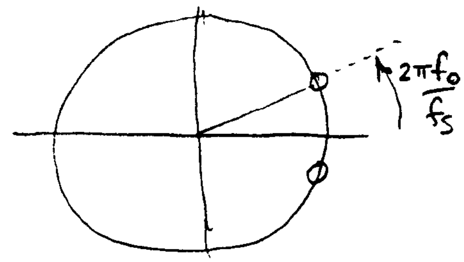
Example: Design an FIR filter with a null at a specified frequency $f_o := 0.2$ normalized by the sampling frequency - that is, a null at 0.2 f_s. The sampling frequency is also normalized with $f_s := 1$

We want zeroes on the unit circle at that sinusoid frequency. From Section 5.2 (using zeroes instead of poles), we construct H(z) as

$$\left(z - e^{j \cdot 2\cdot\pi\cdot\frac{f_o}{f_s}}\right)\cdot\left(z - e^{-j \cdot 2\cdot\pi\cdot\frac{f_o}{f_s}}\right) = z^2 - 2\cdot\cos\left(2\cdot\pi\cdot\frac{f_o}{f_s}\right)\cdot z + 1$$



This filter is not causal (z^2 and z terms mean its impulse response is nonzero for negative time) so delay it by 2 steps: multiply by z^{-2}. Then the z-transform, impulse response and Fourier transform (i.e., frequency response) are

$$H_Z(z) := 1 - 2\cdot\cos\left(2\cdot\pi\cdot\frac{f_o}{f_s}\right)\cdot z^{-1} + z^{-2} \qquad h := \begin{bmatrix} 1 \\ -2\cdot\cos\left(2\cdot\pi\cdot\frac{f_o}{f_s}\right) \\ 1 \end{bmatrix}$$

$$H_F(f) := H_Z\left(e^{j \cdot 2\cdot\pi\cdot\frac{f}{f_s}}\right) \qquad \text{since } z = e^{j \cdot 2\cdot\pi\cdot f\cdot t_s}$$

$$f := -2\cdot f_s, -1.99\cdot f_s .. 2\cdot f_s$$