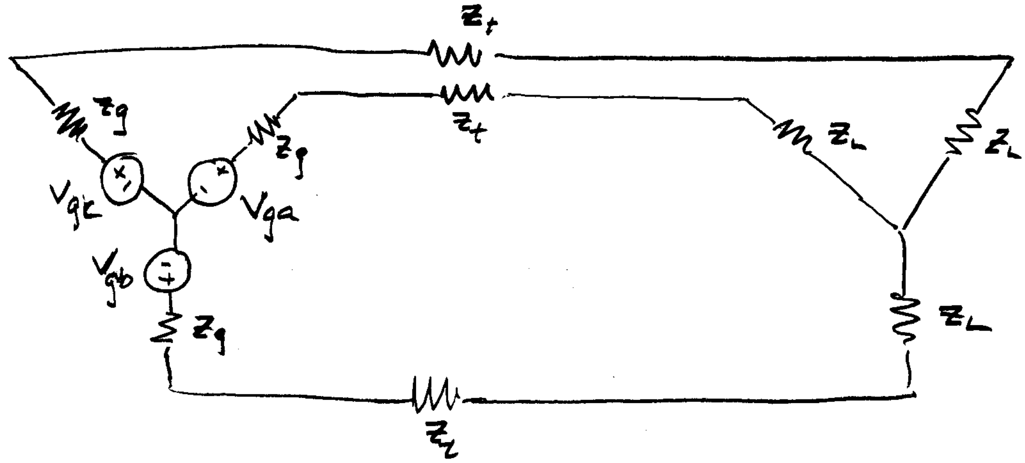


Look up on poles:



- How can we analyze <u>balanced</u> 3-ϕ systems?
  - Select one phase and analyze it.
  - The others are the same, except 120° away.
  - Line to line voltage is $\sqrt{3}$ times phase voltage.
  - Total power transfer is sum of phase pwrs.

- Example (DC+L ex 12.6). Analyze this



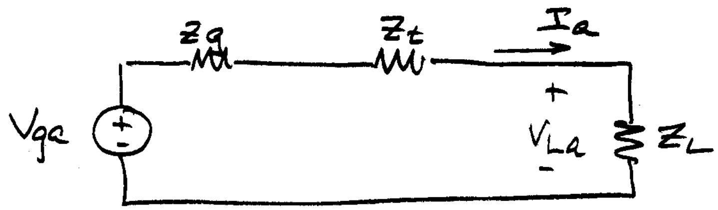
Impedances shown as resistors.

$V_{ga} = 120\angle 0°$ Vrms

$Z_g = 0.2\angle 75°$ $\quad Z_t = 0.3\angle 45°$ $\quad Z_L = 10\angle 20°$

- Find I_a and V_{La} (V_{an} at load).
- Find counterpart quantities in b, c by inspection
- Find line-line voltages V_{ab} V_{bc} V_{ca} at the load.
- Find P.F., P_{La}, total power to load.
- Find VA and VAR at the load
- Find the value of shunt capacitor required to bring load P.F. back to 1.

To solve, draw single-line diagram.



– total $Z_{tot} = Z_g + Z_t + Z_L = 10.4 \angle 21.6°$

so $I_a = \dfrac{V_{ga}}{Z_{tot}} = \dfrac{120 \angle 0}{10.4 \angle 21.6} = 11.55 \angle -21.6$ Arms

and $V_{La} = I_a Z_L = 11.55 \angle -21.6 \cdot 10 \angle 20°$

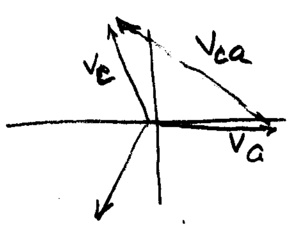
$= 115.49 \angle -1.6°$ Vrms

– The other phases can be obtained by subtracting 120°, 240°

$I_b = 11.55 \angle -141.6°$ $\quad$ $I_c = 11.55 \angle -261.6°$

$V_{Lb} = 115.49 \angle -121.6°$ $\quad$ $V_{Lc} = 115.49 \angle -241.6°$

– Line to line voltages:



$V_{ca} = \sqrt{3} \cdot 115.49 \angle 150 - 1.6°$

$= 200 \angle 148.4°$

– $P.F. = \cos(\angle V_{La} - \angle I_a) = \cos(20°) = 0.94$

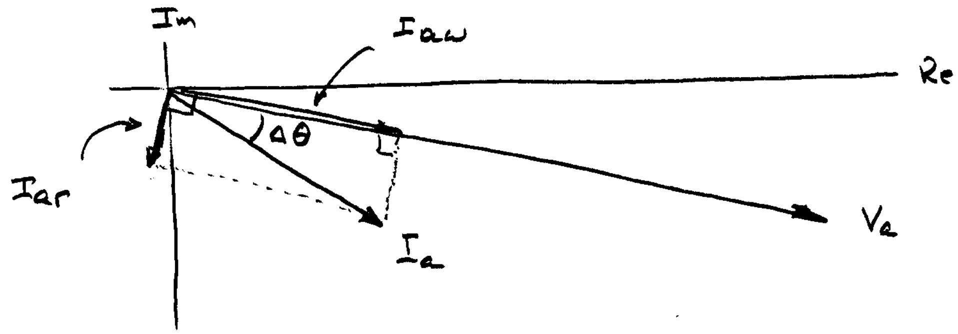
$P_a = \mathrm{Re}[V_{La} I_a^*] = |V_{La}||I_a|$ P.F

no ½ - we used rms quantities

$= 115.49 \cdot 11.55 \cdot 0.94 = 1253$ watt

$P_{total} = 3 P_a = 3.76$ kwatt

– The distinction between VA, watts and VAR is based on decomposition of the current into components parallel to and orthogonal to the voltage.



Not really to scale for this problem.

$$I_a = I_{aw} + I_{ar}$$

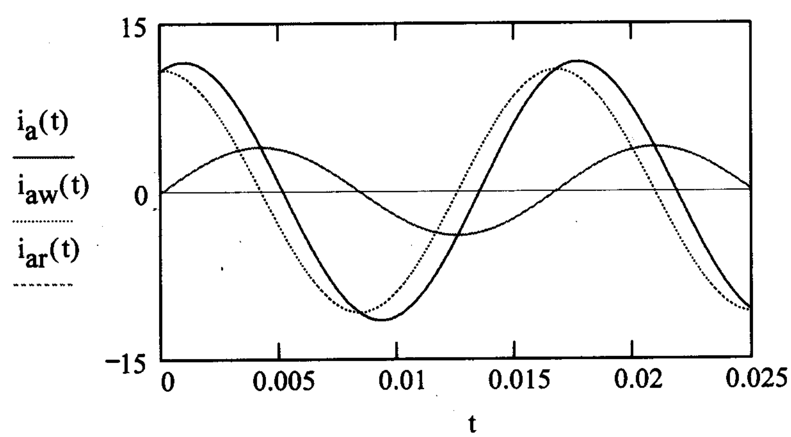
$$= |I_a| \cos(\Delta\theta) \angle -1.6^\circ + |I_a| \sin(\Delta\theta) \angle -91.6^\circ$$

$$= |I_a| \, PF \angle -1.6^\circ + |I_a| \sqrt{1-PF^2} \angle -91.6^\circ$$

$$= 10.9 \angle -1.6^\circ + 3.95 \angle -91.6^\circ$$

or, in the time domain

$$|I_a| \cos(\omega t - 21.6^\circ) = |I_a| \, PF \cos(\omega t - 1.6^\circ) + |I_a| \sqrt{1-PF^2} \cos(\omega t - 91.6^\circ)$$

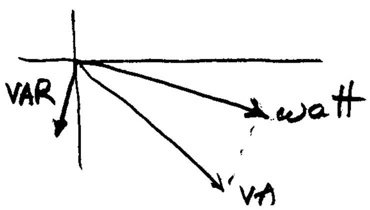


From this, we have instantaneous power

$$P(t) = v_a(t)\, i_a(t) = v_a(t)\, i_{aw}(t) + v_a(t)\, i_{ar}(t)$$

with average value $P_{av} = |V_a||I_{aw}| + 0$

and the power triangle



Short answer to the question:

$$VA = |V_a||I_a| = 115.5 \cdot 11.55 = 1334 \text{ volt amp}$$

of which the real power is

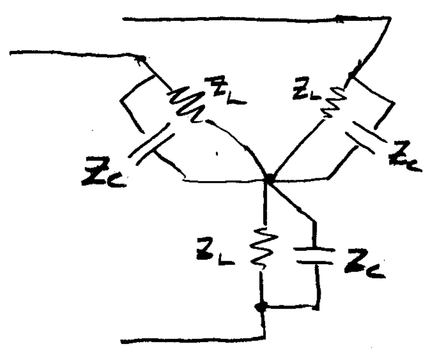
$$\text{watt} = |V_a||I_{aw}| = |V_a||I_a|\, PF = 1253 \text{ watt}$$

and the reactive (wattless) power is

$$VAR = |V_a||I_{ar}| = |V_a||I_a| \sqrt{1 - PF^2} = 115.5 \cdot 11.5 \cdot 0.342$$

$$= 456.3 \text{ volt-amp}$$

— Last question — PF correction by shunt capacitance.



Easiest to work with admittance.

$$Y_L = \frac{1}{Z_L} = 0.1\angle -20^\circ = 0.094 - j0.034$$

$$Y_c = j\omega C$$

We want $Y_L + Y_c$ to be real.

so $\omega C = 0.034$

$$C = \frac{0.034}{377} = 90.2\,\mu F$$