8.4 Three Phase Systems in Δ Format

- We have been connecting phases in Y format

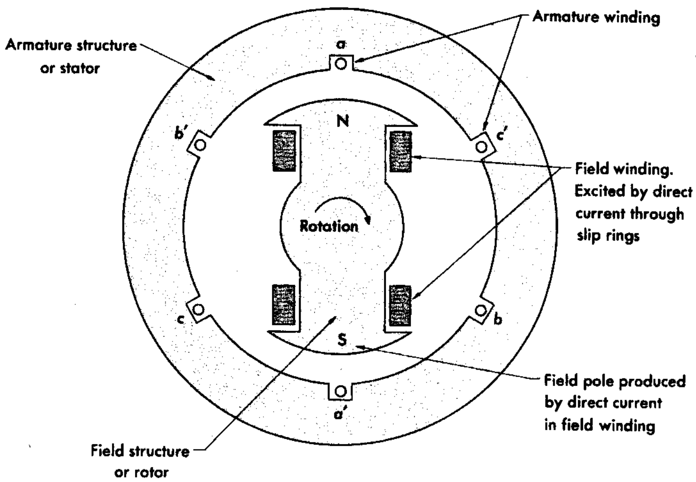
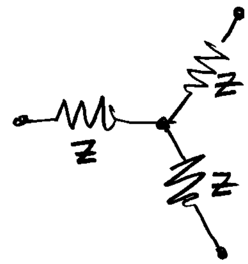
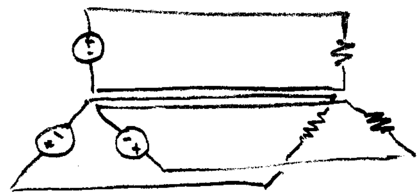


FIGURE 14-10 Elementary three-phase two-pole generator.





but we can also connect in Δ

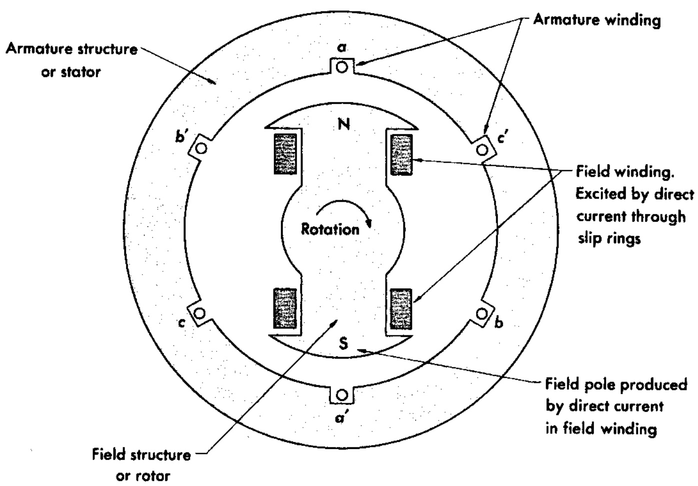
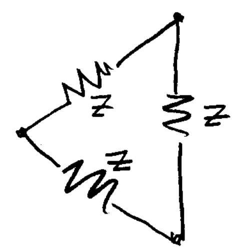
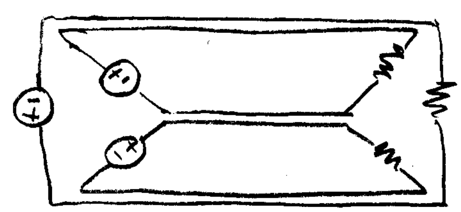


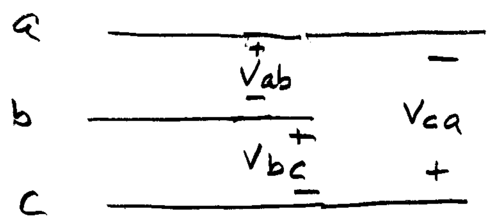
FIGURE 14-10 Elementary three-phase two-pole generator.



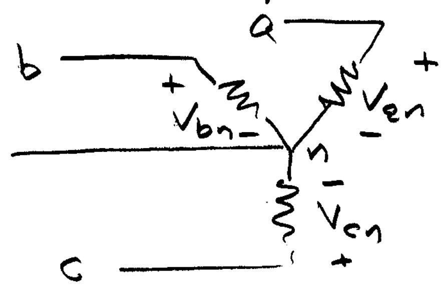


How to analyze them with a common framework?

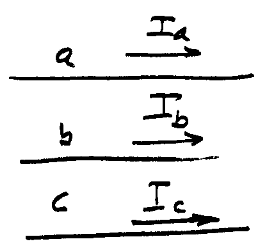
- With two connection alternatives at the source — and at the load — we must be more careful about defining quantities:
  - labels a, b, c identify the lines
  - line to line voltages V_{ab}, V_{bc}, V_{ca}



  - line to neutral voltages



  - line currents



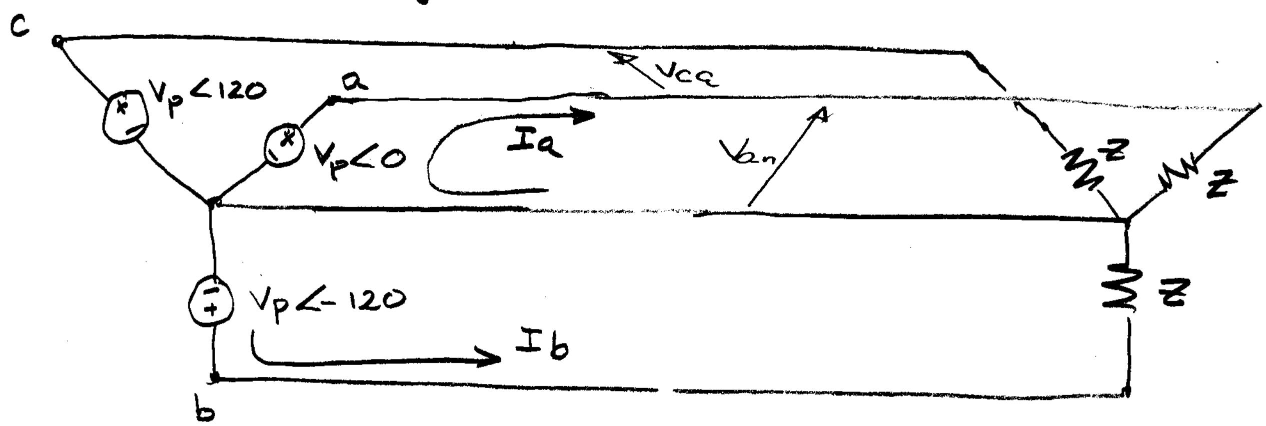
  - phase voltages — less well defined — the voltages produced by the generator.

$V_p \angle 0$ $\quad V_p \angle -120$ $\quad V_p \angle -240$

$= V_p \angle 120$

– phase currents – currents through each of the generator phases

- Here's Y again



– line to neutral voltages equal phase voltages:

$$V_{an} = V_p \angle 0 \qquad V_{bn} = V_p \angle -120 \qquad V_{cn} = V_p \angle 120$$

– line currents equal phase currents:

$$I_a = V_{an}/Z \qquad I_b = V_{bn}/Z \qquad I_c = V_{cn}/Z$$

– power per phase is same, all phases

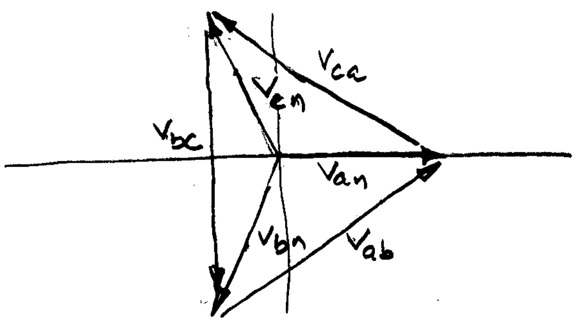
$$P_{phase} = Re\left[V_{phase}\, I_{phase}^*\right] \quad \text{(rms quantities)}.$$

$$= Re\left[V_{an} I_a^*\right] = Re\left[|V_{an}|^2/Z^*\right]$$

$$= \frac{V_p^2}{|Z|} \cdot \underbrace{\cos(\angle Z)}_{pf}$$

$$P_{total} = \frac{3 V_p^2}{|Z|} \cdot p.f.$$

— line to line voltages



$V_{ab} = V_{an} - V_{bn}$

$= \sqrt{3}\, V_p \angle 30°$

$V_{bc} = V_{bn} - V_{cn}$

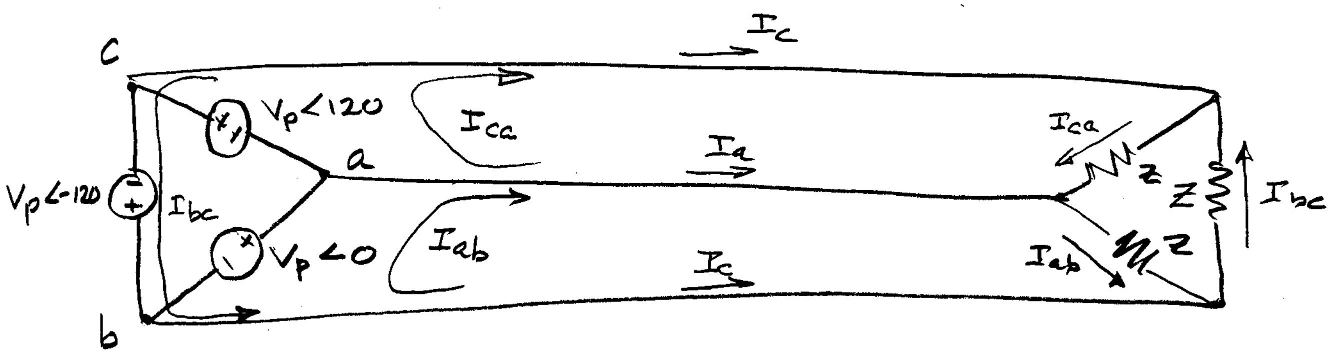
$= \sqrt{3}\, V_p \angle -90°$

$V_{ca} = V_{cn} - V_{an}$

$= \sqrt{3}\, V_p \angle 150°$

line-line voltages are
$\sqrt{3}$ times the line to neutral voltages
and rotated 30°

- Now for Δ connection. First, we'll establish the basic relations, then find a simplification.



— line to line voltage equal phase voltages

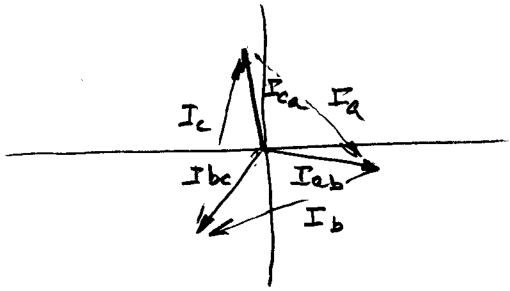
$V_{ab} = V_p \angle 0 \quad V_{bc} = V_p \angle -120 \qquad V_{ca} = V_p \angle 120$

— phase currents are now defined wrt phase voltages

$I_{ab} = V_{ab}/Z \qquad I_{bc} = V_{bc}/Z \qquad I_{ca} = V_{ca}/Z$

– line currents are sums of phase currents

$$I_a = I_{ab} - I_{ca}, \quad I_b = I_{bc} - I_{ab}, \quad I_c = I_{ca} - I_{bc}$$



line currents are $\sqrt{3}$ times phase currents, 30° rotation

– Power per phase

$$P_{ab} = Re\left[V_{ab} I_{ab}^*\right] = Re\left[V_{ab} V_{ab}^* / Z^*\right]$$

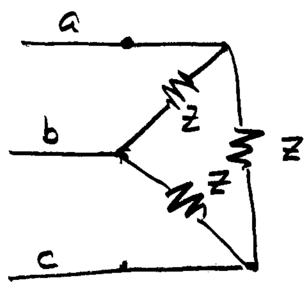
$$= \frac{|V_{ab}|^2}{|Z|} \underbrace{\cos(\angle Z)}_{pf} = \frac{V_p^2}{|Z|} \text{ p.f.}$$

$$P_{tot} = \frac{3V_p^2}{|Z|} \text{ p.f.}$$

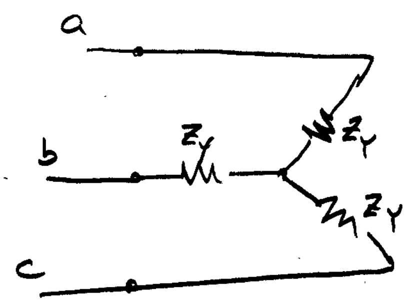
Same as in the Y connection.

- The Δ to Y transformation.
  - We can transform a Δ connected load or generator to an equivalent Y connection. A big help! It lets us analyze single line diagram (easy).
  - First, look at load.

Transform this

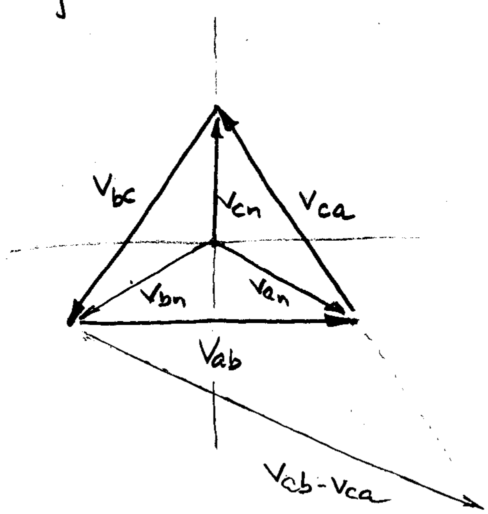


to this



$Z_Y = ?$

Imagine that there is a neutral. Reconstruct Van.

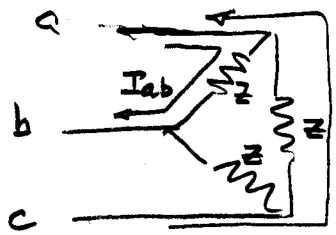


by construction

$$V_{an} = \frac{V_{ab}-V_{ca}}{3}$$

$$\left(\text{note } |V_{ab}-V_{ca}| = \sqrt{3}\,|V_{ab}| = \sqrt{3}\,\sqrt{3}\,|V_{an}|\right)$$

Then for the Δ load, the current in line a is

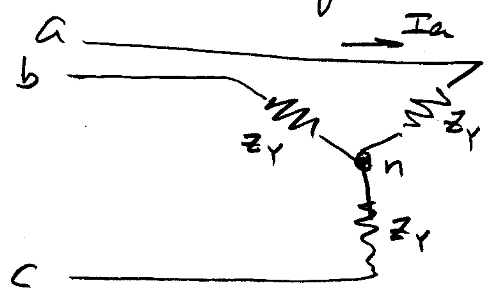


$$I_a = I_{ab} - I_{ca}$$

$$= \frac{V_{ab}}{Z} - \frac{V_{ca}}{Z}$$

$$= \frac{V_{ab} - V_{ca}}{Z}$$

and for an equivalent Y load:

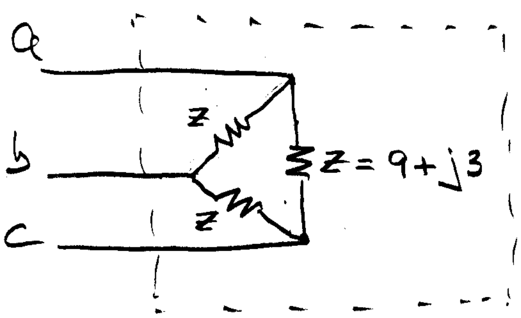


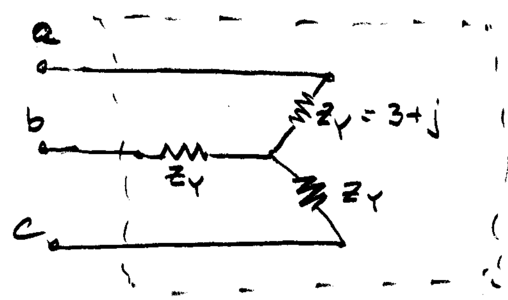
$$I_a = \frac{V_{an}}{Z_Y}$$

$$= \frac{V_{ab} - V_{ca}}{3} \cdot \frac{1}{Z_Y}$$

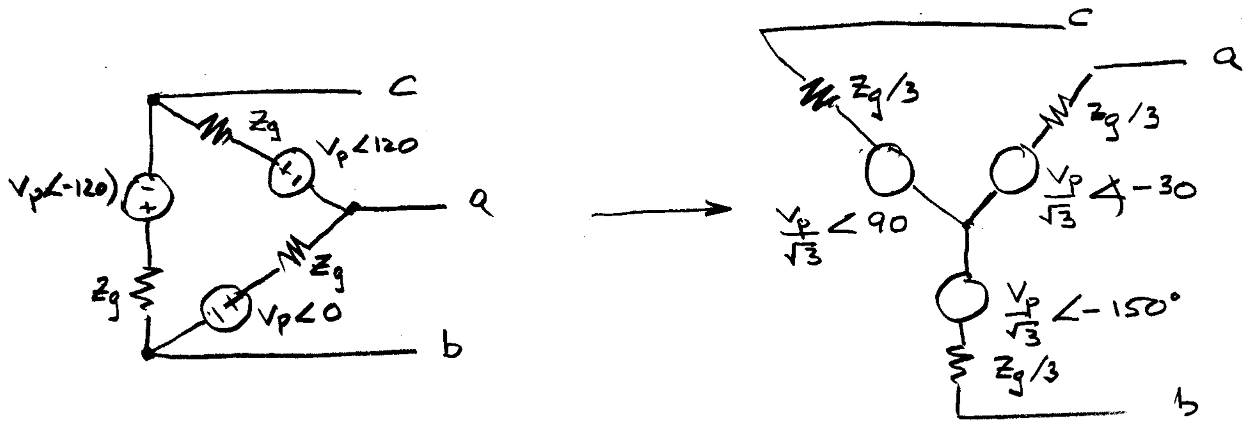
Consequently $Z_Y = \frac{1}{3} Z$ Easy!

<u>example</u> From a terminal perspective, these two loads are equivalent





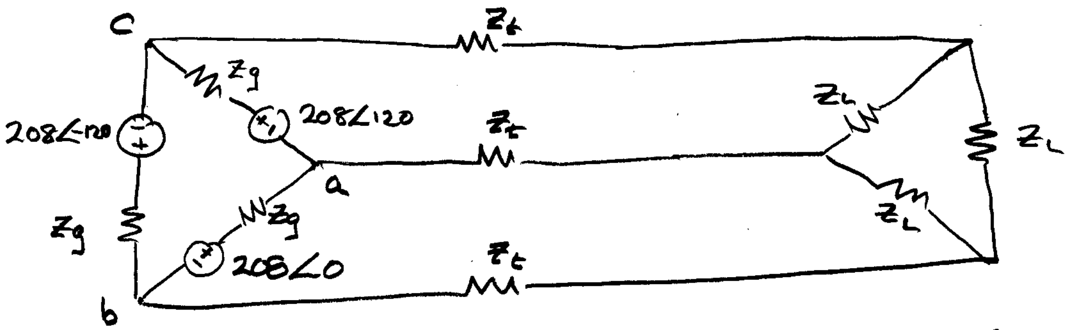
- For a source, including source resistance,



or just give any mutual 120°

- Now we just replace Δ source <u>or</u> load with Y equivalent.

- example. $\Delta - \Delta$

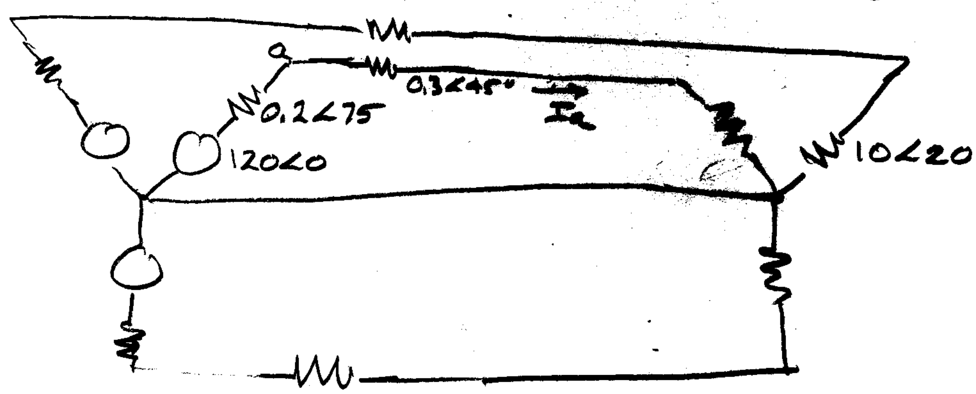


where $Z_g = 0.6\angle 75°$ $Z_t = 0.3\angle 45°$ $Z_L = 30\angle 20°$

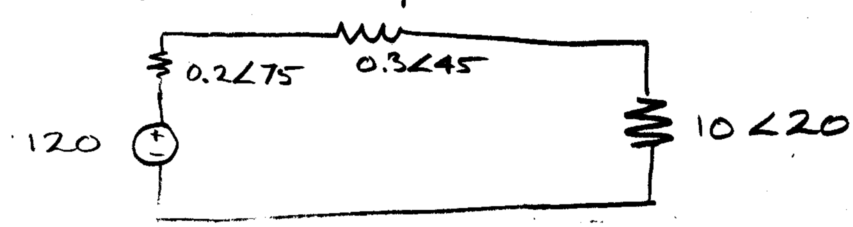
- Find I_a, VA, watts, VAR per phase, P.f.
- Value of shunt capacitor to bring pf at load to 1.

– Transform to Y-Y

$\frac{208}{\sqrt{3}} = 120$ $\quad \frac{Z_g}{3} = 0.2\angle 75°$ $\quad \frac{Z_L}{3} = 10\angle 20$



– Single line diagram



and this is identical to the problem on p. 8.3.10. The only remaining issue is the capacitor. In Y format, we found that C should be 90.2μF (p. 8.3.13). In Δ, the impedance should be 3 times higher, so

$C = 30.6\mu F$

THE END